Participating Insurance Contracts as a Risk Management Tool

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Abstract

The aim of this paper is to investigate optimal combinations of risk management mechanisms and pricing strategies in surplus participating insurance and their effects on insurance demand and policyholders’ benefits in the Solvency II regulatory environment. In general, participation rates and price loadings have an impact on insurers’ safety levels, the contracts’ payoffs and policyholders’ utility. To isolate these effects, we calibrate utility-maximizing consumers and their buying decisions with respect to the granted participation rate and the price loading. Under the condition that a monopolistic insurer, exposed to systematic mortality risk, retains a fixed solvency level, we study the sensitivity of insurance demand, the insurer’s optimal risk strategy and the resulting policyholders’ benefits. Our results are important for insurance company managers and regulators in that they provide a better understanding in the tangle of interactions between insurer’s strategies and consumer’s decisions in the participating insurance under a Solvency II regulatory framework.

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1. Introduction

Surplus participating contracts are well known life insurance products in many countries such as the United States, Japan and several countries in the European Union. In recent years, their specific characteristics have been of great interest for researchers as well as for practitioners. On the one hand, participating insurance contracts give policyholders indirect equity rights on earned profits according to certain bonus schemes. These bonus schemes include a guaranteed interest rate as well as additional embedded options for participation in the insurer’s investment, underwriting and operational surplus. From the insurer perspective, on the other hand, participating insurance products can play an essential role in the insurer’s risk management strategy: the surplus, which is granted additionally to the guarantee, can be used— together with the equity capital—as a safety net in the case of financial distress. Furthermore, an insurer’s pricing strategies and bonus schemes influence the equity capital required to ensure solvency as well as the insurer’s profits. In the literature, the different aspects of surplus participating insurance products and the different contractual features have been highlighted and thoroughly investigated. However, the overall relation between pricing strategy, risk management mechanisms and their impact on insurance demand and policyholders’ benefits was, to our knowledge, not discussed.

From a regulatory point of view, the participating life insurance contracts are also of great importance, especially under the newly developed European regulatory framework, Solvency II, which is to be implemented in 2016. The structure of the participating life insurance contracts with their promised but not guaranteed participation rate allows

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2 Grosen and Jørgensen (2000).
3 There are different regulations considering the minimum participation rate, depending on its origin. For example in Germany, 90 per cent of investment income, 75 per cent of underwriting profit and 50 per cent of other operating surplus must be allocated to policyholders. In France, 90 per cent of the technical result and 85 per cent of the financial result must be shared with policyholders. See Morgan (2010).
insurers to build up so-called “surplus funds”. These surplus funds can be used in case of financial distress as additional safety capital. According to the recent discussion on Solvency II and the conducted Quantitative Impact Study 4, insurers in many European countries report a significant level of surplus funds. In Sweden, for example, undertakings report that up to 99 per cent of their own funds consist of surplus funds, making participating insurance contracts a subject worth further investigating.

The purpose of this paper is to extend the previous literature and to provide insight into the comprehensive tangle of interactions between insurer’s strategies and consumers’ decisions in the context of participating insurance under a Solvency II regulatory framework. We do this by analysing the interaction between an insurer’s pricing policy and risk management strategy, considering undiversifiable mortality risk and changing insurance demand. We answer the question of whether an insurer can replace costly equity capital in its risk management strategy through an optimal trade-off between a higher price loading and a higher participation rate without eroding the insurance demand and its own profits. We identify three effects of the insurers’ decision that have an impact on its shareholder value: the cash flow effect, the demand effect and the risk distribution effect. Furthermore, we show that in its optimal shareholder-value-maximising decision, the insurer transfers all risk management costs for retaining a certain safety level to policyholders. However, policyholders’ utility does not necessarily decrease because of the ex-post allocation of a company’s profits.

The remainder of the paper is organised as follows. Section 2 provides a literature overview. Section 3 describes the theoretical model with the actors and their objective functions. In Section 4, we derive analytical solutions for the case without systematic risk.

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4 The surplus funds are reserves in which accumulated profits are stored until they are ultimately assigned and distributed to policyholders. They are classified as eligible to cover the risk-based capital requirements according to the Solvency II framework. (See Solvency II Framework Directive (2009/138/EC of the European Parliament and the Council of 25 Nov. 2009 on Solvency II [OJ L 335/1 of 17.12.2009], art. 90 and EIOPA (2013a), p.66, p. 291.

In Section 5, the results of the numerically calibrated model are presented. Section 6 discusses the results and relates them to the existing literature. In Section 7, we conclude and provide an outlook for further research.

2. Literature Review

The idea of participating contracts and the mutuality principle was first discussed in the scientific literature in the early 1960s by Borch (1962). These concepts were further developed by Wilson (1968). In the early 1990s, the literature studying the mutuality principle concentrated mainly on incentive problems, contractual design and organizational structure. For example, Smith and Stutzer (1990 and 1995) focus on the participating contractual agreements and show in theoretical models that they can reduce adverse selection and moral hazard problems. Further incentive conflicts are addressed in studies by Mayers and Smith (1981 and 2005). They argue that the mutual form of organization mitigates the owner-customer conflict, but also intensifies the owner-manager problem. Furthermore, Garven and Pottier (1995) show theoretically that participating insurance policies play an essential role in the resolution of the risk-shifting problem. Other authors deal with the mutuality principle in combination with undiversifiable risks. Doherty (1991) and Doherty and Dionne (1993) show theoretically that mutual forms in the sense of a contractual structure are Pareto-superior in the case of undiversifiable risks. Earlier empirical studies provide evidence that after “liability insurance crises”, different forms of mutuality were introduced as a response to aggregate and undiversifiable risks. Zanjani (2004) has also shown - based on the example of the Pennsylvania Fire Insurance Market from 1873 to 1909, that risk-sharing was an important motivation for mutuality in the past.

The systematic mortality risk in life insurance, as a special case of undiversifiable risk, is the risk that there is a change in the mortality rate of all policyholders. This is the

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6 Doherty and Dionne (1993) deliver similar results.
underwriting risk that cannot be diversified through the size of the underwriting portfolio applying the law of large numbers. The general literature on this topic mainly deals with modeling, valuation and hedging of systematic and unsystematic mortality risk. However, few articles study the systematic mortality risk in relation to mutual contractual agreement, such as the participating life insurance, to verify any risk-sharing effects between shareholders and policyholders.

Many contributions concentrate on the pricing and the valuation of different contract characteristics, such as guarantees, bonus schemes and embedded options. Different models with point-to-point guarantee (Briys and de Varenne, 1997; Barbarin and Devolder, 2005; Grosen and Jørgensen, 2002) or cliquet-style guarantee (Grosen and Jørgensen, 2000; Haberman et al., 2003; Kling et al., 2007; Bauer et al., 2006) are studied in financial and/or actuarial approaches. A detailed literature overview can be found in Jørgensen (2004). Further studies draw a comparison between different approaches and different product characteristics (Boyle and Hardy, 1997; Barbarin and Devolder, 2005; Gatzert and Kling, 2007; Zemp, 2011). All of the above-mentioned authors display a profound understanding of the valuation and the pricing of the participating insurance contracts and their product characteristics, but do not consider their effects on insurance demand and the policyholders’ benefits or discuss implications for risk management strategies and underwriting approaches of the providers.

The implementation of risk management strategies and underwriting approaches in participating insurance contracts is more closely studied by only a small number of authors. Gatzert (2008) investigates managerial asset allocation decisions and surplus distribution decisions and their impact on risk-neutral pricing and risk measurements for

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7 Here, we follow the definition by Dahl (2004) and Dahl and Møller (2006). However, a broader term could be aggregate or non-idiosyncratic risk. See Doherty and Dionne (1993).
8 The company guarantees a benefit payment and a participation in surplus only at the time of maturity.
9 In case of a cliquet-style guarantee, at the end of each period the guaranteed interest rate plus part of the insurer’s annual investment income, if it is greater than the guaranteed interest rate, is credited to policyholders’ accounts.
a fixed contractual market value and a fixed safety level. Kling et al. (2007) provide an insight into the interaction of contractual and regulatory parameters and managerial decisions and investigate their influence on the insurer’s default risk. They present parameter combinations that have a huge influence on the insurer’s risk exposure. In the field of the insurance demand for participating contracts, Eling and Kiesenbauer (2012) show empirically, using multivariate regression, that there is a positive dependence between the surplus participation rate and the new business growth in the German life insurance market. This indicates that consumers react to the level of surplus participation. The authors provide evidence consistent with the results of Tekülve (2007). Furthermore, Gatzert et al. (2012) analyse the effect of contractual specifications on policyholders’ welfare in a theoretical model. Looking at the policyholders’ willingness to pay, the authors identify combinations of contract specifications, which increase customer value without destroying contract value from the insurer perspective. All these studies provide an insight into the interaction of the contract specifications of the participating insurance from the managerial and the consumer perspective.

The aim of this paper is to extend the previous literature on participating insurance not only by analysing the comprehensive tangle of interactions between insurer’s strategies and consumers’ decisions, but also by studying the role of the participating insurance contract as a tool in insurer’s risk management strategy and pricing policy.

3. Model Design

In a parsimonious model without time and interest rate effects, we consider a perfectly transparent market with a single product and two stakeholder groups: insurance buyers and a monopolistic insurer. The insurance buyers, who are also the persons insured, are offered a term life insurance contract to protect their beneficiaries from loss of earnings
in case of policyholders’ death.\textsuperscript{10} The contract is sold at a fair premium\textsuperscript{11} plus loading, \(l\), and promises participation in the company’s risk profit with a terminal participation rate, \(\delta\).

\textbf{Insurance Buyers}

The insurance product is offered to a homogeneous group of \(N\) utility-maximising consumers with income, \(z\), initial wealth, \(W\), and identical risk exposure. Individuals buy the insurance product if their expected utility of being insured is higher than or equals the von Neumann-Morgenstern expected utility of being uninsured, given a specific form of their utility function \(U_i[a_i, w_i]\), where \(w_i\) denotes the wealth position and \(a_i\) is the risk aversion coefficient of the \(i^{th}\) individual, \(i \in [1; N]\).\textsuperscript{12} Assuming rational and fully informed consumers, their expected utility depends on the insurance payoff, \(S_i\), and the individuals’ mortality probability. We assume that the possibility of an individual dying follows a Bernoulli distribution with probability \(q\).\textsuperscript{13}

In the event of death, the beneficiaries have three possible outcomes. If the value of the insurer’s overall liabilities, \(L\), does not exceed the premiums gathered, \(P\), the beneficiaries receive the benefit payment, \(B\), plus a share of the insurer’s risk profit, \(PS\), with a participation rate, \(\delta\). If the premiums are not sufficient to cover all liabilities, but the insurer is still solvent, i.e. when the value of the liabilities is lower than the value of the assets, \(A\), but greater than the gathered premiums, the beneficiaries receive only the benefit payment, \(B\). In case of the insolvency, where \(L > A\), the benefit payment is only

\textsuperscript{10} The protection of family members against the loss of future labor earnings is a common motive of single breadwinners for buying a term life insurance. See Lewis (1989). The policyholder is also the person insured here.

\textsuperscript{11} The fair premium is defined as the expected value of a single nominal loss.

\textsuperscript{12} Since the aim of a term life insurance is to secure family dependents, we assume that the so-called bequest function, i.e. the utility of bequest from the bequeather’s point of view, is identical to the utility function for consumption when the policyholder is alive. This is a special case of a positive linear transformation with a coefficient of 1. See Campbell (1980) and Economides (1982).

\textsuperscript{13} The individual mortality probability \(q\) could be a value from a mortality table.
partially covered (see Equation (1)).\(^\text{14}\) If the policyholder survives, he receives a share of the insurer’s risk profit, if there is any (see Equation (2)).\(^\text{15}\)

\[
S^\text{death}_i = \begin{cases} 
B + \delta \cdot PS, & \text{if } L < P \\
B, & \text{if } A > L > P \\
B \cdot A/L, & \text{if } L > A 
\end{cases} \tag{1}
\]

\[
S^{\text{no death}}_i = \begin{cases} 
\delta \cdot PS, & \text{if } L < P \\
0, & \text{otherwise} 
\end{cases} \tag{2}
\]

Depending on the expected utility of his wealth with insurance, \(EU_{LI_i}\), and without insurance, \(EU_{noLI_i}\), the function \(D_i[l, \delta]\) depicts then the buying decision of the \(i^{th}\) individual with respect to the loading and the participation rate.

\[
D_i[l, \delta] = \begin{cases} 
0, & \text{for } EU_{noLI_i}[l, \delta] > EU_{LI_i}[l, \delta] \\
1, & \text{for } EU_{noLI_i}[l, \delta] \leq EU_{LI_i}[l, \delta] 
\end{cases} \tag{3}
\]

Through their buying decisions, consumers generate an aggregate demand function, depending on the loading and the participation rate, \(y[l, \delta]\). The aggregate demand, i.e. number of contracts bought, is then:

\[
y[l, \delta] = \sum_{i=1}^{N} D_i \tag{4}
\]

In the next step, we measure the consumers’ welfare as the consumer surplus, \(CS[l, \delta]\), resulting from a given combination of \(l\) and \(\delta\). It is calculated as the sum of the maximum loading a consumer would pay for the insurance product for a given participation rate and the loading charged by the insurer over all consumers willing to buy the product.\(^\text{16}\)

\(^{14}\) See Ballotta et al. (2006).
\(^{15}\) This could be an ex-ante price reduction for renewing the policy contract after each period.
\(^{16}\) Einav et al. (2010), Schmalensee (1989), Spiegel and Spulber (1994) use similar approaches of representing the consumers’ welfare.
\[ \text{CS}[l, \delta] = \int_l^\infty y[l, \delta] \, dl \] (5)

**Insurer**

The limited liability insurer maximises its shareholder value (\(SHV\)) and makes decisions as to its underwriting strategy under regulatory constraints, i.e. it decides on the participation rate \(\delta\) and the loading \(l\) retaining a certain safety level. Its objective function is the expected value of its equity capital \(E[K_1]\) minus its initial capital endowment \(K_0\), under the constraint that the insurer’s default probability, \(dp\), remains constant. Hence, the probability that the value of the liabilities, \(L\), exceeds the value of the assets, \(A\), does not change, \(\Pr[L \geq A] = \overline{dp}\).\(^{17}\)

\[ SHV = E[K_1] - K_0 \] (6)

Owing to corporate taxation and acquisition expenses, equity endowment is assumed to imply up-front frictional costs which are modelled by a proportional capital charge \(\tau \geq 0\).\(^{18}\) In order to concentrate on the risk profit and its allocation between shareholders and policyholders, we exclude any investment and operational profits. Therefore, the insurer’s assets are assumed to be risk-free. Considering the insurer’s limited liability, the payoff to shareholders is given by the available assets \(A\) minus the claims \(L\), \(\max\{A - L; 0\}\), minus the profit to be shared with policyholders, \(\delta \cdot \max\{P - L; 0\}\) (see Equation (7)).

\[ K_1 = \max\{A - L; 0\} - \delta \cdot \max\{P - L; 0\} \]
\[ = A - (L - \max\{L - A; 0\}) - \delta \cdot \max\{P - L; 0\} \] (7)

\(^{17}\) This is in line with the Solvency II regulatory framework. In its core principle, Solvency II defines a maximum admissible default probability of 0.5 per cent.

\(^{18}\) This is a common approach in the literature; see Zanjani (2002), Froot (2007), Yow and Sherris (2008), and Ibragimov et al. (2010)
We define the single premium, \( p \), as the sum of the fair premium plus a premium mark-up. The fair premium is calculated as the expected value of a single nominal loss without taking the insurer’s default risk into account, i.e. \( B \cdot q \), where \( q \) denotes the individual mortality rate. The premium mark-up is defined as a percentage of the benefit payment, i.e. \( B \cdot l \), where \( l \) stands for the percentage loading. Hence, the premium income of all contracts sold is:

\[
P = y \cdot p \\
= y \cdot B(q + l)
\]  
(8)

The available assets are, then, the sum of the capital endowment, reduced by the frictional costs, \((1 - \tau)K_0\), plus the premium income, \( P \):

\[
A = (1 - \tau)K_0 + y \cdot B(q + l)
\]  
(9)

The insurer’s liabilities, \( L \), equal the individual benefit payment, \( B \), multiplied by the number of occurred insurance events, \( Q \). We define the realised portfolio mortality, \( q_p \), as the ratio between the number of occurred insurance events and the number of contracts, i.e. \( \frac{Q}{y} \). Therefore, the insurer’s nominal liabilities can be represented as follows:

\[
L = B \cdot Q = y \cdot B \cdot q_p
\]  
(10)

Combining the equations above, the expected value of shareholders’ equity equals:

\[
E[K_1] = (1 - \tau)K_0 + y \cdot B(q + l) - E[y \cdot B \cdot q_p] + E[\max(L - A; 0)] - \delta E[\max(P - L; 0)]
\]  
(0.11)

Since the loading, \( l \), and the participation rate, \( \delta \), are the only decision variables and risk management tools, the level of the initial capital endowment, \( K_0 \), is directly related to
these figures under the constraint of a fixed default probability, \(\overline{dp}\).\(^{19}\) Therefore, \(K_0[l, \delta, \overline{dp}]\) corresponds to the required equity capital endowment above the available premiums for a given price loading, participation rate and default probability. It is calculated as the \((1-\overline{dp})\)-quantile of the liability distribution, \(VaR_{(1-\overline{dp})}\), minus the premiums and adjusted for the frictional costs:

\[
K_0[l, \delta, \overline{dp}] = \frac{VaR_{(1-\overline{dp})} - P}{1 - \tau}
\]  
(12)

Representing \(e\) as required equity capital per insured unit, i.e. \(e[l, \delta, \overline{dp}] = \frac{K_0[l, \delta, \overline{dp}]}{y_B}\), and inserting \((0.11)\) into \((6)\), we can rewrite the insurer’s objective function as:

\[
SHV[l, \delta] = y[l, \delta] \cdot B(l - \tau \cdot e[l, \delta, \overline{dp}] + dr[l, \delta] - \delta \cdot pr[l, \delta])
\]

(13)

Where

\[
dr[l, \delta] = \frac{E[\max\{L - A; 0]\]}{y_B} = E[\max\{q_p - (q + l + (1 - \tau)e); 0\}]
\]  
(13a)

\[
pr[l, \delta] = \frac{E[\max\{P - L; 0\}]}{y_B} = E[\max\{q + l - q_p; 0\}]
\]  
(13b)

\[
e[l, \delta, \overline{dp}] = \frac{K_0[l, \delta, \overline{dp}]}{y_B} = \frac{VaR_{(1-\overline{dp})} - (q + l)}{1 - \tau}
\]  
(13c)

dr[l, \delta] denotes the default rate, i.e. the ratio of the value of unpaid claims to the overall sum insured. \(pr[l, \delta]\) denotes the profit rate, i.e. the ratio between the value of the profit to be allocated to the policyholders and the overall sum insured. \(e[l, \delta, \overline{dp}]\) is the required equity capital necessary per insured unit, i.e. the ratio of the insurer’s equity capital to the

\(^{19}\) For sake of simplicity, we assume that the default probability and hence the capital endowment are not decision variables for the insurer. The regulatory admissible default probability is also the fixed default probability, which the insurer retains.
overall sum insured. The insurer’s decision problem is then to find the optimal combination of a loading \( l \) and a participation rate \( \delta \), such that its shareholder value attains its maximum and its default probability remains unchanged.

**Modelling Mortality Risk**

We distinguish between systematic mortality risk, i.e. the risk of deviations of the mortality rate for all policyholders from the projected level, and unsystematic mortality risk, which is the risk of incidental deviations of the portfolio’s mortality rate from its expected value.\(^{20}\)

The unsystematic risk is modelled through a Bernoulli distribution for the individual contracts. For large portfolios without systematic risk, the number of occurred insurance events, \( Q \), is then approximately normally distributed, \( Q \sim N(\mu_Q, \sigma_Q^2) \), where \( \mu_Q = yq \) and \( \sigma_Q^2 = yq(1-q) \). The portfolio mortality rate, \( q_p \), is approximately normally distributed with \( \mu_p = q \) and \( \sigma_p^2 = \frac{q(1-q)}{y} \).

To account for systematic risk resulting from uncertainty about the individual mortality probability, we assume it to be either high or low, \( q = [q^h; q^l] \), with a probability of realization of \( \pi^h \) and \( \pi^l = 1 - \pi^h \) respectively. In this case, the aggregated portfolio mortality no longer follows a normal distribution, but results in a twin peaked distribution.

Since the mortality risk distribution is not normal under the systematic risk assumption, it is not possible to derive analytical solutions regarding the insurance demand and the consumer surplus effects resulting from shareholder value optimization. Therefore, we will derive our analytical solutions for the case without systematic mortality risk (see Section 4). Afterwards, we will provide an insight into the possible consequences for

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\(^{20}\) Similar to Dahl (2004) and Olivieri (2001).
policyholders under systematic mortality risk by means of a numerical simulation (see Section 5).

4. Insurer’s Optimal Decision in the Absence of Systematic Risk

When there is no systematic risk, i.e. there is no risk of changes in the expected individual mortality probability, insurer’s liabilities are normally distributed. This allows us to find closed-form solutions for determining the default rate, \( dr[l, \delta] \), and the profit rate, \( pr[l, \delta] \). Let \( \Phi[\ ] \) and \( \varphi[\ ] \) denote the cumulative distribution function and, respectively, the probability density function of the standard normal distribution. We can then rewrite the insurer’s objective function as follows:

\[
max_{l, \delta} SHV[l, \delta] = yB(l - \tau \cdot e + dr - \delta \cdot pr)
\]

where

\[
dr[l, \delta] = \sigma_p \varphi \left[ \Phi^{-1}[1 - \bar{d}p] \right] - \bar{d}p \cdot \sigma_p \Phi^{-1} [1 - \bar{d}p]
\]

\[
pr[l, \delta] = l \Phi \left[ \frac{l}{\sigma_p} \right] + \sigma_p \varphi \left[ \frac{l}{\sigma_p} \right]
\]

\[
e[l, \delta, \bar{d}p] = \frac{\Phi^{-1} [1 - \bar{d}p]}{1 - \tau} \sigma_p - l
\]

Optimal Loading

For a given participation rate, the first-order condition implies that the optimal loading can be written as:

\[
l^*[\delta] = argmax_l SHV[l, \delta]
\]
\[ = \delta \cdot pr - dr + \tau \cdot e + \frac{y}{-y_t} + \frac{y}{-y_t} \left( \frac{\partial dr}{\partial l} - \frac{\delta}{\partial l} - \frac{\partial pr}{\partial l} - \frac{\tau}{\partial l} \right) \]

Equation (15) points out that the optimal loading includes five components. The first two represent a kind of a fair valuation of the cash flow to policyholders beyond the benefit payment, \( B \), which consists of the value of the profit rate of each contract claims and is adjusted for insurer’s default risk. Under a fixed participation rate, it is easy to verify that the optimal loading is positively related to the value of the profit to be shared and negatively related to the default rate. This consistency holds independent of the demand price elasticity. Furthermore, the loading in both cases is negatively related to the default rate. This result is in line with empirical studies analysing the relation between insurer’s default risk and insurance prices.\(^{23}\) The third component addresses the direct transfer of risk management costs, \( \tau e \), to policyholders. The fourth term is a non-negative profit mark-up, depending on the price sensitivity of the demand function. Unless demand is perfectly price elastic, the profit mark-up will be positive. The fifth term captures two effects of the loading choice on the value of the default rate, the profit rate and the necessary equity capital. The first effect is the direct impact of loading changes on the value of the required equity capital and profit rate through the change of the available surplus funds. Since a positive (negative) change of the loading increases (reduces) the available funds above the expected losses, less (more) equity capital would be necessary to ensure the predefined solvency level, and the value of the profit to be allocated to policyholders increases (decreases). The second effect, subsequently referred to as the risk distribution effect, is a result of the impact of the loading on the aggregate demand and hence on the risk distribution. A variation of the loading changes the demand and, therefore, the size of the portfolio. The modified size itself changes the portfolio risk and the characteristics of the risk distribution.

**Optimal Participation Rate**

The optimization of the participation rate itself is a trade-off. On the one hand, a higher participation rate means lower retained profits for the insurer, and hence lower shareholder value; on the other hand, it makes the insurance product more attractive and enhances the demand, and therefore the overall premium income increases. Additionally, influencing the demand, a change in the participation rate has indirect impact on the risk distribution, hence having an impact on the default and the profit rate (compare Equation (14)).

Let $\delta^*[l^*] = \arg\max_{\delta} SHV[l, \delta]$ denote the insurer’s $SHV$-maximising participation rate. Solving the insurer’s maximization problem (Equation (14)), the first-order condition for the optimal choice of $\delta^*[l^*]$, given that the loading is optimally adjusted, implies that

$$
\frac{y}{-y_l} \left(1 + \frac{\partial dr}{\partial l} - \frac{\tau de}{\partial l} - \delta^* \frac{\partial pr}{\partial l} \right) = \frac{y}{y_\delta} \left( pr - \frac{\partial dr}{\partial \delta} + \frac{\tau de}{\partial \delta} + \delta^* \frac{\partial pr}{\partial \delta} \right)
$$  \hspace{1cm} (16)

The left-hand side of Equation (16) represents the marginal change of insurer’s benefits due to demand reaction and change of the risk distribution when the loading is marginally changed. The right-hand side measures the marginal change of the shareholder value due to marginal changes of the participation rate. In its optimum, the marginal change of the shareholder value due to loading changes equals the marginal change due to participation rate changes.

We can conclude at this point that, in its optimization effort, the insurer considers three opposite impacts on its shareholder value - a cash flow effect, a demand effect and a risk distribution effect - when choosing its optimal combination of loading and participation rate. The cash flow effect captures the direct impact on shareholders’ cash inflow and outflow, such as premium income and retained profits. The demand effect describes the
impact of consumers’ reaction and buying decisions on the $SHV$ level, i.e. a higher demand is related to larger profits. The last effect, the risk distribution effect, is related to the indirect impact of demand changes. The fluctuation of the portfolio size has an impact on the risk, and hence on the standard deviation of the risk distribution. A change of the risk distribution has an influence on the valuation of the default rate, participation rate and the required equity capital. Table 1 summarises these effects for changes of the loading and the participation rate and shows whether it is a positive or negative relation. The combined result of these three effects and their interaction determines the overall change of the shareholder value due to changes of the input parameters.

<table>
<thead>
<tr>
<th>Impact on $\Delta SHV$</th>
<th>Insurer’s Reaction $\Delta l$</th>
<th>$\Delta \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow effect</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Demand effect</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Risk distribution effect</td>
<td>- / +</td>
<td>- / +</td>
</tr>
</tbody>
</table>

+: Positive relation: value increases if parameter increases  
–: Negative relation: value decreases if parameter increases

Table 1: Impact of loading and participation rate changes on the shareholder value.

From the insurer perspective, these results suggest that in its decision problem, the insurer should carefully take consumers’ reaction into account, since the consumers’ willingness to buy drives the insurer’s underwriting profits. The demand effect, direct or indirect, might be so strong that it can outweigh the cash flow effect. Therefore, for the determination of its optimal strategy, the insurer should have an intimate understanding of its existing and future insurance portfolio.

To analyse the participating insurance contract from the policyholder perspective, a calculation of the corresponding consumer surplus for the $SHV$-optimal combination of loading and participation rate is necessary, i.e. $CS[l^*, \delta^*] = \int_{l^*}^{\infty} y[l^*, \delta^*] \, dl$. However, this is only possible if the form of the aggregate function is known. Assuming $N$ consumers with different risk attitude, we cannot derive the demand function (Equation (4))
analytically. Since a derivation of further closed-form solutions is not possible, we conduct a numerical calibration of the theoretical model to understand the interaction of insurer’s decisions and policyholders’ benefits. In addition, we analyse the SHV-optimal combination of participation rate and loading under systematic mortality risk.

5. Insurer’s Optimal Decision in the Presence of Systematic Risk

Starting with the initial theoretical model, we adopt standardised parameters for the individuals’ income, the individual’s initial wealth and the benefit payments. We assume a homogeneous group of 3000 consumers on the market in order to be able to minimise the unsystematic risk for higher demand. The mortality rate, \( q^l = 1.4\% \), is taken from the German mortality table DAV 2008T and corresponds to the second order mortality rate of a 65-year-old man.\(^{24}\) Since we concentrate only on the mortality risk, we exclude the longevity risk and simulate only an increase of the mortality rate, which represents an inconvenient movement for term life contracts. The parameter used for the higher mortality rate, \( q^h \), is 2 per cent, i.e. a 42 per cent increase. This number corresponds to the second order mortality rate of a 68-year-old man. We further assume a frictional cost ratio of \( \tau = 5\% \).\(^{25}\) Furthermore, we assume risk-averse consumers and utilise a specific utility function of the form:

\[
U_i(w) = \begin{cases} 
  w^{1-a_i} & , 0 < a_i < 1 \\
  \ln(w) & , a_i = 1 
\end{cases}
\]

(17)

where \( w \) denotes individuals’ wealth position and \( a_i \) is the risk-aversion parameter. The implemented utility function exhibits a constant relative risk aversion (CRRA).\(^{26}\) The

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\(^{24}\) The second order mortality rate constitutes the experience basis and is free of any extra safety loadings. For derivation of mortality tables, see DAV (2009).

\(^{25}\) Zanjani (2002) reports that the frictional costs for the reinsurance industry can be approximated with 5 per cent

\(^{26}\) Eisenführ et al. (2010), pp. 256-258.
CRRA utility functions are often applied in theoretical studies for derivation of insurance demand functions.\textsuperscript{27} Table 2 summarises the model parameterization.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Notation</th>
<th>Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual’s income</td>
<td>$z$</td>
<td>1</td>
</tr>
<tr>
<td>Benefits</td>
<td>$B$</td>
<td>1</td>
</tr>
<tr>
<td>Individual’s initial wealth</td>
<td>$W$</td>
<td>1</td>
</tr>
<tr>
<td>Consumer population</td>
<td>$N$</td>
<td>3000</td>
</tr>
<tr>
<td>Mortality rate (low)</td>
<td>$q^l$</td>
<td>0.014</td>
</tr>
<tr>
<td>Mortality rate (high)</td>
<td>$q^h$</td>
<td>0.02</td>
</tr>
<tr>
<td>Probability of realisation of mortality rate (low or high)</td>
<td>$\pi^h = \pi^l$</td>
<td>0.5</td>
</tr>
<tr>
<td>Risk aversion parameter</td>
<td>$a_i$</td>
<td>Uniformly distributed [0;1]</td>
</tr>
<tr>
<td>Loading</td>
<td>$l$</td>
<td>0-0.2</td>
</tr>
<tr>
<td>Participation rate</td>
<td>$\delta$</td>
<td>0-100%</td>
</tr>
<tr>
<td>Frictional costs</td>
<td>$\tau$</td>
<td>5%</td>
</tr>
<tr>
<td>Default probability</td>
<td>$\bar{d}\tilde{p}$</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

Table 2: Summary of model parameters.

**Demand Function**

The identification of the demand function and its specifications is the first step in the $SHV$-optimization effort of the insurer, since it is a driving force for its profits. The knowledge and understanding of consumer reaction gives the insurer the possibility to find its optimal $SHV$-maximising strategy.

\textsuperscript{27} For the demand for life insurance, see Campbell (1980) and Economides (1982).
Figure 1: Demand for participating insurance contracts

Therefore, using the simulation tool @Risk, we calibrate the choice preferences of 3000 consumers and generate the resulting aggregate demand function. Figure 1 illustrates that the number of contracts sold is positively related to the participation rate and negatively related to the loading.

Furthermore, the aggregate demand function is imperfectly elastic with regard to the loading and the participation rate, i.e. small changes of the loading and the participation rate do not drop the demand to zero. The reason for this is the monopoly position of the insurer that we assumed at the beginning. For higher participation rates, the demand also reacts extremely slowly to loading changes. This could well be explained by studying how the price elasticity of the aggregate demand function changes for different participation rate levels.

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28 The literature also provides further explanations as to why the demand does not immediately drop to zero in non-monopolistic markets, such as switching and information costs. See D’Arcy and Doherty (1990), Cummins and Danzon (1997), Zanjani (2002), and Yow and Sherris (2008).
Table 3: Price elasticity for an extract from the data

For the sake of clarity, Table 3 shows an extract of the data to illustrate the change of the price elasticity with respect to the participation rate. We observe that the price elasticity decreases when the participation rate increases. The demand function thus becomes flat and consumers react less sensitively to changes of the price loading. The explanation for this is that for greater participation rates, policyholders receive a greater part of the loading paid in advance. The ex-post premium reduction thus increases and therefore makes the insurance product more affordable. This result is in line with the literature studying demand price sensitivity, according to which different characteristics, such as brand effects, switching cost, product features as well as individual characteristics, influence consumers’ sensitivity to price.\(^\text{29}\)

**Optimal Combinations of Loading and Participation Rate**

The next step in the \(SHV\)-optimization process is to identify feasible combinations of loading and participation rate. We define combinations as “feasible” if they lead to a non-negative shareholder value. Therefore, combinations of very low loading and high participation rate,\(^\text{30}\) resulting in negative insurer cash flow in every state of the world, are excluded. A special case is the one in which the participation rate is 1 and all profits are

\(^{29}\) See Strombom et al. (2002), Schut et al. (2003), Erdem et al. (2002).

\(^{30}\) For such combinations, the positive value of the default put option does not compensate for the frictional costs for holding the required capital.
allocated to policyholders. The insurer would bear the entire underwriting risk, but in the case of an advantageous development it transfers all positive cash flows to policyholders, which would be an arbitrage opportunity for policyholders. Additionally, we identify combinations of loading and participation rate for which the consumers are unwilling to buy the product, i.e. the demand decreases to zero, and therefore the $SHV$ is also zero. The resulting shareholder value for the feasible sets of loading and participation rate are illustrated in Figure 2.

![Figure 2: Shareholder value for different combinations of loading and participation rate](image)

For every participation rate, there is only one optimal loading choice, which maximises the shareholder value. All $SHV$-maximising combinations of participation rate and loading are shown in Figure 3. The curve in Figure 3 is a graphical representation of Equation (15), illustrating the $SHV$-maximising loading for a given participation rate. We can easily observe that the $SHV$-maximising loading increases with the increase of the participation rate. Moreover, for higher participation rate levels (higher than 80 per cent), a small change of the participation rate needs an extreme adjustment of the loading in order to attain the maximum possible $SHV$ level for the given participation rate. The reason for this is that for higher participation rates only a very small part of the profit is
distributed to shareholders. Hence, the insurer can increase the participation rate but should compensate for its decreased retained profits with a higher cash inflow through the loading.

![Figure 3: SHV-maximising combinations of loading and participation rate](image)

At this point, we can conclude that expensive insurance contracts are SHV-maximising under consideration of consumers’ reaction. However, the insurer gives a greater part of the gathered loading back to the policyholders and retains only a small part, except for a participation rate of 1. This is the reason why the SHV maximum level increases only moderately compared to the loading increase (see Figure 3). As shown in Figure 4, the maximum achievable shareholder value follows an upward trend up to a participation rate of 0.99, when the global maximum is attained for an extremely high loading. For higher participation rates, the shareholder value decreases abruptly to zero, since no profits are allocated to shareholders.
Figure 4: Maximum achievable $SHV$ for given participation rate and optimally adjusted loading

**Required Equity Capital**

Since we are interested in the question of whether the participating insurance contract can serve as a risk management tool, it is essential for our analysis to study the development of the equity capital required to ensure a certain solvency level with respect to the decision parameters. For a given participation rate, we observe an asymmetric U-shaped development of the required equity capital with respect to the loading, as Figure 5 illustrates. With an increase of the loading, the required equity capital first decreases, since the loading serves as a safety net. For a constant participation rate, however, the demand starts to decrease as a result of the increasing price. Hence, the risk in the portfolio increases. As a result the required equity capital starts to grow with the further increase of the loading because the risk distribution effect outweighs the cash flow effect (compare Equation (13). The maximum $SHV$ for a certain participation rate is attained at lower equity levels before the overall and risk distribution effect becomes stronger than the cash flow effect (see the marks in Figure 5).
These results are transferable for the situation in which the participation rate changes. If the participation rate for a given loading increases, the required equity capital decreases, since the demand increases and the risk in the portfolio becomes lower. Therefore, less capital is necessary in order to ensure the predefined solvency level. Compare for this effect the dotted curve for a participation rate of 0.3 and the gray curve for a participation rate of 0.

In a next step, we look at the required equity capital with respect to the \( SHV \)-optimal combinations of loading and participation rate. As shown in Figure 6, under the calibrated parameters, the required equity capital is a decreasing function of the \( SHV \)-maximising combinations of loading and participation rate. Furthermore, for \( SHV \)-optimal combinations of price loading higher than 0.0135 and a corresponding adjusted participation rate higher than 0.65 no additional equity capital is required to insure the 99.5 per cent solvency level. With the further increase of the loading, the solvency level even improves and the default probability starts to decrease. At some point, the loading is so high that any excess losses over their expected value can be covered and even the undiversifiable systematic mortality risk can be financed through the loading.
From the policyholders’ point of view, an equity capital of zero means avoidance of risk management costs, which are normally transferred to them by the insurer. The policyholders endow the insurer with capital in return for which they participate partially in the company’s profits. From the insurer perspective, an equity capital of zero results in arbitrage effects. These results, however, are driven by the model assumption. For further discussion on the model assumptions, see the discussion in Section 6.

**Consumer Surplus**

To study the implications of the insurer’s optimization for policyholders, we examined the development of the consumer benefits, as depicted in Figure 7. We observe a constant increase in consumer surplus when the participation rate increases and the loading is optimally adjusted. Moreover, the boost in consumer surplus is even higher for participation rate levels higher than 80 per cent, despite the fact that the adjusted loading also increases exponentially (compare Figure 3).
Figure 7: Consumer surplus for given participation rate and optimally adjusted loading

This increase indicates that the benefit through an increase in participation rate cannot be outweighed by the negative effect of the increased loading. This fact is explained by the change in the price elasticity for higher participation rates (compare Table 3). There are thus consumers willing to pay extremely high loadings over the fair premium when they receive them back as an ex-post price reduction through the profit participation. The maximum consumer surplus is attained once again at the maximum possible participation rate of 0.99.

The analysis of the consumer surplus iso-curves provides additional information on how the consumer surplus and the optimal combination of loading and participation rate are related. Figure 8 depicts three consumer surplus iso-curves (black curves) and the $SHV$-maximising combinations of loading and participation rate (grey curve). We can see that every $SHV$-optimal combination dominates every point to its left above the curve and leads to a higher consumer surplus. This can be analytically explained with the slope of the iso-curves which is higher than the slope of the curve depicting the $SHV$-optimal combinations of loading and participation rate.
To sum up, our findings provide an insight into different aspects of the surplus participating insurance and the complex interactions between insurance demand, management decisions and policyholders’ benefits. Starting with the demand for surplus participating insurance products, we can conclude that consumers react differently to the participation rate and to the price loading. The participation rate itself has an impact on consumers’ sensitivity to the loading, making individuals less sensitive to price when they participate more strongly in the company’s profits. Next, we can conclude that a SHV-maximising insurer will also choose consumer-surplus-maximising combinations of loading and participation rate, although both figures will be very high. From the insurer perspective, it is profitable to offer the product at an almost 100 per cent participation rate, since a high participation rate also justifies a high loading, which in combination does not lower the demand sharply and the insurer adds up extra value. Additionally, the insurer does not need to hold extra equity capital to cover its capital requirements for high loadings, hence it does not have extra risk management costs. In this case, the insurer transfers all risk management costs to policyholders. Furthermore, we found that an
insurer’s pricing strategy and decisions have major influence on the risk characteristics of the liability portfolio.

6. Discussion

The aim of this section is to relate our findings to existing studies and to provide a discussion on possible extensions of the model presented.

Only a small number of studies analyse the surplus participating insurance with respect to underwriting and risk management strategies taking policyholders’ reaction into account. Gatzert et al. (2012) make a step in this direction, analysing consumer value for participating life insurance contracts. They show by examples how the consumer value can be increased under a fair contract valuation by varying the contract design consisting of three parameters: guaranteed interest rate, along with annual and terminal surplus participation rate. According to Gatzert et al. (2012), consumers prefer simple contracts consisting only of one of these parameters. We extend this idea by taking behavioural aspects of consumers with different risk appetite into consideration. By modelling a population of consumers with variable risk aversion, we are able to generate an aggregate demand function over this population and to analyse its movement with respect to the calibrated contract parameters, price loading and participation rate. Bohnert and Gatzert (2012) have shown in a recent article that variations of the surplus allocation scheme might have a substantial influence on the insurer’s risk situation and the policyholders’ benefits. Therefore, another possibility to extend our model would be to include different surplus distribution rules, including point-to-point guarantee or cliquet-style guarantee, and to compare consumers’ reaction and insurer’s decisions under these variations.

31 The company guarantees a benefit payment and a participation in surplus only at the time of maturity. See Briys and de Varenne (1997), Barbarin and Devolder (2005), Grosen and Jørgensen (2002).
32 In the case of a cliquet-style guarantee, at the end of each period the guaranteed interest rate plus part of the insurer’s annual investment income, if it is greater than the guaranteed interest rate, is credited to
Let us look at the analysed insurance product from the consumer perspective. Our results show that if the loading is high and the participation rate is adjusted respectively, consumers continue to buy the surplus participating insurance. Therefore, the demand does not drop sharply and the risk in the portfolio does not change considerably. The adjusted participation rate representing an ex-post reimbursement is high enough even to increase policyholders’ utility. However, in our model we do not consider timing effects and opportunity costs that may influence consumers’ decisions. Considering an alternative investment opportunity or a different insurance product, for example, policyholders might choose to invest their initial wealth in this alternative investment instead of buying an expensive insurance product with a high participation rate. Furthermore, varying the form of consumers’ utility function would also influence the aggregate demand. Therefore, a possible extension of this work would be to test different utility functions.

If we look at the results and the model from a policymaker perspective, it would be realistic to discuss critically the monopoly position of the insurer analysed. The investigation of the interactions between insurer’s decisions, insurance demand and policyholders’ implications would be much more realistic if we were to loosen the monopoly restriction. However, the insurance market is not always perfectly competitive and exhibits monopolistic characteristics due to side effects such as switching and information costs. Therefore, we find it reasonable to analyse the insurer’s optimization problem and its implications from a monopoly position in order to accentuate the resulting effects.

In order to represent the interaction between insurer’s price and risk management decisions and insurance demand for participating insurance contracts in an understandable way, we concentrate, in our model, on a simplified version of a term life

policyholders’ accounts. See Grosen and Jørgensen (2000), Haberman et al. (2003), Kling et al. (2007), Bauer et al. (2006)
insurance. Hence, we focus on the mortality risk. The unsystematic mortality risk might be very well predicted and diminished through large portfolios using the law of large numbers. Since we simulate a population of 3000 customers, the unsystematic risk becomes almost negligible. The systematic risk itself can be diminished only to some extent through a large portfolio, but it cannot be fully eliminated. Therefore, our simulation results are mainly driven by the systematic risk. Though the results and any changes analysed are very low in absolute values, our findings depict the main effects of insurer’s decisions on the insurance demand and the policyholder benefits. We show that in the optimum, the insurer has gathered enough capital from the policyholders to finance any adverse development of the systemic risk. A study of different risks and loss distributions would emphasise these findings and provide even further insights into the issue analysed.

There are several limitations inherent to the design of this study. We firstly assume perfect market transparency, i.e. consumers know about the default risk in the liability portfolio and anticipate the profit of the insurer. Secondly, policyholders are aware of the participation rate when making their buying decision. However, there might be an uncertainty about the promised participation rate during the decision-making process. Being unable to anticipate the real participation rate during the buying process, consumers might exhibit varying willingness to buy and hence influence the aggregate demand. Thirdly, the modelling and the analysis in this article are based on a single type of insurance product offered to a homogeneous group of consumers, as can be found in other articles (Gatzert, 2008; Grosen and Jørgensen, 2002; Ballota et al., 2006). Since the insurer’s required equity capital and the value of its default rate and its participation rate depend on the risk in the liability portfolio, it might be interesting to include diversification effects by allowing for heterogeneous policyholders’ groups or combinations of participating contracts with different features. Fourthly, except for the regulation of insurer’s solvency level, we do not consider any regulatory restrictions that
are to be considered in an insurer’s optimization problem. In reality, regulators might put restrictions on the minimum participation rate. In other insurance lines, we also observe price restrictions limiting the maximum admissible price for a given product. Similar restrictions might limit insurer’s admissible decision alternatives.

7. Conclusion

This article investigates the interaction between risk management and price strategies in surplus participating insurance. We study an insurer’s optimal decisions and their impact on insurance demand and policyholders’ benefit. To focus on these effects, we employ a shareholder value optimization model and calibrate utility-maximising consumers and their buying decisions with respect to specific contract parameters.

To this end, we employ a model in which an insurer maximising its shareholder value decides on the optimal combination of price loading and participation rate. In its calculation, the insurer, exposed to systematic mortality risk, considers a regulatory restriction such as a certain solvency level and takes consumers’ reaction into account. The consumers’ willingness-to-buy depends on their utility of the offered insurance contract and its features, i.e. consumers react to the participation rate and the price loading. Analytically, we derive the components that have direct and indirect impact on the insurer’s decisions and discuss their effect on the shareholder value maximization. To evaluate the consumer’s effects, we conduct a numerical calibration of a consumer group with a utility function exhibiting constant relative risk aversion. The consumer group is homogeneous with respect to their initial wealth and risk exposure, but has a range of different risk attitudes.

Our findings show that the interactions between insurer’s decisions, insurance demand and policyholders’ benefits are manifold. We identify three effects of an insurers’

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33 This is the case in Germany.
decision that have an impact on its shareholder value: a cash flow effect, a demand effect and a risk distribution effect. The cash flow effect results from the direct cash inflow from the loading and cash outflow from the profits allocated to policyholders. Whereas the demand effect depicts consumers’ reaction to the loading and the participation rate, the risk distribution effect catches the indirect effect of changes in demand on the risk distribution and therefore on the valuation of the required additional equity capital, the default rate and the participation rate. Our results also show that the insurer finds it optimal to increase the loading and the participation rate almost to the maximum and therefore to transfer all risk management cost to policyholders. Under the model assumptions this is also optimal for policyholders. However, the results are mainly driven by the construction of the model and the assumptions made. Our contribution makes it clear that more research is necessary to identify the detailed effects of different bonus schemes and participation rules on consumers’ decision-making, insurance demand and policyholders’ benefits.

The results of this study are important for insurance companies and have practical implications for insurers’ managers and their pricing policy, but also for product development. The participating contracts represent a product construct that might be also used in other business lines in order to share risk with policyholders. It may also serve as a mechanism to impose discipline on policyholders and hence to mitigate moral hazard problems in insurance lines exposed to information asymmetry. Our findings are also important for policymakers and regulators in that they provide a better understanding of the market for participating insurance products and show situations in which a regulator should intervene in order to fulfil any regulatory target, e.g. product affordability, or to eliminate unwanted effects, e.g. arbitrage.
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