Adverse Selection in Annuity Markets:
Evidence from the British Life Annuity Act of 1808

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July 8, 2007

Abstract

We look for evidence of adverse selection using data from an 1808 Act of British Parliament which effectively opened up a market for life annuities. We develop statistical techniques for analyzing the awkwardly condensed data, and our analysis indicates significant levels of adverse selection. The evidence for adverse selection is particularly strong among a sub-sample of annuitants whose annuities were purchased by pecuniarily-minded speculators. Among the remaining (self-nominated) annuitants we find evidence that, in spite of adverse selection, the market was robust enough to avoid being destroyed by a “death spiral.” This indicates that there was scope for a well-functioning “thick” annuity market, suggesting that adverse selection alone may not explain the paucity of annuity sales in contemporary annuity markets.

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1 Introduction

In 1808, the British Parliament passed the Life Annuity Act, effectively opening a market for government-provided life annuities. The unique features of the Act – including surviving data on the mortality histories of the annuitants – provide an unusual opportunity to explore the empirical importance and consequences of adverse-selection.

Informational asymmetries in general, and adverse selection in particular, have played an important role in economic theory since the seminal works of Akerlof (1970), Spence (1973), and others. Studying the empirical importance of adverse selection has proved challenging, however, for a number reasons. First, as emphasized by Chiappori and Salanié (2000), it is difficult to empirically distinguish between moral hazard and adverse selection.

Second, informational asymmetries between market participants are likely to apply to the econometrician as well: information which is not known by an insurance provider, for example, is unlikely to be observed by the econometrician. Furthermore, econometricians may not even have access to all of the information symmetrically known by the market participants, and, as shown by Dionne et al.’s (2001) comment on Pueltz and Snow (1994), this can confound empirical tests for adverse selection.

Third, the empirical evidence for even the most robust consequences of informational asymmetries has been less than definitive. For example, Cawley and Phillipson (1999) find no evidence of selection in life insurance markets, and Chiappori and Salanie (2000) and Dionne et al. (2001) find no evidence of selection in auto insurance markets. Some economists have recently argued that this absence of evidence may be due to a fourth empirical challenge: the confounding effects of “advantageous” selection (DeMeza and Webb, 2001, Einav and Cohen, 2007 and Finkelstein and McGarry, 2006).

Annuity markets provide a particularly interesting setting to search for adverse selection, not least because these empirical challenges are less acute than in many insurance settings. Indeed, annuity markets are one of the few settings in which direct evidence of adverse selection has been documented (Finkelstein and Poterba, 2002 and 2004). Unfortunately – and in spite of a substantial body of theoretical research suggesting that annuities should play a central role in funding retirement – existing markets for voluntarily purchased private life annuities are most notable for their virtual non-existence. This thinness of annuity markets is known in the literature as the “annuity puzzle.” Since Finkelstein and Poterba’s evidence supporting adverse selec-
tion in annuity markets comes from a U.K. market where annuitization is compulsory, even this evidence must be viewed as limited in applicability.

This paper uses the 1808 Act to provide a novel empirical look at adverse selection. We demonstrate its empirical relevance and importance in the purely voluntary annuity market opened by this Act. Simultaneously, this paper informs the “annuity puzzle” literature by showing that adverse selection was not strong enough to induce the “death spiral” dynamics found by Cutler and Reber (1998) (but not by Buchmueller and DiNardo, 2002) in the health insurance context. Insofar as this finding indicates that markets for private annuities can, in practice, be “thick” and self-sustaining, it suggests that an adverse selection driven “lemons” problem – a lás Akerlof (1970) – cannot, in itself, explain the puzzling dearth of private annuity sales in modern markets.

We proceed as follows. Section 2 describes the annuity puzzle in greater detail and presents a simple illustrative theoretical model of annuity markets. After introducing and describing the 1808 Life Annuity Act in Section 3, we use this model to guide our test for adverse selection and death spiral dynamics. This analysis appears in Section 4. Section 5 offers some concluding remarks. A brief technical appendix describes the formal aspects of our empirical methodology in greater detail.

2 Modern Annuity Markets and the Annuity Puzzle

There is a substantial and growing literature on the so-called “annuity puzzle”: the observation that very few individuals voluntarily choose to annuitize their retirement assets in spite of the benefits economic theory suggests they should provide. On the theoretical side, Yaari (1965) showed that full annuitization should be optimal under specialized circumstances, and Davidoff et al. (2005) have recently shown that the theoretical prediction of significant annuitization by optimizing individuals is quite general. On the empirical side, in contrast, Poterba et al. (2003), Johnson et al. (2004) and others have documented a marked paucity of life annuity purchases by households in the Health and Retirement Survey. This paucity is corroborated by industry sales data which indicate a paltry $5.9 billion in immediate annuity sales in in the U.S. in 2006 (LIMRA, 2007). Nor is this paucity specific to
the U.S. (James and Vittas, 2004).

A number of papers have attempted to explain the annuity puzzle. Potential explanations include pre-existing (public or company pension) annuities, unfavorable pricing resulting from adverse selection or administrative loading (Mitchell et al., 1999), within-household risk pooling for married couples (Brown and Poterba, 2000), bequest motives (Friedman and Warshawsky, 1990), higher returns from alternative assets (Milevsky, 1998), the need for liquidity to cover health shock expenditures (Sinclair and Smetters, 2004), and the option value of delaying annuitization (Dushi and Webb, 2004). Whether some combination of these explanations can fully resolve the puzzle remains an open question.

The typical approach in this literature is to posit that households are life-cycle utility maximizers – typically with constant relative risk aversion, additively separable utility functions – and to compute the value such individuals would place on having access to private annuity markets. There is some empirical support for this approach: Brown (2000) shows that individuals’ stated intentions to annuitize are indeed correlated the theoretical value they would place on annuitization according to such life-cycle models. Nevertheless, a distinct weakness of this approach for addressing the annuity puzzle is that quantitative conclusions regarding the value and levels of annuitization rely heavily on functional form utility assumptions. In contrast, we will be able to explore one of the explanations for the annuity puzzle – adverse selection – with an empirical methodology free from functional form assumptions.

The thinness of private annuity markets calls to mind Akerlof’s (1970) seminal “lemons” paper, in which he showed that an adverse-selection induced “death spiral” can cause an otherwise useful market to fail to exist. Data limitations and the life-cycle functional form approach have thus far prevented research into the potential salience of such a death spiral for modern annuity markets. We will undertake an empirical explanation of this possibility using the 1808 Life Annuity Act.

Towards that end, we now present an abstract model of an annuity market. This purely illustrative model suggests three distinct possibilities: First, the market may exhibit no adverse selection whatsoever. This happens when firms can break even selling annuities to the entire population of potential annuitants. Second, the market may suffer from adverse selection but still remain “thick,” with firms earning normal profits selling annuities to all but a reasonably small number of the least healthy potential annuitants. Finally,
the market may be nearly or completely destroyed by an Akerlovian death spiral. This model and these three possibilities will serve as a guide to the empirical work we present in the subsequent section, where we will find that the market created by the 1808 Act appears to have been of the second variety. We take this as suggestive evidence that adverse selection, while relevant in annuity markets, is unlikely to be the only factor underlying the annuity puzzle.

2.1 A Stylized Model of Adverse Selection and Death Spirals in Annuity Markets

Consider a sequence of periods $t = 0, 1, 2, \ldots$. In period 0, a continuum of individuals retires. Each individual has unit wealth and has the option of exchanging her wealth for a life annuity with a constant per-period payout of $a$. In periods $t = 1, 2, \ldots$, individuals consume, if they are still alive. An individual who purchases an annuity with her wealth will thus have a per-period consumption of $a$ for as long as she lives. Individuals who do not purchase an annuity consume in later periods by saving at an interest rate $r$. We take $r = 0$, purely for simplicity. Also for simplicity, we assume that, conditional on reaching period $t$, an individual $i$ has a privately known probability $S_i$ of surviving to period $t + 1$. Individuals differ only in their survival probabilities, and we take the distribution of $S_i$ to be uniform on $[0, \bar{S}]$, where $\bar{S} < 1$.

Preferences over life contingent consumption streams $c_1, c_2, \ldots, c_t, \ldots$ are given by:

$$V(c_1, c_2, \ldots, c_t, \ldots) = \sum_{t=1}^{\infty} S_i^t \ln(c_t).$$

(1)

Individuals who choose to purchase an annuity thus receive utility

$$V_i^*(a) = \sum_{t=1}^{\infty} S_i^t \ln(a) = \frac{S_i \ln(a)}{1 - S_i}.$$  

(2)

Individuals who do not choose to purchase an annuity solve

$$V_i^A \equiv Max \sum_{t=1}^{\infty} S_i^t \ln(c_t)$$

(3)

s.t. $\sum_{t=1}^{\infty} c_t \leq 1$.  

4
Solving Equation 3 using first order methods yields $c_i^* = S_i^{t-1}(1 - S_i)$, and,

$$V_i^A = \frac{S_i}{1 - S_i} \left( \ln (1 - S_i) + \frac{S_i}{1 - S_i} \ln (S_i) \right).$$

Individuals offered annuities paying $a$ per period will thus choose to annuitize if

$$\ln(a) > \ln (1 - S_i) + \frac{S_i}{1 - S_i} \ln (S_i).$$

The right-hand side of Expression 4 is decreasing in $S_i$. Thus, for any given annuity payment $a < 1$, individuals with survival probabilities above the cutoff value $S^*(a)$ solving

$$\ln(a) = \ln (1 - S^*(a)) + \frac{S^*(a)}{1 - S^*(a)} \ln (S^*(a)),$$

will purchase an annuity, while those with survival probabilities below $S^*(a)$ will choose not to purchase annuities. (If $a > 1$, then the everyone wishes to buy and annuity and we take $S^*(a) = 0$.)

We assume that annuities are sold in a competitive market, so that firms must break even on annuity sales.\(^1\) Selling an annuity with payment $a$ to an individual $i$ yields expected profits $1 - \sum_{t=1}^{\infty} S_t^i a = 1 - \frac{S_i}{1 - S_i} a$. Equilibrium is thus characterized by a value $a$ such that $E \left[ \frac{S_i}{1 - S_i} a \middle| S_i > S^*(a) \right] = 1$.

To solve for equilibrium, we first determine the annuity payment $a^*(\tilde{S})$ that would allow a firm to break even for a given cutoff $\tilde{S}$:

$$a^*(\tilde{S}) = \left[ \int_{\tilde{S}}^{1} \frac{S_i}{1 - S_i} \frac{dS_i}{\tilde{S} - S} \right]^{-1} = \left[ \frac{1}{\tilde{S} - \tilde{S}} \ln \left( \frac{1 - \tilde{S}}{1 - \tilde{S}} \right) - 1 \right]^{-1}.\quad (6)$$

We can find the unique equilibrium using the following iterative technique. First, “guess” that $\tilde{S} = \tilde{S}_0 \equiv 0$, so that everybody annuitizes. Second, use Equation 6 to compute the payout rate $a_0 \equiv a^*(\tilde{S}_0)$ at which firms would break even, given that everyone annuitizes. Third, use Equation 5 to compute $\tilde{S}_1 = S^*(a_0)$ – the shorted-lived individual who would voluntarily buy annuities with this payout rate. If $\tilde{S}_1 = \tilde{S}_0 \equiv 0$, then firms can sell annuities to all individuals and still break even, and the market does not suffer from adverse selection problems. Otherwise, some short-lived individuals – those with $S_i < \tilde{S}_1$ – are driven from the market. In this case, “guess” that $\tilde{S} = \tilde{S}_1$ and repeat the steps by finding the break-even payout rate $a_1 \equiv$

\(^1\) Adding a loading factor does not change the qualitative results.
\(a^*(S_1)\) if only those individuals with \(S_i > \tilde{S}_1\) annuitize, and then the “cutoff” survival probability \(\tilde{S}_2 = S^*(a_1)\). Iterating this process to convergence yields an equilibrium pair \((a_\infty, \tilde{S}_\infty)\). The payout rate \(a_\infty\) allows firms to break even, given that annuities are sold to the set of individuals with \(S_i > \tilde{S}_\infty\) – precisely the set who would voluntarily purchase annuities offering the payout rate \(a_\infty\).

The equilibrium depends on the distribution of risk types – i.e., on \(S\). For low values of \(S\), the market does not suffer from adverse selection. For example, when \(S = .8\), the break-even payout rate for the whole population is \(a_0 = 1.36\), which makes annuities desirable for any rational individual.

For sufficiently high values of \(S\), the market is killed by a “death spiral.” For example, when \(S = .999\), the break-even payout rate for serving the whole market is \(a_0 \approx .17\). A payout rate this low makes annuitization undesirable for all but the longest-lived 37% of individuals \((\tilde{S}_1 \approx .63)\), so the first round of adverse selection “kills” about 63% of the market. To break even with the remaining 37%, firms must lower their payout rate to \(a_1 \approx .067\), which drives another 20% of the population out of the market \((\tilde{S}_2 \approx .83)\). The next round drives another 8% of the population out of the market \((\tilde{S}_3 \approx .91)\), and, in equilibrium, just over 1% of individuals end up buying annuities \((\tilde{S}_\infty \approx .99)\): adverse selection essentially destroys the market.

For intermediate values of \(S\) the market suffers from adverse selection, but the iterative process converges quickly and the market is still “thick” in equilibrium: there is no “death spiral.” When \(S = .85\), for example, the payout rate to serve the whole market is \(a_0 \approx .96\). This payout rate drives individuals with survival probabilities below \(\tilde{S}_1 \approx .0067\) from the market, forcing firms to raise prices by a very small amount, driving a few more individuals from the market, and so forth. But, in equilibrium, a full 99.2% of individuals buy annuities.

This model, while clearly overly simplistic, captures the three key possibilities: adverse selection could be completely absent from annuity markets; adverse selection could be present, but annuity markets could nevertheless function reasonably well; or adverse selection could essentially destroy the annuity market. We now turn to data from U.K. Life Annuity Act of 1808. We will see that the second case appears to have obtained in the annuity market created by this Act.
3 The 1808 Act as a Testing Ground

Prior to the Life Annuity Act of 1808, British government debt was in the form of Consols – coupon bonds with infinite maturity (perpetuities). The explicit goal of the Act was to convert their perpetual obligations to finite-lived ones by allowing the exchange of Consols for life annuities. Since Consols were tradable assets, this effectively opened up a market for government provided life-annuities. Several features of the Act and the market that resulted make it an ideal testing ground for adverse selection and the sustainability of annuity markets.

First, the Act was originally designed to provide annuities at actuarially fair rates. The (semi-annual) payments provided by the annuity received in exchange for a Consol was computed using the market price of Consols – hence, interest rates – and a life table specific to the age of the individual on whose life the annuity payments were contingent (henceforth the nominee). The life table used, known as the Northampton table, was thought to capture the population-average mortality. We can thus test for adverse selection by looking at whether the longevity of the nominees exceeded the population-average longevity predicted by this table. As we will see below, there is indeed striking evidence of this sort of adverse selection: the mortality rates of the nominees was significantly lower than that predicted by the Northampton tables.

The inapplicability of the Northampton tables was suspected from an

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2 There may well have been more subtle motivations leading to its passage. Spencer Perseval, addressing Parliament in 1808 (viz Hendricks, 1856), argued that the Act would allow the government to retire debt at favorable interest rates without causing interest rates to rise – an argument Murphy ridicules as indicating Persival’s desire to “have his cake and eat it too” (Murphy, 1939, page 6) but which is plausible if the government believed it could extract surplus by filling a missing market. Alternatively, there could have been a political desire to align the interests of the retired monied classes with the government by providing them an important service – as argued by Weir (1989) for the French government-issued Tontines of the 18th century.

3 The Northampton table had been well tested by a respected private institution known as The Equitable, which used it in pricing life insurance. Of course, precisely the opposite sort of adverse selection would be predicted to occur in the life insurance market. As such, the simple test we develop for adverse selection in annuity markets will actually be a joint test for adverse selection in at least one of the two insurance markets. Additionally the original pricing of the annuities included a 2% “load” relative to the actuarially fair prices, as computed using the Northampton tables. The Northampton table is published in Baily (1813).
early stage, but it was not until 1823 that the government took active steps to address it. At this point, Parliament commissioned John Finlaison to study the mortality experience of the nominees of this and several earlier (and less significant) life-contingent debt issues. In his report, he developed a new set of life tables, known as the Finlaison tables, based on the observed mortality experiences of the nominees. After some debate and a brief suspension of the life annuity program, Parliament determined to resume it with pricing based on these new tables.

Notice that this history closely mirrors the first steps of the equilibrium computation procedure in the stylized model of section 2.1: pricing annuities fairly for the entire population (i.e., setting the payout at $a_0$ in the notation of Section 2.1) appears to have differentially attracted longer-lived individuals ($S_1 > 0$ in the notation of section 2.1); this forced the government to re-price the annuities ($a_1 < a_0$); and it did so using the mortality rates of the early annuitants ($a_1 = a^*(S_1)$).

These dynamics allow us to test for a “death spiral” in annuity markets: if, subsequent to the re-pricing, the new mortality tables still significantly under-predicted the longevity of the subsequent nominees, then the government would have continued to lose money at the new prices, and additional rounds of re-pricing would have been required. This could potentially have led to an unraveling of much or all of the market. In fact, we will see that this re-pricing actually led to subsequent annuity sales that were reasonably profitable for the government, indicating that the market was not subject to a “death spiral.”

A third feature useful for our study is the fact that the Northampton life tables used for pricing annuities under the original act were pooled-gender tables. In contrast, the Finlaison tables were gender-specific. Since women are and were longer lived than men (especially at the ages at which they’d buy annuities), we can potentially test for adverse selection by comparing the gender composition of purchases prior to and subsequent to re-pricing.

Fourth and finally, the Act had a particularly odd feature that is absent in modern annuity markets: the buyer of an annuity did not have to be the nominee. In other words, any individual could buy an annuity whose payments were contingent on anybody else’s life. This provided an opportunity for “speculation” on lives: investors could find and nominate particu-
larly healthy individuals, buy annuities contingent on their lives, and profit handsomely from the higher-than-average longevity of these nominees – particularly if they bundled a collection of nominees and resold shares of the resulting income streams. Anecdotally supporting the presence of this type of adverse selection is the following quote, take from an informal history of the life annuities sold under the 1808 Act written by John Francis in 1853:

From 1809 this system continued. The speculators soon found out that the Government charge for a life annuity afforded a very remunerative investment, and the insurance offices made considerable profits by purchasing and re-selling them. The Commissioners of Greenwich Hospital also selected many of the most healthy of their pensioners and bought large annuities on them.

We can exploit this feature to study differences between “passive” selection effects among self-nominees and “speculative” selection effects. As we will see shortly, speculative selection appear to have been much stronger: speculators were quite successful at selecting particularly long-lived individuals, and they appear to have profited handsomely at the government’s expense by doing so.

4 Data and Analysis

This section describes the sources and form of the data available for analysis, develops empirical techniques for using it to test for adverse selection and “death spirals,” and it describes the results of that test.

4.1 The Data

Data is available from two reports commissioned by Parliament to examine the profitability of the annuities sold under the 1808 Act. The first is a 1829 report by John Finlaison. This report contains data on annuities sold between 1808 and 1826. The second is an 1860 report by his son, Alexander Glen.

\[5\] There was historical precedent for this sort of speculation. For example, Velde and Weir (1992) describe a scheme known as the “trente demoiselles de Genève” devised by Genevan bankers to nominate classes of healthy young girls to take advantage of a French life-contingent debt issue in the 1770s.
Finlaison. It examines annuities sold between 1808 and 1850. Both reports focus on developing so-called “laws of mortality” (i.e., mortality tables) for annuity buyers. Because of this, they report data only in a highly aggregated form useful for this purpose.

John Finlaison’s 1829 report contains a data set for each gender. Each set consists of three columns of data. The first column is a list of the number of annuities sold between 1808 and 1826 at each nominee age.\(^6\) The second column gives the number of nominee deaths (between 1808 and January, 1826) at each age. The final column reports the distribution of ages, in January 1826, of all nominees still living at that time.

The later report is in a similar format, but it contains four distinct three-column data sets. The first three data sets describe three distinct classes of “the nominees of those parties who speculated in life annuities” (A.G. Finlaison, 1860, page 14), henceforth speculative nominees. The annuities described in these sets were sold not to the nominee himself but rather to an investor or investment group who sought to profit by selecting particularly healthy older men and purchased annuities contingent on their lives. Each of the three data sets refers to annuities purchased after 1828, and each appears to represent one or more distinct investment portfolios. The first contains 353 nominees aged 59 to 64. The second contains 288 nominees aged 73-84, and the third contains 34 nominees aged 85-92.

Additionally, the 1860 report contains data, aggregated by gender, on all nominees between 1808 and the end of 1850, excluding the “speculative” nominees described in the preceding paragraph. We henceforth refer to these nominee classes as “non-speculative,” but it is important to note that that they includes any speculative nominees nominated between 1808 and 1828.

The first column for each of the classes of data in the 1860 report contains the number of nominees at each age between 1808 and 1850. For the non-speculative classes and the youngest speculative class, the second column records the number of nominee deaths between 1808 and May 8, 1854 at each age of death. For the two older speculative classes, the second column reports death ages between 1808 and June 10th, 1856. The final column reports the distribution of ages, on December 31, 1850, of all nominees still

\(^6\)As discussed above, and in contrast with more familiar modern annuity markets, annuity buyers were not constrained to nominate their own lives. That is, the owner of the income stream provided by the annuity did not have to be the individual upon whose life the payments were contingent. The data we analyze is based on the ages of the nominee of a given contract, not the purchaser/owner of the contract.
Table 1: Data Summary

<table>
<thead>
<tr>
<th></th>
<th>1829 Report</th>
<th>1860 Report, Non-Speculative Nominees</th>
<th>1860 Report, Speculative Nominees (Males)</th>
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<td></td>
<td>Purchased, 1808-1826</td>
<td>Living, January 1 1826</td>
<td>Purchased, 1828-1850 Living, May 8 1854 (June 10, 1856)</td>
</tr>
<tr>
<td>Male</td>
<td>2077</td>
<td>1484</td>
<td>Age 59-64 353 298</td>
</tr>
<tr>
<td>Female</td>
<td>4815</td>
<td>3860</td>
<td>Age 73-84 288 1</td>
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<td>Total</td>
<td>6892</td>
<td>5344</td>
<td>Age 85-92 34 0</td>
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Table 1: Summary of the available data. The first data column reports the number of individuals on whom annuities were purchased. The second reports the number still living at the time of observation. The data are broken down into three classes: data on annuities purchased from 1808-1826 (source: J. Finlaison, 1829); data on annuities purchased from 1808-1850 excluding annuities purchased between 1828 and 1850 by “speculators” (source: A.G. Finlaison, 1860); and annuities purchased by speculators between 1828 and 1850.

Table 1 summarizes the data from the two reports. From the 1829 report, we see that, of the 6892 annuities purchased between 1808 and Jan 1, 1826, 2077, or 30%, were male, and 5344, or 77.5% were still alive on Jan 1, 1826. From the 1860 report, we see that a total of 16137 annuities were purchased between 1808 and Dec 31, 1850 (excluding the speculative nominee classes but including the 6892 in earlier report). Of these nominees, 34% were male, and 41% were still living on May 8, 1854. Among the speculative nominees from the 1860 data set, we see that all but one of the nominees from the two
older nominee classes had died by June 10, 1856\textsuperscript{7}, while nearly 85\% of the nominees in the younger class were still living on May 8, 1854.

Table 1 provides the first suggestive evidence of adverse selection: the composition of male and female nominees changed in response to the movement from gender neutral pre-1828 pricing to gender-specific post-1828 pricing. To wit: males constituted approximately 30\% of all early (pre-1826) nominees (i.e., $\frac{2077}{2077+4815}$). In contrast, they accounted for about 41\% of all later nominees (i.e., $\frac{(5542-2077)+353+288+34}{(5542-2077)+353+288+34+(10595-4815)}$). In other words, relatively more favorable pricing for men shifted the purchase patterns away from women and towards men.

Table 2 presents the entire data set for the youngest class of speculative nominees; the data for the other classes of nominees is in similar form, but we omit them to save space. Notice the highly aggregated form of the data: all we observe are the total numbers entered, dead, and living at each age – but not when they were nominated or whether and when any given nominee died.

Figure 1 plots the three columns of data for the non-selected males nominated during the 1808-1850 period. The tall, dark curve graphs the distribution of ages of the nominees at the time of nomination. It shows a modal nomination age of about 60 and a range of nominee ages from the late teens to the early 90s. The second highest curve shows the age, at death, of the males who had died by May 8, 1854. The final curve depicts the distribution of ages on Dec 31, 1850 of those nominees who had not died by May 8, 1854.

4.2 Shortcomings of the Data

Each of the 5542 non-speculative male nominees in the 1808-1850 data set implicitly appears twice in this data: his age at nomination is recorded, and either his age at death or his age in 1850 is recorded. But we lack information providing any connection between these two appearances of this same individual – for example, whether and when any given nominee died.

This form of the data is well suited for estimating a mortality table – as was the purpose of the two reports – but it has significant shortcomings with respect to our goals of testing for adverse selection. In principle, testing for

\textsuperscript{7}The one remaining nominee died at the age of 102 in March, 1857 (A.G. Finlaison, 1860, page 88).
<table>
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<th>Age</th>
<th>Number Nominated (1828-1850)</th>
<th>Number Died (by 1854)</th>
<th>Number Living in 1854 (by age in 1850)</th>
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<tr>
<td>76</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2: Complete data from the youngest speculative nominee class. Column 2 reports the number of nominees of each age (between 1828 and 1850) Column 3 reports the number of these nominees who had died by May 8, 1854 (by age). Column 4 reports the number of the nominees who had not died by May 8, 1854, by age on Dec 31, 1850. Source: A.G. Finlaison (1860).
adverse selection should be relatively straightforward: we want to compare the mortality experience of the nominees with the mortality experience of the population, and ask whether the nominees lived longer than we expected. If we had disaggregated data reporting the purchase age, the purchase year, and the death age, if applicable, of each nominee, testing for adverse selection would thus be completely straightforward. The disaggregated nature of the data makes developing such a test more challenging. We now develop one such test.

4.3 A Consistent Test for Adverse Selection

We wish to test the question: Did nominees live longer than we would have expected? To that end, we first develop a test for the simpler case that applies to the two older nominee classes: when we know the death ages of all nominees (not just those who had died by 1826 or 1854). We then consider how to extend this case to the remaining classes, where a subset of death ages is known and the age distribution of survivors is known.

Take as given a population mortality table \( \overrightarrow{q} = (q_0, q_1, q_2, \cdots, q_{109}, q_{110}) \),

Figure 1: Non-Speculative Male Nominees, 1808-1850. Source: A.G. Finlaison (1860).
where \( q_t \) denotes the age-\( t \) mortality hazard (i.e., the probability of dying before turning \( t + 1 \), conditional on having reached age \( t \)) and we (safely) assume that nobody survives beyond age 110. View the first column on data from each nominee class as a known vector \( \overrightarrow{e} = (e_0, e_1, \ldots, e_{109}, e_{110}) \), describing the distribution of ages at nomination, where \( e_t \) is the number of individuals buying annuities at age \( t \) (or having annuities bought by someone else with payments contingent on their life). Let the random vector \( \overrightarrow{d} = (d_0, d_1, \ldots, d_{109}, d_{110}) \) denote the distribution of the actual death ages of all of these entrants, and let \( \overline{d} \) denote the average death age of the nominees, i.e. \( \overline{d} = \sum_{n=1}^{110} \frac{nd_n}{\sum_{n=1}^{110} d_n} \). When the realized value of \( \overrightarrow{d} \), and hence of \( \overline{d} \), is known, we can simply test whether \( \overline{d} \) is significantly greater than we would have expected, given the ages at nomination and the population mortality table. Specifically, the test statistic

\[
Z = \frac{\overline{d} - E[\overrightarrow{d} | \overrightarrow{e}, \overrightarrow{q}]}{\sqrt{V[\overrightarrow{d} | \overrightarrow{e}, \overrightarrow{q}]}}.
\]  

has a standard normal distribution where \( E[\overrightarrow{d} | \overrightarrow{e}, \overrightarrow{q}] \) and \( V[\overrightarrow{d} | \overrightarrow{e}, \overrightarrow{q}] \) are, respectively, the expected value and variance of \( \overrightarrow{d} \) given the mortality table \( \overrightarrow{q} \) and the nomination ages \( \overrightarrow{e} \). We can explicitly compute these by simulation or with analytic methods. We would thus support the presence of adverse selection if the value of \( Z \) is high (e.g., greater than 1.96 for a evidence at the 5% significance level).

With the exception of the two older classes of speculative nominees, we do not know \( \overrightarrow{d} \). Instead, we know a vector \( \overrightarrow{d^{early}} \) of death ages of individuals who died prior to 1826 or 1850 and a vector \( \overrightarrow{l} \) of ages of the individuals still alive. This requires that we modify the test based on Equation 7. The simpler test essentially involves comparing the actual mean death age with the expected mean death age. The modified test involves comparing the expected mean death age conditional on \( \overrightarrow{d^{early}} \) and \( \overrightarrow{l} \) with the the unconditional expected mean death age. As shown in the appendix, the random variable \( \overrightarrow{Y} \equiv E[\overrightarrow{d} | \overrightarrow{d^{early}}, \overrightarrow{l}, \overrightarrow{q}] \) (which is a function of the random variables \( \overrightarrow{d^{early}}, \overrightarrow{l} \)) is distributed asymptotically normally with a mean of \( E[\overrightarrow{d} | \overrightarrow{q}] \) and a covariance matrix

\[
Var[\overrightarrow{Y} | \overrightarrow{q}] = Var[\overrightarrow{d} | \overrightarrow{q}] - E[Var[\overrightarrow{d^{late}} | \overrightarrow{l}, \overrightarrow{q}]].
\]  

15
Table 3: Testing for Adverse Selection – Northampton Tables

<table>
<thead>
<tr>
<th></th>
<th>E(Years Lived) (’000s)</th>
<th>σ (’000s)</th>
<th>Z-Score</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unconditional</td>
<td>Conditional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>150.2</td>
<td>151.6</td>
<td>0.263</td>
<td>4.97</td>
</tr>
<tr>
<td>Females</td>
<td>343.3</td>
<td>350.7</td>
<td>0.390</td>
<td>18.9</td>
</tr>
</tbody>
</table>

Table 3: Testing for adverse selection among early nominees. This table reports the results of a test for whether the individuals nominated for annuities lived longer than predicted by the Northampton life tables described in the text. The columns labeled “Unconditional” and “Conditional” report the expected number of years lived by the entire class of nominees and the expected number of years lived by the nominees conditional on the observed mortality history through 1826, as reported in Finlaison (1829). The column labeled $\hat{\sigma}$ is the estimated standard deviation of the difference between these two columns. P-value refers to a one tailed test.

The term $\text{Var} \left[ \hat{d} | \hat{q} \right]$ can be directly computed (by simulation), while the latter term is consistently estimated by $\text{Var} \left[ \hat{d}_{\text{late}} | \hat{l}, \hat{q} \right]$ as computed (by simulation) using the realized value of $\hat{l}$ from our data set. Hence, $\sum nY_n$ is distributed normally, with a (computable) mean and a consistently estimable variance. Having fixed a mortality table, our test for adverse selection then amounts to a standard Z-test. Our results are thus based on a series of Z-tests for whether the observed annuitant longevity significantly exceeds the longevity predicted by an exogenously given life table. (See the appendix for formal details.)

4.4 Results

We first test for adverse selection by testing whether early nominees lived longer than the Northampton table – the population average mortality rates used for pricing government-issued annuities prior to 1828 – would have predicted. The results appear in Table 3. We see that the early nominees of both genders lived significantly longer than predicted by the population average mortality tables, with males and females living an estimated aggregate 1400 (151,600 – 150,200) and 7400 additional years, respectively – more than .5 “extra” years per male and 1.5 “extra” years per female. This is strongly suggestive of adverse selection amongst the early nominees.

Our second test amounts to a test for additional adverse selection amongst
annuitants nominated subsequent to repricing in 1828. This involves testing whether various populations of nominees lived longer than predicted by the Finlaison life tables used for pricing after 1828. Table 4 reports the results. The final two rows of the table help to verify the validity of the Finlaison life tables for the early nominees. The Finlaison tables were developed to describe the mortality experience of these populations and, indeed, the tests provide no evidence that these individuals lived longer than these tables predict: males are conditionally predicted to live an estimated aggregate 8 years fewer than the Finlaison tables would suggest, while females are conditionally predicted to live an estimated 43 years less, in aggregate. Both are statistically (and economically) insignificant.

The first two rows of Table 4 report the results of testing whether the “Non-Speculative” nominees from 1808-1850 lived longer than the Finlaison tables would indicate. (Note that the early nominees are a subset of these nominees. Hence, this is a pure test for additional adverse selection amongst post-1828 nominees only insofar as the Finlaison tables accurately describe the mortality of early nominees.) The first row of indicates that non-speculative 1808-1850 nominees are conditionally predicted to live an aggregate of approximately 700 years fewer than would be unconditionally predicted, but this difference is not statistically significant. In contrast, females nominated between 1808 and 1850 are conditionally predicted to live in excess of 6000 fewer years, statistically significantly different from the longevity predicted by the Finlaison tables ($Z = -6.75$). Females nominated after the early data set (after 1826) died an (statistically and economically significant) average of about 1 year sooner than the Finlaison tables would have suggested.

Taken at face value, these results might seem somewhat puzzling. They suggest: (i) the early nominees were adversely selected, relative to the population; (ii) the government re-priced the annuities using a mortality table that correctly captured the mortality of the early nominees; (iii) the subsequent nominees (particularly the females) were advantageously selected relative to the new mortality table. This third observation is in stark contrast to the simple model outlined in Section 2, which suggested at least some additional adverse selection should occur.

These counterintuitive results can be understood by observing that the early nominees include “speculative” purchases. After re-pricing, however, the “speculative” nominees were recorded separately. Hence, the Finlaison tables can correctly capture the mortality rates of the pre-1828 blend of
Table 4: Testing for Adverse Selection – Finlaison Tables

<table>
<thead>
<tr>
<th></th>
<th>Unconditional ('000s)</th>
<th>Conditional ('000s)</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-S Males, 1850</td>
<td>408.4</td>
<td>407.7</td>
<td>6.06</td>
<td>−0.11</td>
</tr>
<tr>
<td>N-S Females, 1850</td>
<td>807.4</td>
<td>801.3</td>
<td>0.915</td>
<td>−6.75</td>
</tr>
<tr>
<td>S Males, 59-64</td>
<td>26.42</td>
<td>26.7</td>
<td>0.0943</td>
<td>3.40</td>
</tr>
<tr>
<td>S Males, 73-84</td>
<td>24.02</td>
<td>24.4</td>
<td>0.0663</td>
<td>5.10</td>
</tr>
<tr>
<td>S Males, 85+</td>
<td>3.056</td>
<td>3.115</td>
<td>0.0112</td>
<td>5.28</td>
</tr>
<tr>
<td>Males, 1826</td>
<td>153.4</td>
<td>153.4</td>
<td>1.38</td>
<td>−0.01</td>
</tr>
<tr>
<td>Females, 1826</td>
<td>366.5</td>
<td>366.4</td>
<td>0.275</td>
<td>−0.16</td>
</tr>
</tbody>
</table>

Table 4: This table reports the results of a test for whether various classes of individuals nominated for annuities lived longer than predicted by the Finlaison life tables described in the text. “N-S” and “S” Refer to non-speculative and speculative classes, respectively. The columns labeled “Unconditional” and “Conditional” report the expected number of years lived by the entire class of nominees and the expected number of years lived by the nominees conditional on the observed mortality history as of 1854, as reported by A.G. Finlaison (1860). The column labeled $\hat{\sigma}$ is the estimated standard deviation of the difference between these two columns. Z and P refer to the Z and P values associated with the 1-tailed test: is the conditional number of years lived significantly longer than the unconditional number of years lived?

speculative and non-speculative nominees, and still understate the mortality of self-selected subset of these early – at least under the plausible assumption that the speculative nominees are longer-lived than the self-selected ones. The self-selected late nominees could therefore have higher mortality rates than the Finlaison tables would indicate, even if they are adversely selected relative to the early non-speculative nominees.

This interpretation explains both the “advantageously selected” post-1828 female nominees and the apparently minimally or non-selected post-1828 male nominees (viz Table 4). Under this interpretation, both result from a combination of two competing forces: first, the post 1828 nominees in this sample are shorter lived than the pre-1828 nominees because they exclude the longer-lived speculative nominees; second, they are longer-lived because they are adversely selected relative to the self-selected pre-1828 nominees. Furthermore, there is a simple explanation for why the post-1828 female nominees appear more “advantageously selected” than the post-1828 male nominees: since females are (and were) longer lived than males (at least at typical annuity-purchasing ages) and early annuity pricing was gender neutral, we would expect the bulk of early speculative nominees to have been female. The first effect is therefore likely to be larger for females than for
males.

Examining the speculative nominees on whom we do have data corroborates this interpretation of our results. The middle 3 rows of Table 4 indicate that all three classes of speculative nominees on whom we have data lived (highly statistically) significantly longer than the Finlaison tables would suggest.\(^8\)

The take-away from all this is that, while the evidence points to adverse selection, it also suggests that the annuity market created by the 1808 Act was not prone to an adverse-selection death spiral. Indeed, in spite of facing worse than actuarially fair annuity prices after 1828, non-selected nominees continued to purchase annuities in significant numbers. (The average annual annuity nominees prior and subsequent to 1826 were virtually identical – 385 versus 383). This suggests that well-functioning annuity markets were possible, even with non-trivial administrative loads.\(^9\)

5 Discussion and Conclusions

The United Kingdom’s 1808 Life Annuity Act effectively opened up a market for government-provided life annuities. Analysis of data on purchases and subsequent mortality rates suggests that the market was characterized by two types of adverse selection: classic “self-selection” whereby individuals purchasing annuities for themselves were healthier than the average individual in the population, and “speculative” selection, whereby pecuniary-minded individuals or institutions took advantage of an odd feature of the act which allowed them to purchase annuities contingent on the lives of others.

---

\(^8\)Speculators appear to have selected older nominees for a simple reason, discussed by Francis (1853): The Finlaison tables were developed based on actual mortality experiences of early nominees. Payouts from early annuities were capped at age 75, which meant that there were extremely few nominees above this age prior to 1828. Finlaison’s mortality estimates for aged individuals are thus derived from the mortality experience of individuals who were nominated at a young age and subsequently grew old – likely significantly underestimating the longevity of selected older lives.

Francis relays anecdotes suggesting that speculators understood this well. Apparently, they even combed the countryside looking for hale older men with valid birth certificates and even resorted to paying local surgeons and pastors to keep them healthy – a rare case of moral hazard in annuity markets!

\(^9\)My back-of-the-envelope calculations suggest that, at least among females, the government earned on the order of 5 cents per non-speculative female nominee after re-pricing in 1828.
Our results indicate that self-selection, while significant, was not sufficient to induce a “death spiral” and undermine the entire market. Indeed, purchases of annuities remained quite robust even after re-pricing rendered them worse than actuarially fair for self-selected nominees. Although not definitive, this suggests that the “annuity puzzle” – the virtual absence of modern annuity markets – cannot be attributed to adverse selection alone.

“Speculative” selection appears to have been quite costly to the British government. Our results indicate that speculators effectively selected particularly long-lived individuals to nominate for annuities and profited considerably from the Act. This recommends care to modern policy makers when contemplating expanding choice in government provided services: when choices can be made by pecuniary-minded individuals or institutions, selection effects may be significantly exacerbated.

References


6 Appendix

This appendix establishes that $\vec{Y} \equiv E\left[\vec{d}|\vec{d}^{\text{early}}, \vec{l'}, \vec{q}\right]$, described in Section 4.3, is normally distributed, and that the asymptotic variance of $\vec{Y}/N$ with a variance which is consistently estimated by $\left(\frac{1}{N^2}\right)\left(\text{Var}\left[\vec{d}|\vec{q}\right] - \text{Var}\left[\vec{d}^{\text{late}}|\vec{l'}, \vec{q}\right]\right)$, where $N$ is the number of annuity buyers.\textsuperscript{10} Since we will take the vector of age-specific mortality hazards $\vec{q}$ as a given throughout this appendix, we will drop it for notational convenience henceforth.

Recall that $\vec{d}$ is a vector-valued random variable describing the number of individuals who died at each age. It is equal to $\vec{d}^{\text{early}} + \vec{d}^{\text{late}}$, where $\vec{d}^{\text{early}}$ is a vector-valued random variable whose elements describe the number of individuals who had died, at each age, by a fixed death observation date (e.g., May 8th, 1854), and $\vec{d}^{\text{late}}$ is, similarly, a vector describing the death ages for those who died after that fixed date. Finally, recall that $\vec{l'}$ is a vector-valued random variable describing the ages, at some (potentially distinct) fixed

\textsuperscript{10}For asymptotics, we have in mind, that the sample of nominees from (e.g.) 1808-1826 was a a representative sample drawn from some distribution of all possible nominees.
date (e.g., Dec 31, 1850) of the individuals who had not died by the death observation date.

First, note that \( E[Y] = \bar{E} \), that \( Y = \bar{E} \), and that

\[
\text{Var}(Y) = \text{Var}(\bar{E}) + \text{Var}(E[\text{date} \mid \bar{l}]) + 2\text{Cov}(\bar{E}, E[\text{date} \mid \bar{l}])
\]

(9)

(N.b.: \( E[\text{date} \mid \bar{l}] \) is a random variable since it depends on the realization of \( \bar{l} \).)

We now prove two simple lemmas using the law of iterated expectations:

**Lemma 1** \( \text{Cov}(\bar{E}, E[\text{date} \mid \bar{l}]) = \text{Cov}(\bar{E}, \text{date}) \).

**Proof.**

\[
\text{Cov}(\bar{E}, E[\text{date} \mid \bar{l}]) = E\left[ \left( \bar{E} - E[\bar{E}] \right) \left( \text{date} - E[\text{date}] \right) \right]
\]

\[
= E\left[ \left( \bar{E} - E[\bar{E}] \right) \left( \text{date} - E[\text{date}] \right) \mid \bar{E}, \bar{l} \right]
\]

\[
= E\left( \bar{E} - E[\bar{E}] \right) E\left( \text{date} - E[\text{date}] \right) \mid \bar{E}, \bar{l}
\]

\[
= \text{Cov}(E[\bar{E}], \text{date} \mid \bar{l}).
\]

**Lemma 2** \( \text{Var}(E[\text{date} \mid \bar{l}]) = \text{Var}(\text{date}) - E[\text{Var}(\text{date} \mid \bar{l})] \).

**Proof.**

\[
\text{Var}(E[\text{date} \mid \bar{l}]) = E\left[ \left( \text{date} - E[\text{date}] \right) \left( \text{date} - E[\text{date}] \right) \mid \bar{l} \right]
\]

\[
= E\left[ \text{date} \mid \bar{l} \right] - E\left[ \text{date} \mid \bar{l} \right] \]

\[
= \text{Var}(\text{date}) - E\left[ \text{Var}(\text{date} \mid \bar{l}) \right].
\]
Proof.

\[
\begin{align*}
\text{Var} \left[ \bar{d}_{\text{late}} \right] &= \mathbb{E} \left[ \left( \bar{d}_{\text{late}} - \mathbb{E} \left[ \bar{d}_{\text{late}} \right] \right) \left( \bar{d}_{\text{late}} - \mathbb{E} \left[ \bar{d}_{\text{late}} \right] \right)' \right] \\
&= \mathbb{E} \left[ \left( \bar{d}_{\text{late}} - \mathbb{E} \left[ \bar{d}_{\text{late}} \right] \right) \left( \bar{d}_{\text{late}} - \mathbb{E} \left[ \bar{d}_{\text{late}} \right] \right)' \right] \mathbb{E} \left[ \mathbb{I} \right] \\
&= \mathbb{E} \left[ \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{E} \left[ \bar{d}_{\text{late}} \right] \mathbb{E} \left[ \mathbb{I} \right] \right] \mathbb{E} \left[ \mathbb{I} \right] \right] - \mathbb{E} \left[ \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{E} \left[ \mathbb{I} \right] \right] \mathbb{E} \left[ \mathbb{I} \right] \right] \\
&\quad + \mathbb{E} \left[ \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{E} \left[ \mathbb{I} \right] \right] \mathbb{E} \left[ \mathbb{I} \right] \right] - \mathbb{E} \left[ \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{E} \left[ \mathbb{I} \right] \right] \mathbb{E} \left[ \mathbb{I} \right] \right] \\
&= \mathbb{E} \left[ \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right] + \mathbb{E} \left[ \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{E} \left[ \mathbb{I} \right] \right] \mathbb{E} \left[ \mathbb{I} \right] \right].
\end{align*}
\]

Re-arranging completes the proof. \(\blacksquare\)

From Lemma 2 and Equation 9, we have:

\[
\text{Var} \left[ \bar{Y} \right] = \text{Var} \left[ \bar{d}_{\text{early}} \right] + \text{Var} \left[ \bar{d}_{\text{late}} \right] - \mathbb{E} \left[ \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right] + 2\text{Cov} \left[ \bar{d}_{\text{early}}, \mathbb{E} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right].
\]

Then, using Lemma 1 and the fact that \(\bar{d} = \bar{d}_{\text{early}} + \bar{d}_{\text{late}}\) gives

\[
\begin{align*}
\text{Var} \left[ \bar{Y} \right] &= \text{Var} \left[ \bar{d}_{\text{early}} \right] + \text{Var} \left[ \bar{d}_{\text{late}} \right] + 2\text{Cov} \left[ \bar{d}_{\text{early}}, \bar{d}_{\text{late}} \right] - \mathbb{E} \left[ \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right] \\
&= \text{Var} \left[ \bar{d} \right] - \mathbb{E} \left[ \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right].
\end{align*}
\]

It remains only to show that \(\mathbb{E} \left[ \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right]\) is consistently estimated by \(\text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \), and this follows immediately from the fact that \(\frac{\bar{d}}{N}\) converges in probability.

Our test is based on the statistic \(\bar{n} \cdot \bar{Y}\) - the expected total number of years lived, conditional on the data, where \(\bar{n}\) denotes a vector \((0, 1, 2, \cdots)\) of ages of death. The mean of this variable is the unconditional expected total number of years lived, \(\bar{n} \cdot \mathbb{E} \left[ \bar{d} \right]\), and \(\bar{n} \cdot \left( \text{Var} \left[ \bar{d} \right] - \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right] \right)\) \(\bar{n}\) consistently estimates its (asymptotic) variance, where the realization of \(\bar{d}_{\text{early}}\) and \(\mathbb{I}\) are observed. The variance matrix \(\text{Var} \left[ \bar{d} \right] - \text{Var} \left[ \bar{d}_{\text{late}} \mathbb{I} \right]\) –
\( \text{Var}\left[ d^{:\text{late}} | d^{:\text{early}}, l \right] \) can be computed, to arbitrary precision, by simulating the distribution of \( \vec{d} \) and of \( d^{:\text{late}} \) conditional on \( l \). This is the approach we take in our empirical work.