Design of Contingent Commission Schedules, Underwriting Quality and “Riskiness” of Lines

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Summary

We derive necessary and sufficient conditions for the design of an insurer’s profit based contingent commissions’ schedule which will optimize a broker-agent’s underwriting effort in absence of moral hazard and adverse selection. The deterministic model indicates that the proportion of commissions contingent upon underwriting profit is concave (diminishing returns) in underwriting quality. This concavity in design arises because the underwriting quality is concave in effort. Assuming that these conditions hold for the contingent commission schedule, we incorporate uncertainty into the underwriting quality and derive the empirically testable prediction that “riskier” lines of business are more likely to be underwritten by “specialist” producers who deliver higher marginal quality with lower levels of effort.

1. Introduction

Initiated by an investigation by New York Attorney General Eliot Spitzer, insurers’ use of contingent commissions has come under intense scrutiny over the past several years.¹ Contingent commissions are primarily bonuses that insurance companies pay brokers and independent agents (“producers”) for doing business with the insurer.² These payments are made in addition to the customary fixed percentage of premium, the premium-based commission. The main complaint with contingent commissions is that they generate a conflict of interest for producers, although Kurland (1996) fails to find evidence of it. In a

¹ The press statement regarding Eliot Spitzer’s investigation of the insurance industry can be located at http://www.oag.state.ny.us/press/statements/insurance_investigation_testimony.pdf.
² The difference between brokers and independent agents is the degree that they are legal agents of the policyholder or the insurer. Brokers are legally described as agents of the consumer, while independent agents are depicted as agents of the insurer. In practice, however, the difference between these intermediaries is minimal (Cummins and Doherty, 2006).
related study, Howe et al (1994) finds negative correlation between customer orientation of insurance salespersons and unethical behavior. Still, Spitzer’s investigation alleges that producers “routinely mislead their clients about the true nature of contingent commissions.” As a result of Spitzer’s probe and those of other regulators, insurance brokerages have paid more than $1 billion in fines and penalties and seven states have passed legislation on these commissions (Fitzpatrick, 2006).³ On the company side, many firms have abandoned contingent commissions altogether.

Despite Spitzer’s statement that contingent commissions “provide little or no value or services to insurance carriers,” contingent commissions continue to be a significant part of compensation in the insurance industry.⁴ Contingent commissions in the insurance industry are of two flavors: volume-based and profit-based. Volume-based contingent commissions vary depending on the amount of business that a particular producer refers to the insurer, while profit-based contingent commissions adjust with changes in the insurer’s profitability (Wilder, 2002). This paper focuses upon profit-based contingent commissions.⁵ The purpose of research is to examine the optimal structure of contingent commissions in the property and liability (P/L) industry and to investigate whether contingent commissions add value to the insurance intermediary process. This paper achieves the purpose by developing an effort based model of a profit maximizing producer who doubles as the “first underwriter”. Empirical testing of the predictions of the model is yet to be completed.

Compensation in insurance distribution channels is a relatively unexplored area whereas several papers have endeavored to explain the coexistence of direct writers (DWs)

³ Federal officials have pointed to the state’s perceived slow response as testimony to the problems of regulating insurance at the state level.
⁴ See BestWire Services dated 05/08/2006.
⁵ Regan and Tennyson (1996) find that insurers pay a larger proportion of agent commissions on a profit-contingent basis.
and independent agents (IAs). These theories rely on search costs (Posey and Yavas, 1995; and Posey and Tennyson, 1998); information asymmetry (Marvel, 1982); vertical integration (Grossman and Hart, 1986); monitoring (Kim et al., 1996); transaction costs (Regan, 1997); and risk selection and classification (Regan and Tennyson, 1996). Regan (1997) and Regan and Tennyson (1996) bring together agent efforts, risk complexity and compensation. While Regan and Tennyson (1996) conclude that IAs are preferred when the subjective information provided by them is important to profitable underwriting, Regan (1997) argues that DWs are more likely when relationship-specific investments are important. While these two papers focus upon the choice of distribution system, the model in our proposed paper will focus on the shape of contingent compensation schedule, linking it to effort and the “riskiness” of the type of insurance. We study the design of the profit based contingent commissions’ schedule, deriving the conditions that should be imposed on it, so as to ensure that a risk neutral producer expends optimal effort at underwriting. We ignore information asymmetry and observability issues and derive our results in a simple one period deterministic framework. We then build uncertainty into the model so as to derive results linking “riskiness” of lines and agent underwriting effort directly.

The goal of the model is to derive the optimal shape of the contingent commission schedule as a function of underwriting quality where the producer acts as the “first underwriter”. We will then incorporate uncertainty into the quality of underwriting with the assumption that riskier lines of business are associated with more uncertain underwriting quality. The purpose of incorporating uncertainty into the model is to allow for a deterioration of risk (in the FOSD sense) and then to determine the resulting comparative statics. The main contributions of our model are that it will disaggregate producers
according to “riskiness” of lines and establish a direct link between “riskiness” of lines and contingent commissions.

Our model yields the following empirically testable predictions: (1) the percentage of contingent commissions an insurer employs is concave in their need for underwriting quality; (2) inherently riskier lines are underwritten by “specialist” producers who deliver higher marginal quality at lower levels of effort; and (3) the optimal design of the contingent commission schedule suggests that insurers that write “riskier” lines will rely relatively more on contingent commissions. We note that (3) is a direct implication of (1) and (2).

The organization of the paper is as follows: In the following section we undertake a literature review and shed some light on the compensation earned by BA firms. In section 3, we present the assumptions of the model and results of the base model. In section 4, we extend the base model to incorporate uncertainty into the model. Section 5 presents the conclusions and the agenda for future research.

2. Literature Review

Compensation for independent insurance intermediaries (BA firms) comprises of three revenue streams. One is the fee they charge the clients and remaining two are the commissions they receive which are based on the policies they sell and/or commissions based on performance. The performance metric could be the policies renewed or the profitability of the business. Cummins and Doherty (2006) document that contingent commission structures are usually progressive in the sense that the marginal rate of commission increases with the level of activity; and the percentage commission rate increases as higher profit triggers are obtained.

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6 In this paper we study the profit based contingent commission.
Design of compensation schemes for risk-averse agents has been explored in various streams of economics. Holmstrom and Milgrom (1987) develop a total compensation model for risk-averse agents possessing constant absolute risk aversion in a multi-period setting in a framework that incorporates unobservability of effort. The optimal total compensation plan in their framework is linear because the principal desires to reduce the inter-temporal variability in the agent’s effort. The multi-period model reduces to a single period optimization in their framework. Basu et al (1985), studies compensation for the sales force in a static framework when the selling environment is uncertain. The authors show that the optimal mix of salary and compensation can be convex, linear or concave dependent upon the rate of change of risk tolerance of the selling agent. Lal and Srinivasan (1993), and Kalra et al (2003) extend the model of Basu et al (1985) in different directions.

While compensation studies cited heretofore focus on optimal total compensation or combinations of the various components of compensation (salary, fee, commissions), they do so from the perspective of a risk-averse individual agent whose utility function governs their behavior. However, a BA firm is considered risk neutral and for a BA firm these analyses fall short. Moreover, insurance intermediaries are in the business of selling protection for fortuitous losses hence the analysis differs from that of the consumption of a tangible product. Therefore, the model presented below differs from the studies cited earlier in several respects: i) The BA producer in our model is a risk neutral rather than a risk-averse agent; ii) we focus on the BA firm as a profit maximizing intermediary without modeling the interaction between the insurer and the producer. This simplification helps us avoid the information asymmetry problems besotting the principal-agent relationship; iii) we study the shape of the profit based commission schedule peculiar to the insurance industry, while studies cited above focus on sales commissions which are volume based and can only be
equated to premium based commissions; iv) we do not factor in the customer-producer relationship in our model as some of the studies above do when they account for customer satisfaction related compensation. The reasons for this last simplification are two-fold: one, for an property-liability insurer client satisfaction based agent compensation is of secondary importance especially if the producer owns the expirations list; and two, evidence of conflict of interest when contingent commissions are paid is absent (see Kurland, 1996; and Howe et al, 1994) even if such conflicts can theoretically exist under the assumption of producer myopia.

The insurance industry comprises both direct writers (DWs) and independent agents (IAs). Several reasons have been put forth for the co-existence of these distribution systems. For example, Posey and Yavas (1995) and Posey and Tennyson (1998) offer a search cost based explanation, wherein shopping with independent agents entails a single search while searching with DWs requires a sequential search. Posey and Tennyson (1998) find that producers with high production costs and consumers with high search costs utilize independent agency while low cost producers and consumers employ the direct writing system. Other explanations for the coexistence of both channels rely on incentives and information asymmetry. Marvel (1982) argues when information asymmetry can be best reduced by advertising, carriers resort to direct underwriting rather than hiring independent agents. Grossman and Hart (1986) rely on the services provided by the independent agents and conclude that when their services are most valued they end up owning the “expirations” list, otherwise the distribution channel gets integrated with the producer firm. Kim et al (1996) focus on the monitoring role of the agents. When there is information opacity between consumers and insurance companies, agents tend to get employed to act as
delegated monitors. Their model predicts that there exists a relationship between organizational form of the carrier and the distribution channel employed.

We hope to build upon studies by Regan and Tennyson (1996) and Regan (1997). Regan and Tennyson (1996) model the effort of agent across distribution systems. They focus on the role played by the agents in risk selection and classification (underwriting). Through the efforts an independent agent makes as a “first underwriter”, and the bargaining ability whereby she can place the risk with one among several companies, the agent can extract a share of the residual profits, which a tied agent cannot do. If we couple this with Basu et al (1985), then we can hypothesize that tied agents will have less contingent commission based compensation than independent agents. The marginal cost of compensating an independent agent for information gathering effort is thus lower. Hence the authors predict that when subjective information provided by the insurance agent is important to profitable underwriting then independent insurance agents are more efficient.

While this paper addresses the issue of compensation, it is only for direct writers (DW) vs. independent agents (IA). On the other hand this paper concentrates only on the profit-based commission for the broker-agents. And the role of independent agent as “first underwriters” described in Regan and Tennyson (1996) occupies a central place in our model.

Regan (1997) proposes a transactions cost based explanation for the choice of distribution systems. Under this theory, when transactions costs are high vertical integration of supply chain takes place. The paper argues when products are complex or when the environment is uncertain, the agency system prevails. The agent thus fulfils the roles of both an arbiter in insurer/customer conflicts and of an information intermediary. The paper models complexity of lines. However, the issue addressed is whether a more complex line
will be underwritten by an IA or a DW. In our paper we address the question, among independent BA firms, which firm will underwrite what line of business.

Both the papers above combined together build upon agent efforts, risk complexity and compensation, but the questions they address are different. The authors are addressing these issues from the point of view of costs of DWs and IAs. The model in this paper however, focuses on the design of contingent compensation contracts, linking it to effort and the “riskiness” of the line, which to our knowledge is an open issue (Cummins and Doherty (2006)). The purpose of this paper is to fill this gap.

3. Assumptions and Base Model Results

We model the behavior of a broker-agent (BA) firm that acts as “first underwriter” for the insurance carrier. While individual risk-averse individuals make up the BA firm, we assume the firm to be risk neutral. The dual role of selling and underwriting referred to in Regan and Tennyson (1996) is assumed to be carried out at the level of the firm. Consequently, the distinction between BA firms in which individual agents double up as both sellers and underwriters and BA firms that have two types of teams- one that generates sales and the other that underwrites the business- is irrelevant to our model. However, what is relevant is the two types of effort exerted by the BA firm, underwriting and selling. We model the trade-off between these two types of efforts.

Revenues to the BA firm that accrue from the insurer comprise of a fixed fee, sales commissions (premium based) and profit based commissions. We assume that the BA firm operates on the frontier of effort. Greater selling effort lowers underwriting effort, so that while premium based commissions increase, the profit based commissions fall. The converse also holds true. The underwriting effort leads to risk classification and risk transformation
and the quality of risk underwritten and profit based commission generated from it depends upon the efforts expended at underwriting.

We make the following assumptions for the BA firm:

1. The contingent commission is expressed as a fraction of the underwriting profit, $f_{\pi_u}$ of underwriting profit $\pi_u$. Previous studies (Cummins and Doherty, 2006) have documented that the relationship between $f_{\pi_u}$ and $\pi_u$ is convex. \(^7\) So we assume a sufficiently smooth function, $f_{\pi_u}(.), f'_{\pi_u}>0$ and $f''_{\pi_u}>0$.

2. We model the effort spent in underwriting as follows. The greater the effort exerted by the underwriting department of BA firm, the better the quality of the transformed risk that the carrier underwrites. This quality is reflected in the lower claim costs experienced and hence higher profits earned by the insurer. Let the quality of the transformed risk be $q(e_u)$, which shows that the quality ($q$) is a function of the underwriting effort ($e_u$) applied. We assume diminishing marginal returns of the final quality of the transformed risk as a function of effort, so that $q'(e_u)>0$ and $q''(e_u)<0$.

3. The cost of information gathering and analysis incurred by the BA firm’s underwriting services is modeled as an indirect function of effort $e_u$.

Underwriting effort determines the quality of the underwritten transformed risk. We assume the underwriting costs are represented by $C(q(e_u))$ and for ease of algebra we assume $C(q)$ is linear in $q$ with a non-zero positive marginal cost.

\(^7\) Similar convexity has been reported by Frank (1984) in the real estate industry.
4. The revenues and costs associated with sales effort are modeled as follows. For a given quality of risk, the BA firm is assumed to be a price taker (the carriers quote their prices as per a fixed schedule). If the number of exposures placed with a carrier is \( S \) at unit price \( p \), the revenue earned from sales is \( f_s p S \), where \( f_s \) denotes the sales commission as a percentage of the premiums generated denoted by \( P(\equiv pS) \). The cost of generating sales \( S \) is \( C(S) \) and \( C'(S) > 0, C''(S) > 0 \). Again, we assume that the number of exposures \( S \) is a function of the sales effort \( e_s \), i.e., \( S \equiv S(e_s) \). We assume \( S'(e_s) > 0, S''(e_s) = 0 \). \(^8\)

In effect, cost of generating sales is an indirect function of sales effort \( e_s \), i.e., \( C(S) \equiv C(S(e_s)) \)

5. When the quality of underwriting is \( q(e_u) \), then the claim costs are \( Q(q(e_u)) \). We assume the claim costs are monotonically decreasing and convex in quality of underwriting, so that \( Q'(\cdot) < 0, Q''(\cdot) > 0 \).

The profit function of the BA firm can now be written as follows:

\[
\Pi_{BA} = F_0 + f_s p S(e_s) + f_{\pi_u} (P - Q(q(e_u))) - C(q(e_u)) - C(S(e_s))
\]

(1)

where \( F_0 \) is a fixed fee charged to the customers by the BA firm for services rendered. \( F_0 \) serves the purpose of meeting the participation constraint of the BA firm. The BA firm maximizes this objective function over two variables, \( e_s \) and \( e_u \). Note that in (1) while \( f_s \) is a scalar \( f_{\pi_u} (P - Q(q(e_u))) \) is a function of underwriting profit \( \pi_u (= P - Q(q(e_u))) \).

The first order conditions for the unconstrained problem (1) are given by:

\(^8\) See Brown and Peterson (1994) for the validity of this assumption.
\[
\frac{\partial \Pi_{B,A}}{\partial e_i} = \delta'(e_i) \left[ f, \frac{\partial f_{\pi_w}}{\partial \pi_w} + p - C'(S) \right] = 0 \tag{2}
\]

\[
\frac{\partial \Pi_{B,A}}{\partial e_a} = -\frac{\partial f_{\pi_w}}{\partial \pi_w} \cdot \frac{\partial q}{\partial e_a} - \frac{\partial C}{\partial q} \cdot \frac{\partial q}{\partial e_a} = 0 \tag{3}
\]

Equation (2) and (3) imply the profit maximizing optimal level of sales \( e_i^* \) and underwriting effort \( e_a^* \) exist subject to the verification of the second order conditions. Equation (2) generates a straightforward interpretation, namely, sales are generated to a point where the marginal benefit equals marginal cost.

To verify the second order conditions, we need to look at the following Hessian matrix, \( H \):

\[
H = \begin{pmatrix}
\frac{\partial^2 \Pi_{B,A}}{\partial e_i^2} & \frac{\partial^2 \Pi_{B,A}}{\partial e_i \partial e_a} \\
\frac{\partial^2 \Pi_{B,A}}{\partial e_a \partial e_i} & \frac{\partial^2 \Pi_{B,A}}{\partial e_a^2}
\end{pmatrix}
\]

We consider the following individual second order conditions:

\[
\frac{\partial^2 \Pi_{B,A}}{\partial e_i^2} = \delta'(e_i)^2 \left[ p^2 \frac{\partial^2 f_{\pi_w}}{\partial \pi_w^2} - C''(S) \right] \tag{4}
\]

\[
\frac{\partial^2 \Pi_{B,A}}{\partial e_i \partial e_a} = \delta' S'(e_i) \frac{\partial^2 f_{\pi_w}}{\partial \pi_w^2} \left( \frac{\partial Q}{\partial q} \cdot \frac{\partial q}{\partial e_a} \right) \tag{5}
\]

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\( ^9 \) For notational convenience, we assume \( \frac{df_{\pi_w}}{d\pi_w} = \frac{\partial f_{\pi_w}}{\partial \pi_w} \) throughout.
\[
\frac{\partial^2 \Pi_{BA}}{\partial e_a^2} = \frac{\partial}{\partial e_a} \left[ -\frac{\partial f_{\pi_e}}{\partial \pi_e} \frac{\partial Q}{\partial e_a} \cdot \frac{\partial q}{\partial e_a} - \frac{\partial C}{\partial e_a} \cdot \frac{\partial q}{\partial e_a} \right]
\] (6)

Equation (6) can be rearranged\(^\text{10}\) to yield the following form:

\[
\frac{\partial^2 \Pi_{BA}}{\partial e_a^2} = (\frac{\partial q}{\partial e_a})^2 \frac{\partial^2 f_{\pi_e}}{\partial q^2} \frac{\partial q}{\partial e_a} + \frac{\partial^2 q}{\partial e_a^2} \frac{\partial f_{\pi_e}}{\partial q} - \frac{\partial^2 q}{\partial e_a^2} \frac{\partial C}{\partial q}
\] (7)

For optimality of \(e^*_v\) and \(e^*_s\) we need to impose some restrictions on s.o.c. (4), (5) and (7).

We notice that the term in (4) is negative if the curvature of \(f_{\pi_e}\) is less than \(\frac{C''(S)}{p^2}\), while (7) can be easily verified to be negative. So a necessary condition for the optimum \(e^*_v\) and \(e^*_s\) to exist is the least upper bound on the curvature of the fraction paid as contingent commissions equals \(\frac{C''(S)}{p^2}\). This restriction ensures elements along the principal diagonal are negative.

For sufficiency of the negative semi-definiteness of the Hessian matrix, the second principal minor should be \(\geq 0\). Since \(\Pi_{BA}\) is \(C^2\), by Young’s theorem, \(\frac{\partial^2 \Pi_{BA}}{\partial e_v \partial e_a} = \frac{\partial^2 \Pi_{BA}}{\partial e_a \partial e_v}\).

The second principal minor can now be written as follows:

\[
S'(e_r)^2 \left\{ \left[ (p^2 \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2} - C''(S)) \left( \frac{\partial q}{\partial e_a} \frac{\partial^2 f_{\pi_e}}{\partial q^2} + \frac{\partial^2 q}{\partial e_a^2} \frac{\partial f_{\pi_e}}{\partial q} - \frac{\partial^2 q}{\partial e_a^2} \frac{\partial C}{\partial q} \right) \right] - \left[ p \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2} \left( \frac{\partial Q}{\partial q} \frac{\partial q}{\partial e_a} \right) \right]^2 \right\}
\] (8)

We look at the sign of the two terms inside of the curly brackets in equation (8). The equation is of the form \(S'(\cdot)^2 \{\text{Term I}-\text{Term II}\}\). Equation (8) will be positive overall if

\(^{10}\) See appendix for the algebra.
\{\text{Term I} - \text{Term II}\} \text{ is positive. We now explain the conditions under which this will indeed be so.}

Term II is unambiguously positive while Term I is a product of two terms. By the necessary second order condition for maximization (equation (4)), \(p^2 \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2} - C''(S)\) is negative. However, overall Sgn (Term I) is positive if the following conditions are met:

\[ i) \frac{\partial f_{\pi_e}}{\partial q} > \frac{\partial C}{\partial q} > 0; \text{ and } ii) \frac{\partial^2 f_{\pi_e}}{\partial q^2} < 0, \quad (9) \]

Condition (i) of (9) implies at the margin, the percent of commission accruing to the BA firm as a result of improving quality of underwriting is greater than the marginal cost. Thus (i) and (ii) together imply the contingent commission schedule be concave in the underwriting quality of the BA firm. Conditions (9) then make the second part of Term I, \(\left(\frac{\partial q}{\partial \pi_e}\right)^2 \cdot \frac{\partial^2 f_{\pi_e}}{\partial q^2} + \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2} \cdot \frac{\partial f_{\pi_e}}{\partial q} - \frac{\partial^2 q}{\partial \pi_e^2} \cdot \frac{\partial C}{\partial q}\) negative as well, thus making the overall Term I positive.

Still, even with conditions enumerated in equation (9), equation (8) is not unambiguously positive unless Term II turns out to be smaller than Term I in magnitude. Heuristic arguments show this case is more likely when the cost curve for sales effort is sufficiently flat & the rate of change of claims costs with underwriting effort is not very high. Since least upper bound for \(p \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2}\) is given by \(\frac{C''(S)}{p}\), flatness of \(C(S)\) ensures \(p \frac{\partial^2 f_{\pi_e}}{\partial \pi_e^2}\) is small in magnitude. Simultaneously, if \(\frac{\partial Q}{\partial q} \cdot \frac{\partial q}{\partial \pi_e}\) is small as well, Term II, which is the square of the product of these two terms is expected to be small. Consequently, for sufficiently flat sales and claims cost curves, Term II will be smaller than Term I. In such
cases, conditions enumerated in equation (9) form the sufficient conditions for exertion of optimal levels of effort by a profit maximizing BA firm. In other words, when cost curves are sufficiently flat, the fraction paid as profit based commission should be concave in underwriting quality for optimal amounts of sales and underwriting effort to be expended by the BA firm.

This result may seem counter-intuitive at first glance.\textsuperscript{11} However, when we model quality of underwriting as a function of effort so that quality has diminishing returns to effort, the result obtained in (9) does not seem counter-intuitive. The source of the concavity in the commissions schedule lies in the concavity of the quality of underwriting (as a function of the effort).

4. Extended Model: Uncertain Quality of Underwriting

We now extend the model to incorporate uncertainty into the quality determination function, assuming that the contingent payment schedule meets the conditions enumerated above.

Let the quality have a deterministic component which depends upon the underwriting effort and a random component $\tilde{\theta}$, so that $\tilde{q}(\epsilon_s) = q(\epsilon_s) + \tilde{\theta}$. Let the domain of $\tilde{\theta}$ coincide with the domain for effort and let the density function of $\tilde{\theta}$ be given by $f_{\tilde{\theta}}$.\textsuperscript{12} We limit ourselves to the uncertainty inherent in underwriting quality and do not delve deeper into the source of this randomness. However, one among several plausible explanations\textsuperscript{13} for

\textsuperscript{11} From the cursory standpoint of underwriting quality alone, the fraction paid as contingent commission should be convex for aligning incentives of producers with carrier firms.

\textsuperscript{12} With this assumption $E_q \equiv E_{\tilde{\theta}}$.

\textsuperscript{13} Other explanations can be i) a technology constraint for uncovering the “true” quality of underlying risk; ii) lack of experience with the line of business; or iii) costly verification of the true quality of underlying risk exposure. However, we draw upon Regan (1997) and Regan and Tennyson (1996) for the explanation in the text.
this randomness is the asymmetry of information between the BA firm and the client; and our results can be explained in light of this interpretation. Regardless of the source of this randomness, it can be expected that the “riskier” (in the likelihood order sense) the line of business, the “greater” this randomness, and $\tilde{\theta}$ captures this in a 1-1 onto manner.

The expected quality of underwriting is given by $E(\tilde{q}(\epsilon_s)) = q(\epsilon_s) + \mu_\theta$, where $E$ denotes the expectations operator. The expected profit function of the broker agent can now be written as:

$$E(\Pi_{BA}) = F_\theta + f_s p S(\epsilon_s) + E_\theta [f_{\pi_s} (pS(\cdot) - Q(\tilde{q}(\epsilon_s))) - E_\theta [C(\tilde{q}(\epsilon_s))] - C(S(\cdot)) \quad (10)$$

The first order condition w.r.t. $\epsilon_s$ for the function above is now obtained as:

$$\frac{\partial E(\Pi_{BA})}{\partial \epsilon_s} = f_s p + \frac{\partial}{\partial \epsilon_s} E_\theta [f_{\pi_s} (pS(\cdot) - Q(\tilde{q}(\epsilon_s))) - C'(S(\cdot)) = 0 \quad (11)$$

Assuming $f_{\pi_s}$ satisfies the Lipschitz type conditions (Casella and Berger, 2002, pp 70) in both $S$ and $Q(\cdot)$, equation (11) can be rewritten as

$$\frac{\partial E(\Pi_{BA})}{\partial \epsilon_s} = f_s p + E_\theta [p \frac{\partial f_{\pi_s} (\cdot)}{\partial \pi_s}] - C'(S(\cdot)) = 0 \quad (12)$$

Equation (12) is similar to equation (2), with expectations incorporated. We now take the partial derivative of equation (10) w.r.t. $\epsilon_s$ and under assumption of the same Lipschitz type conditions on $C(\tilde{q}(\epsilon_s))$, we get

$$\frac{\partial E(\Pi_{BA})}{\partial \epsilon_s} = -E_\theta \frac{\partial f_{\pi_s}}{\partial \pi_s} \frac{\partial q}{\partial \epsilon_s} - E_\theta \frac{\partial C}{\partial q} \frac{\partial q}{\partial \epsilon_s} = 0 \quad (13)$$

Again, we see that this condition is similar to the one in equation (3) with expectations incorporated. Solving for (12) and (13), we obtain the optimal quantity of sales and
underwriting efforts expended in case of uncertain quality of underwriting (assuming that the second order conditions hold under expectations as before). We denote the optimal sales and underwriting efforts by $e_s^*$ and $e_u^*$ respectively.

We now turn to the comparative static exercise and focus on the changes in optimal underwriting effort $e_u^*$ when there occurs deterioration in $\theta$ to $\tilde{\theta}$. Let the corresponding cumulative distributions functions over the support be $F_\theta$ and $F_{\tilde{\theta}}$. We assume that $F_{\tilde{\theta}}$ first order stochastically dominates $F_\theta$ so that $F_{\tilde{\theta}} \leq F_\theta \forall e_s$. For all non-decreasing functions $U(.)$ this holds iff $E_\theta[U(.)] \geq E_{\tilde{\theta}}[U(.)]$ (Levy, 1992). Let us analyze the difference in the f.o.c for the two risks, $\tilde{\theta}$ and $\tilde{\theta}'$. Taking the difference in (13) for each, we get

$$-E_\theta[\frac{\partial f_\pi}{\partial q} \cdot \frac{\partial q}{\partial e_u}] - E_\theta[\frac{\partial C}{\partial q} \cdot \frac{\partial q}{\partial e_u}] + E_{\tilde{\theta}}[\frac{\partial f_\pi}{\partial q} \cdot \frac{\partial q}{\partial e_u}] + E_{\tilde{\theta}'}[\frac{\partial C}{\partial q} \cdot \frac{\partial q}{\partial e_u}]$$

$$= \frac{\partial f_{\pi}}{\partial q} E_{\tilde{\theta}}(-\frac{\partial Q}{\partial q}) \cdot \frac{\partial q}{\partial e_u} - \frac{\partial f_{\pi}}{\partial q} E_{\tilde{\theta}'}(-\frac{\partial Q}{\partial q}) \cdot \frac{\partial q}{\partial e_u}$$

$$= \frac{\partial f_{\pi}}{\partial q} E_{\tilde{\theta}}(-\frac{\partial Q}{\partial q}) \cdot \frac{\partial q}{\partial e_u} - \frac{\partial f_{\pi}}{\partial q} E_{\tilde{\theta}'}(-\frac{\partial Q}{\partial q}) \cdot \frac{\partial q}{\partial e_u}$$

Note that $Q(.)$ is decreasing and convex by assumption so $-Q(.)$ is increasing (non-decreasing) and concave. Also, since $C(.)$ has constant marginal cost,
Since, \( (-\frac{\partial Q}{\partial q}) \) is a monotonic transformation of \( \tilde{q} \), the FSD property is preserved (see Levy and Wiener, 1998). Consequently, \( E_{\theta}(-\frac{\partial Q}{\partial q}) \geq E_{\theta'}(-\frac{\partial Q}{\partial q}) \). Now, if the fraction paid out as contingent commission is the same at the margin for both \( \tilde{\theta} \) and \( \tilde{\theta}' \), then (16) will equal zero (since it signifies first order conditions) \( \text{iff} \ \frac{\partial q}{\partial \epsilon_u} \text{under} \ \tilde{\theta} \) is less than or equal to \( \frac{\partial q}{\partial \epsilon_u} \text{under} \ \tilde{\theta}' \). This is possible only if the marginal quality under \( \tilde{\theta}' \) is higher than the marginal quality under \( \tilde{\theta} \). Since marginal quality is decreasing in effort, this implies the optimal level of effort exerted under \( \tilde{\theta}' \) (first order deterioration of risk) is less than the optimal level of effort exerted under \( \tilde{\theta} \).

Note that the result above is in terms of marginal quality, and not the level of quality. So if we consider two BA firms, one underwriting line of business with risk \( \tilde{\theta} \) and the other with risk \( \tilde{\theta}' \) (where \( \tilde{\theta} \) dominates \( \tilde{\theta}' \) in the FSD sense) and if both firms are earning equal marginal contingent commissions on profit, then the firm which underwrites the deteriorated risk \( \tilde{\theta}' \) optimally, is the one that delivers higher marginal quality at lower levels of effort. Since this is more likely to be the case if the BA firm is a specialist, we are more likely to find specialist BA firms acting as first underwriters in lines that are “riskier”.

5. Conclusions and Future Research

Before we conclude, something must be said about the model. The model ignores moral hazard and observability issues. Moreover, it looks at BA firms as unconstrained profit
maximizers. While we agree that the model makes simplifying assumptions, nonetheless it serves an important purpose – that of filling a gap in the literature. Moreover, since we obtain our results in terms of curvature of the commission schedule, we feel incorporating unobservable effort will only affect the magnitude of curvature without altering its sign. However, it will add a considerable amount of algebra. Same can be said about adding a constraint to the profit maximization problem of the BA firm.

The paper derives the shape of the contingent commission schedule as a function of the underwriting quality, when the BA agent acts as the “first underwriter” for the insurance company. This design turns out to be concave in underwriting quality. While this seems counterintuitive at first, in reality it is not so. We note that convex commission schedules affect the revenue part of the profit function, while underwriting quality affects the cost part of the profit function. The schedule turns out to be concave because the underwriting quality is concave in effort. We then incorporate uncertainty into the quality of underwriting with the assumption, the riskier the line of business – the more uncertain the quality of underwriting. When we allow for a FSD deterioration of risk, the comparative statics exercise reveals that more uncertain line of business will be underwritten by a producer who can deliver higher marginal quality with lower amounts of effort, whom we call a “specialist”.

Future research will focus on testing the empirical predictions of this model.
Appendix

We suppress the subscript $u$ in $e_u$ for the derivation below.

Expanding equation (6), we get,

$$\frac{\partial^2 \Pi_{BA}}{\partial e^2} = \left\{ \frac{\partial^2 f_{\pi_w}}{\partial \pi_w^2} \frac{\partial \pi_w}{\partial e} \left( \frac{\partial Q}{\partial q} \frac{\partial q}{\partial e} \right) + \frac{\partial f_{\pi_w}}{\partial \pi_w} \left( \frac{\partial^2 Q}{\partial q^2} \frac{\partial q}{\partial e} + \frac{\partial Q}{\partial q} \frac{\partial^2 q}{\partial e^2} \right) \right\}$$

$$- \frac{\partial^2 C}{\partial q^2} \left( \frac{\partial q}{\partial e} \right)^2 - \frac{\partial C}{\partial q} \left( \frac{\partial^2 q}{\partial e^2} \right)$$

(A1)

$\therefore C(q)$ is linear, so the second partial is zero. Again, $\therefore \frac{\partial \pi_w}{\partial e} = -\frac{\partial Q}{\partial q} \frac{\partial q}{\partial e}$, rewriting (A1):

$$\frac{\partial^2 \Pi_{BA}}{\partial e^2} = \frac{\partial^2 f_{\pi_w}}{\partial \pi_w^2} \left( \frac{\partial Q}{\partial q} \frac{\partial q}{\partial e} \right)^2 - \frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial^2 Q}{\partial q^2} \left( \frac{\partial q}{\partial e} \right)^2 - \frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} \frac{\partial^2 q}{\partial e^2} - \frac{\partial C}{\partial q} \left( \frac{\partial^2 q}{\partial e^2} \right)$$

$$= \left( \frac{\partial q}{\partial e} \right)^2 \left[ \frac{\partial^2 f_{\pi_w}}{\partial \pi_w^2} \left( \frac{\partial Q}{\partial q} \right)^2 - \frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial^2 Q}{\partial q^2} \right] + \frac{\partial^2 q}{\partial e^2} \left[ -\frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} - \frac{\partial C}{\partial q} \right]$$

(A2)

$$= \left( \frac{\partial q}{\partial e} \right)^2 \left[ -\frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} \right] + \frac{\partial^2 q}{\partial e^2} \left[ -\frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} - \frac{\partial C}{\partial q} \right]$$

(A3)

Now, (A3) can be rearranged to give equation (7) in the text in the following manner

$$\therefore -\frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} = \frac{\partial f_{\pi_w}}{\partial q}$$

(A4)

$$\therefore \frac{\partial^2 \Pi_{BA}}{\partial e^2} = \left( \frac{\partial q}{\partial e} \right)^2 \left[ -\frac{\partial f_{\pi_w}}{\partial \pi_w} \frac{\partial Q}{\partial q} \right] + \frac{\partial^2 q}{\partial e^2} \frac{\partial}{\partial q} \left[ f_{\pi_w} - C \right].$$

Making repeated use of (A4), we get equation (7) in the text.
References


