Hedging Portfolios of Financial Guarantees *

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July 23, 2007

*We thank Salma Mefteh and participants at the 2007 annual meeting of the European Financial Management Association in Vienna for their helpful comments and suggestions. We acknowledge financial support from the Institut de Finance Mathématique de Montreal (IFM2), the Fonds Conrad Leblanc, the Fonds Québécois de la Recherche sur la Société et la Culture (FQRSC) and the Social Sciences and Humanities Research Council of Canada (SSHRC). This paper All errors are the authors’ sole responsibility.

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Abstract

We propose a framework à la Davis et al. (1993) and Whalley and Wilmott (1997) to study dynamic hedging strategies on portfolios of financial guarantees in the presence of transaction costs. We contrast four dynamic hedging strategies including a utility-based dynamic hedging strategy, in conjunction with using an asset-based index, with the strategy of no hedging. For the proposed utility-based strategy, the portfolio rebalancing is triggered by the tradeoff between transaction costs and utility gains. Overall, using a Froot and Stein (1998) and Perold (2005) type of risk-adjusted performance measurement metric, we find the utility-based strategy to be a good compromise between the delta hedging strategy and the passive stance of doing nothing. This result is even stronger with higher transaction costs. However, if the insured firms assets are not traded or in a high transaction costs environment, the guarantor can use an index-based security as hedging instrument.

JEL classification code: G11, G13, G22.
Keywords: Financial guarantee, Credit insurance, Dynamic hedging, Portfolio replication.
1 Introduction

Enterprise risk management is nowadays a must for all corporations especially financial institutions. In this article, we use a risk management framework à la Merton and Perold (1993) and Froot and Stein (1998) to study hedging strategies by financial guarantee providers who hold invariably portfolios composed of several financial guarantee contracts. For instance, firms in the financial services industry can diversify away the systematic risk and/or insure (reinsure), hedge, retain (e.g., Bodie and Merton (1999)), and undertake alternative risk transfer (e.g., Banks (2004)). However, these risk management strategies cannot be implemented at no cost and perfectly. Further, in the domain of hedging a portfolio of financial guarantees, which is the focus of our study, it is widely recognized that the portfolio credit risk cannot be completely diversified away. Therefore, financial guarantee providers need to find strategies to enhance their risk-adjusted returns.

Following Leland (1985), there is a significant literature on hedging derivatives with the same underlying asset and with transaction costs. Davis et al. (1993) propose a utility-based hedging model with negative exponential utility function. Later, Whalley and Wilmott (1997) use an asymptotic approach to hedge a call option. Alternatively, Edirisinghe et al. (1993) propose a dynamic programming approach with binomial trees to hedge an option with transaction costs. To overcome huge transaction costs associated with dynamic hedging, Derman et al. (1994) and Carr and Gupta (1998) propose static hedging models. Unfortunately, to our knowledge, there are no studies on hedging portfolios of financial guarantees with several underlying assets in the presence of transaction costs, which is the contribution of this paper.\footnote{There is a substantial literature on dynamic hedging and replication of derivatives under transaction costs both in discrete and continuous time, e.g., Avellaneda and Paras (1994), Boyle and Vorst (1992), Clewlow and Hodges (1997), Zakamouline (2005) among others.}

There are limitations with the existing models. Indeed, in many real life situations, the portfolio to be hedged contains several underlying securities, hence many sources of risk to hedge. Moreover, the portfolio manager has to rebalance the portfolio repeatedly. On the one hand, the dynamic hedging methodology of Hodges and Neuberger (1989),
Davis et al. (1993) and Whalley and Wilmott (1997) is less appropriate since the introduction of other sources of risk makes the problem more complex and no analytical solution can be derived. The approach of Edirisinghe et al. (1993) becomes cumbersome since the calculation time evolves exponentially with the number of rebalancing points and multiple risks. One the other hand, the static hedging approach of Derman et al. (1994) and Carr and Gupta (1998) which requires many replicating instruments with specific characteristics that may not be available in the market makes the hedging much less efficient.

In this paper, we study relatively simple hedging strategies by an insurer holding a portfolio composed of multiple financial guarantee contracts. Albeit complex, the hedging exercise is illustrated in this paper by ways of a portfolio of two financial guarantees, many rebalancing dates and non-zero transaction costs. Ideally, the guarantor will use the underlying assets to hedge. However, in some cases, either the underlying assets are not available for trade in the market (for example non-publicly available firms where equities are not traded) or it can be too costly to trade all the required underlyings. In these situations, one may prefer to use a sector-based index instrument for hedging. Naturally, the guarantor gains by using an index instrument for cross-hedging closely related to his activities or highly correlated to his portfolio. As a matter of fact, Ramaswami (1991) and Ramaswamy (2002, 2005) among others exploit the insight that when the put is in-the-money, it behaves as equity, then hedging the default risk of the bond is tantamount to hedging equity risk. Therefore, we consider five strategies: (i) the doing nothing strategy, (ii) the dynamic delta hedging (consisting of creating an option position synthetically) and (iii) utility-based hedging strategies using the underlying assets, (iv) the dynamic delta hedging and (v) utility-based hedging strategies using a security-based index as alternative hedging instrument.

In the spirit of Merton and Perold (1993), Froot and Stein (1998), and Perold (2005), to compare the performance of our five strategies, we use the relatively modern performance metric, the so-called risk-adjusted performance measure or RAPM, which is defined as the ratio of the portfolio expected return over its value at risk (VaR). To
better apprehend the impact of the parameters on our hedging strategies, we focus our numerical exercises on a portfolio composed of two financial guarantees. Overall, based on our parameters values, we found the utility-based hedging with the underlying assets to be a better compromise between the delta hedging strategy and the passive stance of doing nothing. This result remains stronger even with higher transaction costs. However, if the insured firms assets are not traded or in a high transaction costs environment, the guarantor can use an index-based security as hedging instrument.

Institutionally, managing and hedging portfolios of financial guarantees require the guarantor to set reserves and economic capital. Setting a risk-based capital or capital at risk allows us to capture the changes in the capital allocation associated with the hedging decisions. By doing so, we capture the portfolio diversification feature and price the risk associated with the tails of the distribution inherent to credit risk. Unlike Smith and Stulz (1985) and Morellec and Smith (2006), the focus of our paper is to study hedging strategies of portfolios of financial guarantees. However, the implications of our study are consistent with their assertion that hedging can increase firm value.

The rest of the paper is structured as follows. In section 2, we present the model. In section 3, we discuss the dynamic hedging strategies. In section 4, we present the simulation parameters and discuss the results. Section 5 concludes.

2 General model

Before presenting the general model of the multiple-asset risk sources case, we present a one risky asset portfolio case to capture the essence of the hedging problem.\(^4\)

\(^3\)See “Moody’s Portfolio Risk Model for Financial Guarantors: Special Comment”, Moody’s Investors Services, Global Credit Research, July 2000, by R. Cantor, J. Dorer, L. Levenstein and S. Qian.

\(^4\)Note that among other approaches to derive the optimal hedging strategies, there are (i) the mean-variance framework which minimizes the portfolio total variance for a given expected return (e.g., Ederington (1979), Schweizer (1992)), (ii) the tracking error approach mimics the optimal portfolio with lower transaction costs (e.g., Jamshidian and Zhu (1990), Edirisinghe et al. (1993)), (iii) the utility-based framework, where the rebalancing decision and the optimal allocations are chosen based on the utility maximization (e.g., Davis et al. (1993), Avellaneda and Paras (1994)). As stated before, our approach is in the spirit of the utility-based framework.
2.1 The single underlying asset portfolio case

To gain the insight of our paper, we first start by providing the model with only one underlying asset. We present a utility-based dynamic hedging model to replicate a single option with one underlying asset.

We consider a guaranteed risky firm which asset, $S_t$, process is described as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where $W_t$ is a standard Brownian motion, and $\mu$ the drift and $\sigma$ the returns’ volatility are constant. We also consider an riskless bond, $B_t$, with process

$$dB_t = r B_t dt,$$

where $r$ is the constant risk-free rate.

We define $y_t$ the quantity of risky asset and $B_t$ the amount of risk-free asset held by the guarantor. The transaction costs are assumed to be proportional to the value of the asset. We use $\theta$ to designate the proportion of transaction costs. The function $L$ represents the value of the risky investment:

$$L(y_t, S_t) = \begin{cases} 
(1 + \theta) y_t S_t, & \text{if } y_t < 0 \\
(1 - \theta) y_t S_t, & \text{if } y_t > 0.
\end{cases}$$

We want to maximize the utility function of the guarantor with respect to the cash-flows he will receive or pay at the maturity $T$ of the guarantee. Let’s assume that the guarantor has underwritten a guarantee contract on the firm’s total debt $K$ (equivalent to a short put on the firm asset). Thus, we can define the net-wealth function of the guarantor as follows:

$$\Phi(T, B_T, y_T, S_T) = B_T + 1_{\{S_T < K\}}[L(y_T + 1, S_T) - K] + 1_{\{S_T \geq K\}}L(y_T, S_T).$$

The indirect utility function is defined as the maximum expected utility of the guarantor with respect to the hedging strategies

$$V(B) = \sup_{\psi \in \Omega(B)} \mathbb{E}[U(\Phi(T, B_T, y_T, S_T))],$$

where $\Omega(B)$ represents the set of possible strategies for a guarantor endowed with $B$ amount and $\psi$ is a strategy.
The optimal hedging strategy is obtained by solving a dynamic programming problem with the following indirect utility function

\[
V(t, y_t, B_t, S_t) = \max_m \left\{ \max V(t, y_t + m\delta, B_t - (1 + \theta)m\delta S_t, S_t), \right.
\]
\[
\left. \max V(t, y_t - m\delta, B_t + (1 - \theta)m\delta S_t, S_t), \right. \left. \right\}
\]
\[
\mathbb{E}[V(t + \Delta t, B_t \exp(r\Delta t), S_t(1 + \tilde{r}_S))],
\]

where the maximum is done with respect to \( m \) with values in \{0, 1, 2, \ldots, \infty\}, \( \delta \) represents the portion of the asset that can be traded and \( \tilde{r}_S \) is the movement coefficient of the stock price. For example, \( \tilde{r}_S \) can be assumed to be binomial, i.e.,

\[
\tilde{r}_S = \begin{cases} 
  u & \text{with probability } p \\
  -d & \text{with probability } 1 - p 
\end{cases}, \quad \text{with } d, u > 0.
\]

The two terms of the utility function \( V \) are

\[
V(t, y_t + m\delta, B_t - (1 + \theta)m\delta S_t, S_t) = 
\mathbb{E}[V(t + \Delta t, y_t + m\delta, (B_t - (1 + \theta)m\delta S_t) \exp(r\Delta t), S_t(1 + \tilde{r}_S))]
\]

and

\[
V(t, y_t - m\delta, B_t + (1 - \theta)m\delta S_t, S_t) = 
\mathbb{E}[V(t + \Delta t, y_t - m\delta, (B_t + (1 - \theta)m\delta S_t) \exp(r\Delta t), S_t(1 + \tilde{r}_S))].
\]

As in Edirisinghe et al. (1993), using a Markov chain decomposition, we need to compute several expectations. For each \( y \), we need to compute the maximum value of the utility function with respect to all the strategies available to the guarantor.

For illustrative purpose, Figure 1 presents three trade regions for the replication of a put option. This figure has been obtained using a similar model as in Whalley and Wilmott (1997) who solve analytically the problem for a European call option. They provide asymptotic approximations of the Bellman-Jacobi equation of Davis et al. (1993) by characterizing the optimal hedge and the trade regions. In the top region, it is optimal to sell the underlying asset. In the bottom region, it is optimal to buy the underlying asset. And in the middle region, it is optimal to not trade because of the transaction costs.
2.2 The multiple-underlying assets portfolio case

Although, guarantors manage portfolios of more than two underlying assets, to address the main focus of the paper, we study a portfolio with two underlying securities. The simulation results obtained with two underlying assets, which is already complex, convey the main message of the paper without loss of insight.

We consider a riskless asset $B_t$ with process

$$dB_t = rB_t dt,$$

(9)

where $r$ is the constant risk-free interest rate, and two risky securities $S_{1,t}$ and $S_{2,t}$ representing the assets of two client firms. The processes of the two firms assets are

$$dS_{i,t} = \mu_i S_{i,t}dt + \sigma_i S_{i,t}dW_{i,t}, \quad i = 1, 2,$$

(10)

where the constants $\mu_i$ and $\sigma_i$ are the instantaneous returns and returns’ volatilities of the firms assets.

The guarantor underwrites separate guarantee contract with each client firm. Thus, each firm holds a put option written by the guarantor with exercise price the face value of its debt $K_i$. One special feature of the guarantee business is that guarantors usually hold portfolios composed of insured firms operating in the same industrial sector or having some common characteristics. Therefore, we assume the existence an index-based security $I_t$ which can be the index of the industry, and its process is given by

$$dI_t = \mu_I I_t dt + \sigma_I I_t dW_{I,t},$$

(11)

where the constants $\mu_I$ and $\sigma_I$ are the instantaneous return and returns’ volatility of the industry index. We also consider a market index $M_t$ with dynamics given by

$$dM_t = \mu_M M_t dt + \sigma_M M_t dW_{M,t},$$

(12)

where the constants $\mu_M$ and $\sigma_M$ are the instantaneous return and returns’ volatility of the market index.

The above securities returns are correlated through their Brownian motions $dW_{i,t}$ as follows: $\rho_{i,j} = corr(dW_{i,t}, dW_{j,t})$, where $i$ and $j$ designated securities $i$ and $j$. 
As stipulated earlier, the guarantor underwrites two put options to the client firms with initial values \( P_{1,0} \) and \( P_{2,0} \). Hence, its portfolio value is

\[
P_t = P_{1,t} + P_{2,t}.
\]

(13)

Applying Ito’s lemma to this expression yields

\[
dP_t = \left( \frac{\partial P_{1,t}}{\partial t} + \frac{\partial P_{1,t}}{\partial S_{1,t}} S_{1,t} \mu_1 + \frac{1}{2} \frac{\partial^2 P_{1,t}}{\partial S_{1,t}^2} \sigma_1^2 S_{1,t}^2 \right) dt + \frac{\partial P_{1,t}}{\partial S_{1,t}} \sigma_1 S_{1,t} dW_{1,t}
\]

\[
+ \left( \frac{\partial P_{2,t}}{\partial t} + \frac{\partial P_{2,t}}{\partial S_{2,t}} S_{2,t} \mu_2 + \frac{1}{2} \frac{\partial^2 P_{2,t}}{\partial S_{2,t}^2} \sigma_2^2 S_{2,t}^2 \right) dt + \frac{\partial P_{2,t}}{\partial S_{2,t}} \sigma_2 S_{2,t} dW_{2,t}
\]

\[
= \mu_P P_t dt + \frac{\partial P_{1,t}}{\partial S_{1,t}} \sigma_1 S_{1,t} dW_{1,t} + \frac{\partial P_{2,t}}{\partial S_{2,t}} \sigma_2 S_{2,t} dW_{2,t},
\]

(14)

where \( \mu_P \) is the drift of the portfolio returns obtained by summing the terms before the \( dt \) and dividing the sum by \( P_t \). This equation highlights explicitly the exposition of the returns to the underlying risk sources \( dW_{1,t} \) and \( dW_{2,t} \).

The dynamic delta hedging strategy of the portfolio consists of trading \( \frac{\partial P_{1,t}}{\partial S_{1,t}} \) of \( S_{1,t} \) and \( \frac{\partial P_{2,t}}{\partial S_{2,t}} \) of \( S_{2,t} \). Abstracting from the transaction costs, this means that we need to hold delta quantity of each asset in order to delta-hedge.\(^5\)

In some cases, either the underlying assets are not available for trade in the market (for example non-publicly available firms where equities are not traded) or it can be too costly to trade all the required underlying. In these situations, one may prefer to use a sector-based index instrument for hedging. As a matter of fact, Ramaswami (1991) and Ramaswamy (2002, 2005) among others exploit the insight that when the put is in-the-money, the put behaves as equity, then hedging the default risk of the bond is tantamount to hedging equity risk. Naturally, the guarantor gains by hedging using an index instrument closely related to his activities or highly correlated to his portfolio. To see that, let’s decompose the underlying risk sources as follows:

\[
dW_{i,t} = \rho_{i,t} dW_{i,t} + \sqrt{1 - \rho_{i,t}^2} dZ_{i,t}, \quad i = 1, 2,
\]

(15)

\(^5\)In incomplete markets, under stochastic volatility and/or stochastic risk-free interest rate, one may need to use the other Greeks of the option for hedging such as the gamma-hedging and the number of asset required for the replication increases, which we left for further study.
where \( \{dZ_i,t, \ i = 1,2\} \) are two independent Brownian motions independent from \( dW_{I,t} \).

The exposure of the portfolio to the index risk \( dW_{I,t} \) is given by

\[
\frac{\partial P_{1,t}}{\partial S_{1,t}} S_{1,t} \sigma_{11} + \frac{\partial P_{2,t}}{\partial S_{2,t}} S_{2,t} \sigma_{22}.
\]

(16)

Comparing this expression with the dynamic of the index given in equation (11), the guarantor needs to trade the following amount of the index

\[
\frac{\frac{\partial P_{1,t}}{\partial S_{1,t}} S_{1,t} \sigma_{11} + \frac{\partial P_{2,t}}{\partial S_{2,t}} S_{2,t} \sigma_{22}}{I_t \sigma_I}.
\]

(17)

Doing so, he benefits from the correlation between the portfolio and the index.

The next section presents the hedging strategies used to manage the portfolio of guarantees.

3 Hedging strategies

We present below the replication strategies used to hedge the portfolio composed of two guarantee contracts. Hereafter, we use interchangeably replication or hedging or rebalancing to designate the same action. We consider 24 rebalancing dates over the year, i.e., twice per month, and the time step is denoted by \( \Delta t \). Let’s denote by \( E_{L_i,t} \) the expected loss by the insured firm \( i \) at time \( t \), \( U_{L_i,t} \) its unexpected loss (set at 5% confidence level for purpose) corresponding to the value at risk, \( \text{VaR}_{i,t} \).

At the signature of the two guarantee contracts, the guarantor charges the following premium to the client firm \( i \)

\[
PREM_i = (1 + \varepsilon_i) \times E_{L_i,0} + H_i \times U_{L_i,0},
\]

(18)

where \( \varepsilon_i \) is a loading coefficient capturing all market imperfections and \( H_i \) represents the hurdle rate for the guarantee contract \( i \) as suggested by Marrison (2002) who analyses project finance guarantee portfolios. The total premium raised by the guarantor from the firms is

\[
\sum_i PREM_i = \sum_i ( (1 + \varepsilon_i) \times E_{L_i,0} + H_i \times U_{L_i,0}),
\]

(19)

We assume that \( \sum_i \varepsilon_i \times E_{L_i,0} \) is used to cover the current operating expenses and other fees related to the signature of the guarantee contracts. Thus, the \( \varepsilon_i \) are chosen in order to break even these fees.
Following the practice of capital at risk, in addition to the portfolio total expected loss, $EL_0 = \sum_i EL_{i,0}$, the guarantor has to set aside economic capital equals to the total unexpected loss of the portfolio $UL_0$. Therefore, the guarantor’s shareholders provide $UL_0 - \sum_i H_i \times UL_{i,0}$ to raise the economic capital level to $UL_0$. In sum, the guarantor collects the two premiums and manages its guarantee portfolio up to the maturity of the guarantees. The total amount available for investment is then $EL_0 + UL_0$.

Our set-up assumes that the guarantor invests the portfolio total expected loss amount raised from the firms, $EL_0$, in a reserve account earning the risk-free interest rate $r$ to cover future expected losses. The rest of the premium, the capital at risk $UL_0$, is invested at the cost of capital, $r_G$, given as follows

$$r_Gdt = rdt + \beta_{GM}((\mu_M - r)dt + \sigma_M dW_M). \quad (20)$$

This is the ICAPM (Intertemporal Capital Asset Pricing Model) type cost of capital and asserts that the cost of capital is equal to the risk-free rate plus the guarantor’s beta times the market excess return.

At each rebalancing date, the guarantor reevaluates the current value of the portfolio expected loss $EL_t = EL_{1,t} + EL_{2,t}$ and replenishes or reduces its reserve account balance accordingly. Therefore, the guarantor’s reserve account balance is set equal to the portfolio total expected losses and is assumed to be invested in the risk-free bond. The rest of its wealth is invested at the rate $r_G$. This purports to reflect sound risk management practice through the use of capital at risk to cover unexpected losses.

Next, we present the passive strategy and four hedging strategies. The passive strategy of doing no hedging is the benchmark.

### 3.1 Strategy 0: The passive strategy of doing no hedging

The first strategy called the passive strategy consists of not hedging at all. However, both the reserve and risky accounts are reshuffled in order to maintain the reserve account to the level of the total expected loss.
3.2 Strategy 1: The dynamic delta hedging using the underlying assets

This strategy called delta hedging consists of performing the delta replication at the rebalancing dates using the insured firms assets. We denote by \( \Delta_{i,t} \) the delta of stock \( i \) at time \( t \). Here, we compute the delta of the portfolio and make the required trades on the underlying assets with transaction costs to obtain the hedged portfolio. We assume the transaction costs to be proportional to the trading amount, and the proportion coefficient \( \theta \) is the same when buying or selling the securities. For example, at \( t + \Delta t \), the transaction costs on trading stock \( i \) are

\[
\theta \times |\Delta_{i,t+\Delta t} - \Delta_{i,t}| \times S_{i,t+\Delta t}.
\]

(21)

3.3 Strategy 2: The utility-based dynamic hedging using the underlying assets

This strategy called utility-based hedging consists of using the utility maximization to determine the appropriate hedging dates and uses the underlying assets as hedging instruments. At each potential rebalancing date, the guarantor must decide to rebalance or not it portfolio fully. In other words in this framework it is possible to have no replication at all at some dates.

The rebalancing decision is based on the following indirect utility function

\[
V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t}\left[U(\Phi(T, B_{t+\Delta t}e^{\tau T}, \Delta_{1,t+\Delta t}, S_{1,T}, \Delta_{2,t+\Delta t}, S_{2,T}, B_{G_{t+\Delta t}} e^{\tau r_G}))\right],
\]

(22)

where \( T \) represents the same maturity of the two individual guarantee contracts, \( \tau = T - t - \Delta t \) is the time to maturity and the function \( \Phi(.) \) is a function of our underlying two state variables among others. \( B_{t+\Delta t} \) is the reserve account balance at time \( t + \Delta t \) multiplied by the compound factor to obtain its time \( T \) value, \( \Delta_{i,t+\Delta t} \) is the number of stock \( i \) held, \( S_{i,t+\Delta t} \) is the price of stock \( i \), and \( B_{G_{t+\Delta t}} \) represents the amount invested in the risky account (earning the rate of return \( r_G \)) multiplied by the corresponding compound factor.

Moreover, this is a self financing exercise because, at each rebalancing date, the total investment in the risk-free and risky accounts are equal to the previous time total
investment value minus the total transaction costs. The reserve account at \( t + \Delta t \) is

\[
B_{t+\Delta t} = EL_{1,t+\Delta t} + EL_{2,t+\Delta t}
\]  

(23)

and the sum of the reserve account \( B_{t+\Delta t} \) and the risky account \( B_{t+\Delta t}^G \) is

\[
B_{t+\Delta t} + B_{t+\Delta t}^G = B_t e^{r\Delta t} + B_t^G e^{r^G \Delta t} \\
- (\Delta_{1,t+\Delta t} - \Delta_{1,t}) S_{1,t+\Delta t} - \theta |\Delta_{1,t+\Delta t} - \Delta_{1,t}| S_{1,t+\Delta t} \\
- (\Delta_{2,t+\Delta t} - \Delta_{2,t}) S_{2,t+\Delta t} - \theta |\Delta_{2,t+\Delta t} - \Delta_{2,t}| S_{2,t+\Delta t}.
\]  

(24)

As stated above, the guarantor does not need to trade necessarily in the two stocks simultaneously, the decision to trade one or both underlying assets will be based on the indirect utility function.

Note that, in a single risky asset environment, as in Edirisinghe et al. (1993), the indirect utility function can be computed relatively easy using binomial trees. For two risky assets case where the assets correlation matters, it is important to look at all the possibilities, i.e., buying and selling portions of the two assets. The computation time of the dynamic programming approach in this case is too long and inefficient. This is why we follow the simulation approach.

In this utility-based dynamic hedging strategy, at each rebalancing date, the guarantor weighs the following four possible exclusive choices using the expected utility maximization:

- **Choice 1** - Delta-replicate the portfolio using only stock 1, or
- **Choice 2** - Delta-replicate the portfolio using only stock 2, or
- **Choice 3** - Delta-replicate the portfolio using both stocks simultaneously, or
- **Choice 4** - Do not hedge the portfolio.

For **Choice 1**, the decision function in equation (22) is simplified as follows

\[
V_{t+\Delta t} = \max E_{t+\Delta t} \left[ U \left( \Phi(T, B_{t+\Delta t} e^{r^G \Delta t}, \Delta_{1,t+\Delta t}, S_{1,t}, \Delta_{2,t}, S_{2,t}, B_{t+\Delta t}^G e^{r^G \Delta t}) \right) \right],
\]  

(25)

and the total investment in equation (24) becomes

\[
B_{t+\Delta t} + B_{t+\Delta t}^G = B_t e^{r\Delta t} + B_t^G e^{r^G \Delta t} \\
- (\Delta_{1,t+\Delta t} - \Delta_{1,t}) S_{1,t+\Delta t} - \theta |\Delta_{1,t+\Delta t} - \Delta_{1,t}| S_{1,t+\Delta t}.
\]  

(26)
For Choice 2, the equations are similar to the ones of Choice 1, except that stock 1 is replaced by stock 2. For Choice 3, the decision function and the sum of the investment accounts are given respectively by equations (22) and (24). Finally, for Choice 4 when there is no trade, the decision function is

\[
V_{t+\Delta t} = \max E_{t+\Delta t} \left[ U \left( \Phi(T, B_{t+\Delta t} e^{r \tau}, \Delta_1, \Delta_2, S_1, S_2, B_{t+\Delta t}^G e^{r \tau}) \right) \right], \tag{27}
\]

and the investment account value is

\[
B_{t+\Delta t} + B_{t+\Delta t}^G = B_t e^{r \Delta t} + B_t^G e^{r \tau \Delta t}. \tag{28}
\]

Comparing these four decision functions, the guarantor decides what transactions to undertake at time \( t + \Delta t \) in order to maximize his expected utility with the portfolio held at that time.

In the above description, we have introduced two dynamic hedging strategies (Strategy 1 and Strategy 2) using the portfolio underlying assets. However, sometimes it could be too costly and/or impractical (e.g., not traded, overly illiquid, institutional constraints, etc.) to replicate the portfolio using the underlying assets. One may then resort to use an security-based index hedging instrument such as the sector index \( I_t \). As indicated earlier, in the financial guarantee business, it is common to see a guarantor specializing in particular industries. For that purpose, next, we introduce two additional strategies using an index hedging instrument.

### 3.4 Strategy 3: The dynamic delta hedging using an index instrument

This strategy consists of using the index to replicate the portfolio. It is similar in spirit to the hedging Strategy 1. At each rebalancing date, the industry risk is completely eliminated by the delta replication, but the residual (if any) firms idiosyncratic risks remain. Intuitively, this strategy can be attractive compared to Strategy 1 since less replication costs are required to delta-hedge using the sector index hedge instrument. However, the guarantor’s portfolio risk may be higher since we do not hedge completely the total risk of the portfolio unless the portfolio is perfectly correlated with the index.
3.5 Strategy 4: The utility-based dynamic hedging using an index instrument

This strategy uses the utility-based hedging but with the index as hedging instrument. In this strategy, there are only two possible exclusive choices:

Choice 1- Delta-replicate the portfolio using the index, or
Choice 2- Do not hedge the portfolio.

We then have to compare only two decision functions. For Choice 1, the decision function is

\[ V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t} \left[ U \left( \Phi(T, B_{t+\Delta t} e^{r^G \tau}, \Delta_{I,t+\Delta t}, I_T, B_{t+\Delta t}^G e^{r^G \tau}) \right) \right], \tag{29} \]

where \( \Delta_{I,t+\Delta t} \) is the number of index held, and the total investment is

\[ B_{t+\Delta t} + B^G_{t+\Delta t} = B_t e^{r^G \Delta t} + B^G_t e^{r^G \tau} - (\Delta_{I,t+\Delta t} - \Delta_{I,t}) I_{t+\Delta t} - \theta |\Delta_{I,t+\Delta t} - \Delta_{I,t}| I_{t+\Delta t}. \tag{30} \]

This represents the guarantor’s investment in the reserve and the risky accounts minus the transaction costs. For Choice 2, the decision function is

\[ V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t} [ U(\Phi(T, B_{t+\Delta t} e^{r^G \tau}, \Delta_{I,t}, I_T, B_{t+\Delta t}^G e^{r^G \tau})) ]; \tag{31} \]

and the total investment is

\[ B_{t+\Delta t} + B^G_{t+\Delta t} = B_t e^{r^G \Delta t} + B^G_t e^{r^G \tau}. \tag{32} \]

At each rebalancing date, the guarantor delta-replicates or not the portfolio based on the decision functions values.

3.6 Computing the RAPM

We now need to compute the risk-adjusted performance measurement (RAPM) of the guarantor. As mentioned in the introduction, rather than benchmarking as done in the portfolio performance measurement literature which requires the construction of a proper benchmark portfolio, here we simply compare different strategies using the RAPM metric. To do that, we proceed as follows. The proceeds of the guarantee net of the
loading fees, which are $\sum_i \epsilon_i \times EL_i$, is\(^6\)

\[ \sum_i (EL_{i,0} + H_i \times UL_{i,0}). \]  

(33)

Since the guarantor is short of two puts, we have the following payoff

\[ -\sum_i P_i. \]  

(34)

This payoff is equal to the sum of the two expected losses: $-\sum_i EL_{i,0}$. Combining equations (33) and (34) gives the net value

\[ \sum_i H_i \times UL_{i,0}. \]  

(35)

At the maturity of the guarantee contracts, the net gain to the guarantor is given by the total investment value (reserves and risky accounts) plus the value of the guarantee portfolio minus the realized guarantee payments made. Since the guarantor’s shareholders initial capital contribution is $UL_0 - \sum_i H_i \times UL_{i,0}$, we can compute the return as follows

\[
R = \frac{\text{Guarantee portfolio value} + \text{Investment value} - \text{Realized guarantee payments}}{UL_0 - \sum_i H_i \times UL_{i,0}} - 1. 
\]

(36)

Assuming $H_i = H$ for all $i$ and $UL_0 = \sum_i UL_{i,0}$ (e.g., perfect correlation between firms), the above expression of $R$ can be approximated as follows

\[
R \approx \frac{(1 + r)(UL_0 + EL_0) - (1 + r) \sum_i EL_{i,0}}{(1 - H)UL_0} - 1 = \frac{(1 + r)UL_0}{(1 - H)UL_0} - 1 = \frac{1 + r}{1 - H} - 1. 
\]

For example, using the following parameters values, $r = 0.05$ and $H = 0.10$, we obtain a return $R = 17\%$, which is near the average return found in our numerical experiments given below. Note that the return would be higher if the firms were not perfectly correlated since $UL_0 < \sum_i UL_{i,0}$.

To obtain the risk-adjusted performance measure, RAPM, we compute the value at risk, VaR, of the returns for each strategy and the RAPM is defined as follows:

\[
RAPM = \frac{R}{VaR}. 
\]

(37)

\(^6\)Stated otherwise, the $\epsilon_i$ are chosen in order to break even these fees. For comparative purpose between the strategies, in the simulations section below, we assumes $\epsilon_i = 0$. However, while positive values of $\epsilon_i$ will render the insurer more viable, it will not affect our qualitative results.
As discussed earlier, the numerator is the portfolio return and the denominator is our chosen risk metric namely the value at risk rather than the traditional standard deviation of returns. Note that, if we use as risk metrics the standard deviation (respectively the semi variance), we would have obtained roughly the Sharpe ratio (respectively the Sortino ratio).

In the next section, we run several simulations and report the returns obtained.

4 Simulations

As in Pellizzari (2005), we will run numerical simulations to obtain our results, however, we differs from this paper since its focus is on static hedging, while we conduct dynamic hedging.

4.1 Simulating the trading regions

Figure 2 plots the trading regions for negative exponential and power utility-based hedging and Figure 3 plots the trading regions for negative and positive correlations between the insured firms under the exponential utility-based dynamic hedging regime. To generate the graphs of these figures, we set exogenously the number of the two shares held: $\Delta_1 = -0.2316$ of stock and $\Delta_2 = -0.1737$ of stock 2, and we search for the no-trade regions corresponding to the white regions in the graphs. As expected, in the utility-based dynamic hedging, it is not always optimal to trade and the optimal decision depends not only on the utility function specification but also the parameters values.

4.2 Simulation results with positive correlation between the insured firms

In this section, we implement the strategies described above. The baseline parameters values used for the simulations are: $\sigma_I = 0.3$, $\sigma_1 = 0.25$, $\sigma_2 = 0.4$, and $\sigma_M = 0.15$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.1$ the guarantor’s beta with the market, $H_1 = H_2 = 0.1$ the hurdle rates, $K_1 = K_2 = 100$ the
firms debt face values. We use the following negative exponential utility function for the guarantor

\[ U(x) = -e^{-\lambda x}, \]  

with risk aversion coefficient \( \lambda = 1/100 \). The negative exponential utility function exhibits the feature of constant absolute risk aversion and is widely used for its simplicity.

We assume the following positive correlations between the securities returns: \( \rho_{I,1} = 0.5, \rho_{I,2} = 0.7, \rho_{I,M} = 0.3, \rho_{1,M} = 0.35, \rho_{2,M} = 0.25, \rho_{1,2} = 0.5 \). Since we are using the risk neutral probabilities, the drift of the securities returns will be equal to the risk free rate \( r = 0.05 \).

In our framework, we assume the same transaction costs structure as in Leland (1985), therefore Black and Scholes (1973) formula hold provided we use the modified volatility, \( \sigma^*_i \), of the hedging instruments derived by Leland (1985). For long call and put positions, the modified volatility is

\[ \sigma^*_i = \sigma_i \left(1 - \sqrt{\frac{8}{\pi \Delta t}} \frac{\theta}{\sigma_i} \right)^{1/2}, \]  

and for short call and put positions, it is

\[ \sigma^*_i = \sigma_i \left(1 + \sqrt{\frac{8}{\pi \Delta t}} \frac{\theta}{\sigma_i} \right)^{1/2}. \]  

We run 10 000 simulations (including 5000 antithetic variables) using the risk neutral probabilities to obtain the returns distributions of each of the following strategies described above:

- **Strategy 0**: No hedging,
- **Strategy 1**: Dynamic delta hedging with the underlying assets,
- **Strategy 2**: Utility-based dynamic hedging with the underlying assets,
- **Strategy 3**: Dynamic delta hedging with a security-based index hedging instrument,
- **Strategy 4**: Utility-based dynamic hedging with a security-based index hedging instrument.

Table 1 presents the expected returns, \( R \), the value at risk, VaR, and the risk-adjusted performance measure, RAPM, of the five strategies. From Panel 1 of Table 1 with
transaction costs proportion $\theta = 0.50\%$, comparing Strategies 0, 1, and 2, we observe that Strategy 2 performs better than the other strategies. This means that the utility-based hedging strategy with the underlying assets is better than doing nothing or delta-replicating. In the case of the use of a security-based index hedging instrument, we observe that the utility-based hedging strategy with the index (Strategies 4) is better than the simple dynamic delta hedging (Strategy 3).

From Panel 2 of Table 1, we observe changes in the strategies returns when the transaction costs proportion doubles. We observe that the delta hedging strategy produces on average lower returns with higher transaction costs. Compared to the hedging strategies using the underlying assets, delta-replicating with the index produces higher absolute returns, which is expected since less transaction costs incur when transacting with the index.

In this positive correlation scenario, the utility-based hedging strategy produces on average better results since positive correlation increases the future cash flows of the portfolio. Thus, even with high transaction costs, replication can result in increased utility relative to the passive strategy of doing nothing.

The graphs in Figure 4 show the distribution of the strategies returns. For all the strategies, we observe the skewness in the portfolio distribution, with more skewness in Strategy 0 relative to the other strategies (hedging strategies). Among the hedging strategies, the skewness is less in the delta hedging strategies than in the utility-based strategies. Intuitively, the utility-based strategy produces two simultaneous effects: the reduction of the portfolio risk from hedging and the gains in return from the skewness, hence a combination of the no hedging strategy and the delta-replication.

4.3 Simulation results with negative correlation between the insured firms

We use the same baseline parameters values except for the securities correlations. Here, we assume a negative correlation between the insured firms 1 and 2: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.5$, $\rho_{1,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = -0.2$. This can happen for example if the firms do not belong to the same industry. As in the previous case, we run our simulation
under risk neutral probabilities.

Table 2 presents the results for the five strategies. From Panel 1 of Table 2 with transaction costs proportion $\theta = 0.50\%$, we observe that the worst RAPM are obtained with the simple dynamic delta hedging. Although, this strategy has the lowest risks, its returns are too small. As in the positive correlation case discussed above, the best strategy is the utility-based dynamic hedging.

From Panel 2 of Table 2 with transaction costs proportion $\theta = 1.00\%$, with regard to the distribution of returns, the same comments can be made as discussed in Table 1, i.e., the simple dynamic delta hedging strategy produces much lower returns with higher transaction costs. Again, compared to the hedging strategies using the underlying assets, delta-replicating with the index produces higher absolute returns. Nonetheless, with the utility-based hedging strategy, it is better to hedge using the index when the transaction costs proportion is high.

The strategies returns distributions are given by the graphs of Figure 5. We observe the same trend as the one discussed in the case of Figure 4.

### 4.4 Impact of the utility specification on the utility-based dynamic hedging strategy

Here, we study the sensitivity of the utility-based hedging results with respect to the following utility functions specification: negative exponential utility, $U(x) = -e^{-\lambda x}$, with $\lambda = 1/100$ (low absolute risk aversion) or $1/10$ (high absolute risk aversion), power utility, $U(x) = \frac{(x + 50)^{1-\gamma}}{1-\gamma}$, with $\gamma = 2$ (low relative risk aversion) or $4$ (high relative risk aversion), and log-utility function $U(x) = \log(x + 50)$ (obtain with $\gamma = 1$ from the power utility).

We use the same baseline parameters values and the following correlations between the securities returns: $\rho_{1,1} = 0.5$, $\rho_{1,2} = 0.5$, $\rho_{1,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$.

Table 3 presents the simulation results for the five specified utility functions. As we can observe from the table, for the negative exponential utility specification, when the absolute risk aversion coefficient increases, the return and risk of the portfolio decrease.
The same trend is observed with the power utility function with respect to the relative risk aversion coefficient. These results are in line with the risk-return paradigm. However, the risk-adjusted return is higher with lower risk aversion coefficients.

The graphs of Figure 6 plot the returns distributions for different utility functions. The graphs have been generated for the utility-based dynamic hedging strategy using the underlying assets. As expected, the returns distributions are more skewed for low risk aversion coefficients.

5 Conclusion

In this paper, we study dynamic hedging strategies for portfolios of financial guarantees with transaction costs. By considering multiple-risk sources within portfolios of financial guarantees, we extend previous works on dynamic hedging with transaction costs, e.g., Hodges and Neuberger (1989) and Whalley and Wilmott (1997).

Consistent with the capital at risk practice, e.g., Froot and Stein (1998), Merton and Perold (1993) and Perold (2005), we use the expected losses as well as the unexpected losses or Value at Risk in order to capture the changes of capital allocation associated with the hedging strategies. We examine five hedging strategies: (i) the doing nothing strategy, (ii) the dynamic delta hedging and (iii) utility-based hedging strategies using the underlying assets, (iv) the dynamic delta hedging and (v) utility-based hedging strategies using a security-based index hedging instrument; and compare their performance using the modern concept of risk-adjusted performance measurement (RAPM) consisting of the ratio of the expected return over the value at risk (VaR) of the portfolio.

To better apprehend the impact of the parameters on our hedging strategies, we focus our numerical exercises on a portfolio composed of two financial guarantees. Based on our parameters values, we found that the utility-based hedging strategy with the underlying assets is a better compromise between the delta hedging strategy and the passive stance of doing nothing. This result remains stronger even with higher transaction costs. However, if the insured firms assets are not traded or in a high transaction costs environment, the guarantor can use a security-based index as hedging instrument.

Even in the case of a portfolio of two financial guarantees, the numerical exercise re-
quires substantial amount of computation time, especially if one uses non risk-neutralized probabilities where prices and deltas have to be computed numerically. A challenging avenue for future research will be to study the interactions between the capital structure, the capital requirements, the hedging strategies and the institution performance under the framework of portfolios of more than two guarantees.
References


Figure 1: The trading regions when hedging a put option in the presence of transaction costs

We use the following parameters values: \( K = 100, T = 0.2, r = 0.05, \sigma = 0.2, \mu = 0.07 \) and \( \theta = 0.01 \).
Figure 2: Trading regions with negative exponential and power utility functions

These graphs have been generated using the following baseline parameters values: \( K_1 = 100 \), \( K_2 = 100 \), \( S_{1,0} = 105 \), \( S_{2,0} = 110 \), \( \tau = 1 \), \( r = 0.05 \), \( \sigma_1 = 0.15 \), \( \sigma_2 = 0.17 \), \( \rho_{1,2} = 0 \), \( \Delta_1 = -0.2316 \) and \( \Delta_2 = -0.1737 \). For the top graph, we use the negative exponential utility function \( U(x) = -e^{-\lambda x} \) with \( \lambda = 1/10 \) and for the bottom graph, we use the power utility function \( U(x) = \frac{x^{-\gamma}}{1-\gamma} \) with \( \gamma = 2 \).

Negative exponential utility

Power utility
Figure 3: Trading regions for negative and positive correlations between the insured firms

These graphs have been generated using the following baseline parameters values: $K_1 = 100$, $K_2 = 100$, $S_{1,0} = 105$, $S_{2,0} = 110$, $\tau = 1$, $r = 0.05$, $\sigma_1 = 0.15$, $\sigma_2 = 0.17$, $\Delta_1 = -0.2316$ and $\Delta_2 = -0.1737$. We use a negative exponential utility function $U(x) = -e^{-\lambda x}$ with $\lambda = 1/10$. For the top graph, the correlation between the insured firms is $\rho_{1,2} = -0.20$, and for the bottom graph, it is $\rho_{1,2} = 0.50$. 

\[ \rho_{1,2} = -0.2 \]
Table 1: Strategies returns, VaR and RAPM for positive correlation between the insured returns

These tables have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.25$, $\sigma_2 = 0.4$, and $\sigma_M = 0.15$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.1$ the guarantor’s beta with the market, $H_1 = H_2 = 0.1$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative exponential utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume the following positive correlations between the securities returns: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.7$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$. In Panel 1, we use the transaction costs proportion $\theta = 0.5\%$ and in Panel b, $\theta = 1.0\%$.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Strategy 0</th>
<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Strategy 3</th>
<th>Strategy 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.1944</td>
<td>0.1414</td>
<td>0.1768</td>
<td>0.1619</td>
<td>0.1967</td>
</tr>
<tr>
<td>VaR</td>
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<td>0.9390</td>
<td>0.9680</td>
<td>1.1824</td>
<td>1.3149</td>
</tr>
<tr>
<td>R/VaR</td>
<td>0.1500</td>
<td>0.1506</td>
<td>0.1826</td>
<td>0.1369</td>
<td>0.1496</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Strategy 0</th>
<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Strategy 3</th>
<th>Strategy 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.1944</td>
<td>0.0871</td>
<td>0.1757</td>
<td>0.1279</td>
<td>0.1533</td>
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<tr>
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<td>0.9917</td>
<td>1.1546</td>
<td>1.1358</td>
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<tr>
<td>R/VaR</td>
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<td>0.0936</td>
<td>0.1772</td>
<td>0.1108</td>
<td>0.1350</td>
</tr>
</tbody>
</table>
Figure 4: Strategies returns distributions with positive correlation between the insured firms

These graphs have been generated by simulations using the following baseline parameters values: \( \sigma_I = 0.3, \sigma_1 = 0.25, \sigma_2 = 0.4, \) and \( \sigma_M = 0.15 \) for the securities returns volatilities, \( \mu_I = 0.10, \mu_1 = 0.08, \mu_2 = 0.12 \) and \( \mu_M = 0.10 \) are the instantaneous mean returns of the securities, \( S_{1,0} = S_{2,0} = 100 \) the firms initial values, \( I_0 = 100 \) the index initial value, \( M_0 = 100 \) the market initial value, \( \beta_{GM} = 1.1 \) the guarantor’s beta with the market, \( H_1 = H_2 = 0.1 \) the hurdle rates, \( K_1 = K_2 = 100 \) the firms debt face values. We use the negative exponential utility function for the guarantor \( U(x) = -e^{-\lambda x} \), with constant risk aversion coefficient \( \lambda = 1/100 \). The risk free rate \( r = 0.05 \). We assume the following positive correlations between the securities returns: \( \rho_{I,1} = 0.5, \rho_{I,2} = 0.7, \rho_{I,M} = 0.3, \rho_{1,M} = 0.35, \rho_{2,M} = 0.25, \rho_{1,2} = 0.5. \) We use the transaction costs proportion \( \theta = 0.75\% \).

Strategy 0: No hedging

Strategy 1: Delta hedging with the assets

Strategy 2: Utility hedging with the assets

Strategy 3: Delta hedging with the index

Strategy 4: Utility hedging with the index
Table 2: Strategies returns, VaR and RAPM for negative correlation between the insured firms

These tables have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.25$, $\sigma_2 = 0.4$, and $\sigma_M = 0.15$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.1$ the guarantor’s beta with the market, $H_1 = H_2 = 0.1$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative exponential utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume firm 1 to be negatively correlated with firm 2: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.5$, $\rho_{1,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = -0.2$. In Panel 1, we use the transaction costs proportion $\theta = 0.50\%$ and in Panel b, $\theta = 1.00\%$.

Panel 1: The transaction costs portion $\theta = 0.50\%$

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Strategy 0</th>
<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Strategy 3</th>
<th>Strategy 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.2582</td>
<td>0.1856</td>
<td>0.2457</td>
<td>0.2162</td>
<td>0.2519</td>
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<tr>
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<td>1.1900</td>
<td>1.2193</td>
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<td>R/VaR</td>
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<td>0.1560</td>
<td>0.2015</td>
<td>0.1843</td>
<td>0.1903</td>
</tr>
</tbody>
</table>

Panel 2: The transaction costs portion $\theta = 1.00\%$

<table>
<thead>
<tr>
<th>Strategy</th>
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<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Strategy 3</th>
<th>Strategy 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.2582</td>
<td>0.1021</td>
<td>0.2567</td>
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<tr>
<td>VaR</td>
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<tr>
<td>R/VaR</td>
<td>0.1864</td>
<td>0.0884</td>
<td>0.1880</td>
<td>0.1423</td>
<td>0.1903</td>
</tr>
</tbody>
</table>
Figure 5: **Strategies returns distribution with negative correlation between the insured firms**

These graphs have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.25$, $\sigma_2 = 0.4$, and $\sigma_M = 0.15$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.1$ the guarantor’s beta with the market, $H_1 = H_2 = 0.1$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative exponential utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume firm 1 to be negatively correlated with firm 2: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.5$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = -0.2$. We use the transaction costs proportion $\theta = 0.75\%$.

**Strategy 0: No hedging**

**Strategy 1: Delta hedging with the assets**

**Strategy 2: Utility hedging with the assets**

**Strategy 3: Delta hedging with the index**

**Strategy 4: Utility hedging with the index**
Table 3: Utility-based hedging returns, VaR and RAPM with different utility functions

These tables have been generated by simulations using the following baseline parameters values: \( \sigma_I = 0.3, \sigma_1 = 0.25, \sigma_2 = 0.4, \) and \( \sigma_M = 0.15 \) for the securities returns volatilities, \( \mu_I = 0.10, \mu_1 = 0.08, \mu_2 = 0.12 \) and \( \mu_M = 0.10 \) are the instantaneous mean returns of the securities, \( S_{1,0} = S_{2,0} = 100 \) the firms initial values, \( I_0 = 100 \) the index initial value, \( M_0 = 100 \) the market initial value, \( \beta_{GM} = 1.1 \) the guarantor’s beta with the market, \( H_1 = H_2 = 0.1 \) the hurdle rates, \( K_1 = K_2 = 100 \) the firms debt face values. The risk free rate \( r = 0.05 \). We use the following correlations between securities returns: \( \rho_{I,1} = 0.5, \rho_{I,2} = 0.5, \rho_{I,M} = 0.3, \rho_{1,M} = 0.35, \rho_{2,M} = 0.25, \rho_{1,2} = 0.5. \) We use the transaction costs proportion \( \theta = 0.75\% \).

Panel 1: Utility-based dynamic hedging with the underlying assets

<table>
<thead>
<tr>
<th>Utility function</th>
<th>(-e^{-\frac{x}{100}})</th>
<th>(-e^{-\frac{x}{10}})</th>
<th>(\log(x + 50))</th>
<th>(-(x + 50)^{-1})</th>
<th>(-\frac{(x+50)^{-3}}{3})</th>
</tr>
</thead>
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<tr>
<td>R</td>
<td>0.1761</td>
<td>0.1350</td>
<td>0.1700</td>
<td>0.1538</td>
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<tr>
<td>VaR</td>
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<td>0.8627</td>
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<td>R/VaR</td>
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<td>0.1598</td>
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<td>0.1711</td>
<td>0.1622</td>
</tr>
</tbody>
</table>

Panel 2: Utility-based dynamic hedging with the index

<table>
<thead>
<tr>
<th>Utility function</th>
<th>(-e^{-\frac{x}{100}})</th>
<th>(-e^{-\frac{x}{10}})</th>
<th>(\log(x + 50))</th>
<th>(-(x + 50)^{-1})</th>
<th>(-\frac{(x+50)^{-3}}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.1871</td>
<td>0.1551</td>
<td>0.1734</td>
<td>0.1704</td>
<td>0.1601</td>
</tr>
<tr>
<td>VaR</td>
<td>1.2974</td>
<td>1.1209</td>
<td>1.1643</td>
<td>1.1556</td>
<td>1.1255</td>
</tr>
<tr>
<td>R/VaR</td>
<td>0.1442</td>
<td>0.1384</td>
<td>0.1489</td>
<td>0.1475</td>
<td>0.1422</td>
</tr>
</tbody>
</table>
Figure 6: Utility-based strategies returns distribution for different utility functions

We plot the graphs for the utility-based dynamic hedging strategy using the underlying assets. These graphs have been generated by simulations using the following baseline parameters values: $\sigma_1 = 0.3$, $\sigma_2 = 0.25$, $\sigma_2 = 0.4$, and $\sigma_M = 0.15$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.1$ the guarantor’s beta with the market, $H_1 = H_2 = 0.1$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. The risk free rate $r = 0.05$. We use the following correlations between the securities returns: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.5$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$. The transaction costs proportion $\theta = 0.75\%$.

$$U(x) = \log(x + 50)$$

$$U(x) = -\frac{x^{-1}}{1}$$

$$U(x) = -\frac{x^{-3}}{3}$$

$$U(x) = -e^{-\frac{1}{100}}$$

$$U(x) = -e^{-\frac{1}{10}}$$