ENHANCED ANNUITIES, INDIVIDUAL UNDERWRITING, AND ADVERSE SELECTION

A SOLUTION FOR THE ANNUITY PUZZLE?

Gudrun Hoermann
Jochen Ruß

ABSTRACT

We analyze the effect of enhanced annuities on an insurer conducting individual underwriting. We use a frailty model for the heterogeneity of the insured population and model the individual underwriting by a random variable that positively correlates with the corresponding frailty factor. For a given annuity product we analyze the effect of the quality of the underwriting on the insurer’s profit/loss situation and the impact of adverse selection effects.

1. INTRODUCTION

In many countries, e.g. the U.K. and Germany, there are tax incentives that encourage owners of old age provision contracts to receive their benefits in the form of a lifelong annuity as opposed to a lump sum. There even exist state subsidized or tax sheltered product lines where annuitization is compulsory.

From an economical point of view it may make sense to provide incentives for annuitization. However, in a market where only so-called standard annuities are

1 Gudrun Hoermann is with the University of St. Gallen, Institute of Insurance Economics, Kirchlistrasse 2, 9010 St. Gallen, Switzerland; Internet: www.ivwhsg.ch, Email: gudrun.hoermann@unisg.ch. Jochen Ruß is managing director of the Institute for Financial and Actuarial Science and lecturer at the University of Ulm, Helmholtzstrasse 22, 89081 Ulm, Germany; Internet: www.ifr-ulm.de, Email: j.russ@ifa-ulm.de.
offered, such regulations result in significant disadvantages for insured persons whose life expectancy at the time of annuitization is below average.

With standard annuities, the annuity paid depends only on the amount of money that is annuitized, the age of the insured at the time of annuitization and the insured’s gender. Thus, the value for money of a standard annuity is the higher, the longer the life expectancy of an insured person. If certain tax incentives favour annuitization, then people with a reduced life expectancy only have the choice between an annuitization at “unfair” rates and a lump sum benefit that triggers some sort of tax disadvantage. In product lines with compulsory annuitization, impaired people are even forced to annuitize at unfavourable rates, i.e. the present value of the expected annuity benefits may be significantly lower than the amount to be annuitized.

This discrimination could be prevented if so-called enhanced annuities\(^2\) were offered, i.e. products where the annuity paid is the larger, the lower the insured’s life expectancy.

In many insurance markets for term life and disability insurance, it is already a common feature to offer so-called preferred life products, where the premium is lower for “good risks”, i.e. insureds with low mortality or morbidity rates. In the annuity business, obviously impaired persons are good risks from the insurer’s perspective. Therefore, enhanced annuities are sometimes also referred to as impaired annuities.

For immediate enhanced annuities, the relation of premium and annuity depends on the health of the insured at the time the contract is taken out. In deferred annuities, however, the insurer has to perform some kind of underwriting at the end of the deferment period. If the insured person does not submit to the underwriting proceeding, she would receive the annuity corresponding to the healthiest class of insureds.

The fact that enhanced annuities are still very uncommon in many insurance markets could also explain the so-called annuity puzzle: E.g. Dushi and Webb

(2004) found that only 10.2% of seniors in the USA have annuitized (a portion of) their wealth. This is always considered a surprise in the academic literature since e.g. Brown et al. (2005) showed that under usual assumptions (partial) annuitization increases expected utility. However, this analysis assumes that future mortality rates are known and depend only on age and gender of the insured person. In other words, they suppose that the value for money of annuities is essentially the same for all potential insureds. Yet in reality, strong selection effects can be observed: Persons who elected to annuitize part of their wealth have significantly lower than average mortality rates, i.e. higher than average life expectancies. We can conclude that out of those individuals who receive a good value for money when purchasing an annuity, significantly more than 10.2% annuitize, whilst only a very small portion of people with low life expectancy do. If enhanced annuities were offered, everybody would get a “fair deal” when purchasing an annuity. Thus, the acceptance of annuities should increase.

Existing literature about enhanced annuities primarily concentrates on practical issues of the enhanced annuity market, mainly in the U.K., and deals e.g. with its development, its size or potential, different types of enhanced annuities, the description of possible underwriting methods and underwriting challenges, as well as further issues like tax considerations, distribution channels, reinsurance, etc.3 Beyond that, Ainslie (2000) provides a quantitative analysis of potential adverse selection effects on the standard annuity market by determining some critical size of the enhanced annuity market. He considers a hypothetical portfolio of males aged 65. The heterogeneity of their health is modelled using a normal distribution for the mortality. For different parameter combinations for this normal distribution he determines the portion of pensioners buying enhanced annuities (instead of standard annuities). Besides, Jones and Richards (2004) e.g. emphasize the risk of underwriting enhanced annuities: "The greatest risk for enhanced annuities lies in underwriting,..."4 but do not perform quantitative analyses.

Moreover, there have been no attempts to develop a model that describes the individual underwriting of enhanced annuities, the quality of such an

---

underwriting and quantifies the resulting effects on the insurer’s profit/loss situation. Furthermore, the impact of adverse selection resulting from competitors offering enhanced annuities on an insurer not offering such products has not been investigated, yet.

The present paper fills this gap. We present quantitative analyses of the effect enhanced annuities have on an insurer conducting an individual underwriting. First, we specify the heterogeneity of the insured persons in Section 2.1 by modeling the distribution of the degree of impairment within a population using a frailty model for individual mortality rates. Subsequently, in Section 2.2, we present our model for the individual underwriting. The result of the underwriting is a stochastic frailty factor that correlates with the actual frailty factor of the insured person. The correlation coefficient is our measure for the quality of the underwriting. In Section 2.3, we specify the considered insurance product and the community of insureds. In addition, Section 2.4 explains how adverse selection effects can be analyzed within our model framework. In Section 3 we present numerical results that are derived using Monte Carlo methods. After specifying the parameters for our analyses in Section 3.1, we show results for three model companies in Section 3.2. By calculating the empirical profit distribution of each of the three companies we analyze the effect of enhanced annuities and of the quality of the underwriting on the insurer’s profit/loss situation and assess the impact of adverse selection effects on companies who do not offer enhanced annuities when other insurers in the market do. We conclude with a summary in Section 4.

2. MODEL FRAMEWORK

2.1 Individual Mortality Rates

We define \((x)\) and \((y)\) as a male or female person age \(x \in \mathbb{N}_0\) or \(y \in \mathbb{N}_0\), respectively. In what follows we restrict ourselves to the case of a male insured. Age at death of the considered person is modeled by the random variable \(X \geq 0\). The random variable \(K(x) = X - x\), \(X > x\) describes the remaining lifetime of \((x)\). Its distribution function \(_qx\) at a point \(k \in \mathbb{N}_0\) is denoted by

\[
_k q_x = F_{K(x)}(k) = P(K(x) \leq k | X > x) = 1 - P_x,
\]  

(1)
where \( \kappa p_x \) is the \( k \)-year survival probability of \((x)\).

To specify heterogeneity in the insurance portfolio we use a frailty model, i.e. an individual factor\(^5\) (also referred to as mortality multiplier) by which the actual mortality of each person differs from a given standard mortality table.\(^6\) Probabilities given in the standard mortality table will be denoted with a "´".\(^7\) Thus, the one-year individual mortality rate for a given insured with mortality multiplier \( d \) is given by

\[
q_x = \begin{cases} 
  d \cdot q_x', & \text{if } d \cdot q_x' \leq 1 \\
  1, & \text{otherwise}
\end{cases} \quad \text{with } x \in \{0, \ldots, \omega\}.
\]

The individual mortality rates \( q_x \) determine the distribution of the annuity payments, i.e. the insurer’s liabilities.

The parameter \( d \) describes the state of health of an individual:

- For \( 0 < d < 1 \), we have \( q_x < q_x' \). The individual has an above average life expectancy.
- For \( d = 1 \), we have \( q_x = q_x' \). The individual mortality rates coincide with the mortality table.
- For \( 1 < d \), we have \( q_x > q_x' \). The individual is impaired.

For \( d < 1 \) we let \( q_\omega = 1 \), where \( \omega \) is the so-called limiting age of the standard mortality table, i.e. the age that according to that table cannot be survived. Thus, the remaining probability mass is assigned to the mortality rate of the last year, and we obtain a truncated distribution.

---

\(^5\) Such a factor (often analogously applied to the continuous force of mortality) is usually called "frailty factor", cf. Vaupel/Manton/Stallard (1979), p. 440; cf. also Jones (1998), p. 81.

\(^6\) Cf., e.g., Pitacco (2003), p. 13. This kind of modeling assumption is reasonable, since many impairments generally increase mortality over a longer period of time, as e.g. cardiovascular disease, which according to the WHO Europe (2004), p. 12 or PAN American Health Organization (2006), p. 7 makes up the main cause of death. However, in practical implementations, one might prefer to use a different “shape” of extra mortality for different impairments, e.g. so-called flat extras, i.e. an additive extra mortality over several years only.

\(^7\) Cf., e.g., Haberman (1982), p. 211.
If we randomly select an individual from some population, the corresponding $d$ is modeled as a realization of a random variable $D$. Slightly simplified, the distribution $F_D$ of $D$ describes, which portion of the general population is in which state of health. More precisely, it specifies the portion of individuals whose mortality is lower or higher than a certain percentage of standard mortality. For this distribution $F_D$, we make the following assumptions, that have been proposed in the literature before:

- The distribution $F_D$ is continuous such that finest nuances of the state of health and of the remaining life expectancy are possible.
- Its domain is positive ($d \geq 0$), as there are no negative probabilities.
- The probability density function is "flat" at zero and equal to zero for $d = 0$, since mortality rates near zero are unrealistic.
- The distribution is right-skewed, i.e. very high values of $d$ can occur, however, they are bounded below by zero.
- Over the population, the expected value $E(D) = 1$, i.e. the standard mortality table describes an "average individual".

### 2.2 Individual Underwriting

The purpose of the individual underwriting is to assign each insured an estimate $\hat{d}$ for the frailty factor $d$, i.e. to determine the so-called pricing mortality rates $\hat{q}_x$, used by the insurer for premium calculation:

$$\hat{q}_x = \begin{cases} \hat{d} \cdot q'_x, & \hat{d} \cdot q'_x \leq 1 \\ 1, & \text{otherwise} \end{cases} \quad \text{with } x \in \{0, ..., \omega\}.$$  

Again, we let $\hat{q}_x := 1$ for $\hat{d} < 1$.

We model the parameter $\hat{d}$ as a realization of a random variable $\hat{D}$ and assume $D$ and $\hat{D}$ to be identically distributed: $D \sim \hat{D}$. This means, that there is no

---

14 This is a so-called "numerical rating system", cf. Pitacco (2003), p. 13.
systematic underwriting error, i.e. the mortality estimation of the underwriting over the whole population is not just correct on average but also with respect to the portion of people that are identified to lie in a certain impairment range. Furthermore, in order to focus on the pure effect of introducing individual underwriting, we do not consider any safety loadings. Finally, the random variables $D$ and $\hat{D}$ are supposed to be positively correlated with a correlation coefficient $0 \leq \rho_{D,\hat{D}} \leq 1$. This correlation coefficient is a measure for the quality of the individual underwriting: the larger $\rho_{D,\hat{D}}$, the smaller the mean deviation between $d$ and $\hat{d}$. The condition $\rho_{D,\hat{D}} = 1$ implies $d = \hat{d}$, i.e. the hypothetical case of a "perfect" underwriting.

Note that this model for individual underwriting is continuous in the sense that all values for $\hat{d}$ are possible and symmetric in the sense that insureds with increased and reduced life expectancy are treated analogously. In practical implementation, for the sake of tractability one might prefer a discrete model, where e.g. only integer multiples of 0.25 are admissible values for $\hat{d}$. Alternatively or additionally, an asymmetric model might be favored, where all insureds that have an above average life expectancy are clustered in one group and individual underwriting is only performed for impaired persons.

2.3 The Considered Annuity Contract and the Portfolio of Insureds

To simplify notation, in what follows we concentrate on a simple immediate lifelong annuity where the annual annuity paid to the insured is calculated from the single premium using pricing mortality rates. We do not consider any charges. Our findings also apply to a deferred annuity where the underwriting takes place at the end of the deferment period.

We distinguish between three model companies: Company A does not perform individual underwriting. The annuity is always calculated using the standard mortality table (i.e. the mortality rates $q_x'$). Company B conducts an individual underwriting as described in Section 2.2, the quality of which is characterized by the correlation coefficient $\rho_{D,\hat{D}}$. For the sake of comparison, we also consider the

---

15 Safety loadings could also be considered in the model, e.g. by assuming that the pricing rates are the mortality rates that result from the underwriting multiplied by a certain factor.
hypothetical case of an insurer with "perfect" individual underwriting (company C)\(^\text{16}\) where \(d\) and \(\hat{d}\), and thus actual and estimated mortality rates coincide.

In what follows, formulas for the calculation of benefits and for the analysis of the insurer’s profit and loss are derived for the three companies. For an immediate lifelong annuity paying the annual amount \(\mathcal{A}_t\) to a male insured aged \(x\), the present value of future benefits\(^\text{17}\) is given by:

\[
\mathcal{B}_t = \mathcal{A}_t, \quad [j] = A, B \text{ or } C,
\]

with \(v = \frac{1}{1+r}\), where \(r\) denotes the guaranteed rate of return\(^\text{18}\), and

\[
K(x) \sim F_{K(x)}(k) = q_x = 1 - p_x = 1 - \prod_{l=0}^{k-1}(1 - D \cdot q'_{x+l}), \quad k \in \mathbb{N}_0 \ (\text{cf. (1)}).
\]

For a single premium \(P\), company A would pay an annual annuity amounting to

\[
\mathcal{A}_A = \frac{P}{\sum_{k=0}^{\varphi-x} v^k p_x} = \frac{P}{\sum_{k=0}^{\varphi-x} v^k \prod_{l=0}^{k-1}(1 - 1 \cdot q'_{x+l})},
\]

which is the same for all insured persons of equal age and gender since it is based only on the standard mortality rates.

Company B determines the annuity amount \(\mathcal{A}_B\) using the estimated mortality:

\[
\mathcal{A}_B = \frac{P}{\sum_{k=0}^{\varphi-x} v^k \hat{p}_x} = \frac{P}{\sum_{k=0}^{\varphi-x} v^k \prod_{l=0}^{k-1}(1 - \hat{D} \cdot q'_{x+l})},
\]

Therefore \(\mathcal{A}_B\) is a random variable.

Finally, for company C, we get:

\(^{16}\) Which obviously corresponds to company B with \(\rho_{D, \hat{D}} = 1\).


\[
A_c = \frac{P}{\sum_{k=0}^{n-1} v^k p_x} = \frac{P}{\sum_{k=0}^{n-1} v^k \prod_{l=0}^{k-1} (1 - D \cdot q'_{x+l})}.
\]

We analyze a portfolio of \( n \) insureds \( i, i = 1, \ldots, n \) of the same gender and age, who are randomly selected from some general population and thereby implicitly assume, that a suitable standard mortality table is chosen, i.e. a table that describes an average individual of this general population. We denote all figures that refer to a specific insured or contract with a corresponding index \( i \): \( D_i \) denotes the mortality multiplier of person \( i \), and \( \hat{D}_i \) denotes the mortality multiplier estimated by the insurer’s underwriting for that person, etc. The present value of future profits of the policy of person \( i \) in the observed portfolio is given by the difference between the single premium and the present value of future benefits:

\[
\Pi_i = P - \mathcal{B}_i, \quad \mathcal{B}_i = A, B \text{ or } C, \quad i = 1, \ldots, n.
\]

The cumulated present value of future profits of company A, B and C, respectively (\( \Pi(A), \Pi(B) \) and \( \Pi(C) \)) then results to

\[
\Pi(\cdot) = \sum_{i=1}^{n} \Pi_i.
\]

We denote the corresponding distribution function by \( F_{\Pi(\cdot)} \). In Section 3, properties of this distribution are analyzed using Monte Carlo simulation methods.

2.4 The Impact of Adverse Selection

If some insurers in the market were to offer enhanced annuities, then in theory impaired persons would prefer such a product to a standard annuity. Therefore, no impaired person would buy a contract from an insurer offering only standard annuities. Thus, company A would end up with a portfolio of insureds with increased average life expectancy and suffer from a loss. Realistically, because of market imperfections, only a portion of impaired persons would apply for an enhanced annuity. Therefore, we assume that \( s\% \) of all persons with a mortality
multiplier $d$ exceeding a threshold value $d^*$ (referred to as the selection barrier) avoid the standard insurer $A$. The resulting modified distribution of the frailty factors $\tilde{D}$ in company A’s portfolio is denoted by $F_\theta$. Therefore, the quantities calculated above will change as follows:

$$B_{A_{\text{sel}}} = A_A \sum_{k=0}^{\tilde{K}(x)} k^k \text{ with } \tilde{K}(x) \sim F_{\tilde{K}(x)}(k) = q_k x = 1 - \prod_{l=0}^{k-1} (1 - \tilde{D} \cdot q_{x+l})$$

$$\Pi(A_{\text{sel}}) = \sum_{i=1}^{n} \Pi_{A_{\text{sel}},i} \text{ with } \Pi_{A_{\text{sel}},i} = P - B_{A_{\text{sel}},i}, \quad i = 1, \ldots, n.$$ 

Again, the corresponding distribution $F_{\Pi(A_{\text{sel}})}$ can be approximated using simulation techniques.

3. **Numerical Analyses**

3.1 **Specification of Parameters and Simulation Details**

We use the DAV2004R table as standard mortality table throughout our analyses.\(^{19}\)

To our knowledge, there are no data available that could be linked to the distribution $F_D$ in the population, especially with regard to higher age groups. Furthermore, based on information from direct insurers regarding the proportion of different insurance ratings, at the most rough inferences could be made to the health distribution of younger applicants\(^{20}\), whereby an assumption for senior citizens would have to be made. Therefore, in our analyses, we choose a distribution which has – for a suitable choice of parameters – the characteristics listed in Section 2.1 and which is commonly used in the literature to describe the distribution of the mortality multiplier in the general population: We let $D$

---

\(^{19}\) The DAV2004R mortality table is the most current German annuity table for ages $x$ and $y$ from 0 to 121. We use the table with age adjustment for the years of birth from 1910 to 2020. For further information, cf. DAV (2005).

follow a Gamma-distribution\textsuperscript{21}, \( D \sim \Gamma(\alpha, \beta, \gamma) \). Density function, expected value and variance are then given by:

\[
f_{\Gamma(\alpha, \beta, \gamma)}(d) = \frac{1}{\Gamma(\alpha) \beta^\alpha} (d-\gamma)^{\alpha-1} e^{-\frac{d-\gamma}{\beta}}, \quad E(D) = \alpha \beta + \gamma, \quad \text{and} \quad \text{Var}(D) = \alpha \beta^2
\]

for \( d \geq \gamma, \gamma \in \mathbb{R}^+, \alpha, \beta > 0 \).

The fact that certain accidents are inevitable and thus mortality rates close to zero are unrealistic supports a positive third parameter \( \gamma \).\textsuperscript{22} Taking into account the properties mentioned in Section 2.1, a plausible parameter combination is \( \alpha = 2, \beta = \frac{1}{4}, \text{and} \gamma = \frac{1}{2} \).

Here we would like to point out again that the distribution as well as its parameterization is not supported by epidemiological/medical data. It is rather motivated by the desired properties listed above. Therefore, we considered a variety of different reasonable parameter values, i.e. alternative parameter values that also lead to a distribution that fulfils the characteristics demanded in Section 2.1, all of which produced similar outcomes.

We examine an insurance portfolio consisting of 100 male insured persons at the age of 65 (cf. Table 1). The premium paid by each insured is $100,000, and we use the current guaranteed interest rate for German annuity insurance products of 2.25%. We also assume that the insurer always earns exactly this rate on the assets backing the annuity, i.e. the profit/loss considered stems only from a deviation of the actual from the based on pricing rates expected number of deaths. Such a deviation can occur due to pure random effects, as well as due to a difference between individual and pricing mortality rates. For insurer C only random effects are observed since pricing mortality rates and actual rates coincide.

Unless stated otherwise, we fix the following values for the various parameters in our numerical analyses:


\textsuperscript{22} Cf. Hougaard (1984), p. 79.
### TABLE 1: Specification of simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (and gender)</td>
<td>$x$</td>
<td>65</td>
</tr>
<tr>
<td>Guaranteed interest rate</td>
<td>$r$</td>
<td>2.25%</td>
</tr>
<tr>
<td>Single premium</td>
<td>$P$</td>
<td>$100,000$</td>
</tr>
<tr>
<td>Number of insureds</td>
<td>$n$</td>
<td>100</td>
</tr>
<tr>
<td>Number of simulations</td>
<td>$m$</td>
<td>10,000</td>
</tr>
<tr>
<td>Selection barrier *</td>
<td>$d$</td>
<td>1.5</td>
</tr>
<tr>
<td>Correlation coefficient (company B only)</td>
<td>$\hat{\rho}_{D,D}$</td>
<td>Different values are analyzed</td>
</tr>
<tr>
<td>Selection intensity</td>
<td>$s$</td>
<td>Different values are analyzed</td>
</tr>
</tbody>
</table>

As mentioned above, we use Monte Carlo methods to analyze the distribution of the insurer’s profit. For each insured, three random numbers have to be generated: $d$ specifies the individual mortality, i.e. the probability distribution of the remaining lifetime and thus the benefits; $\hat{d}$ is the mortality multiplier resulting from the individual underwriting and determines the pricing rate and therefore the annuity amount; finally, a random number $u$ specifies the realization of the time of death.

The random numbers $d$, $\hat{d}$ and $u$ are realizations of the random variables $D$, $\hat{D}$ and $U$, where the first two are $F_D$-distributed and correlated with $\rho_{D,D}$\(^\text{23}\) and the latter follows a continuous uniform distribution $U(0,1)$. From $u$, the year of death $\kappa$ is calculated as the first year in which the value of the probability distribution of the remaining lifetime exceeds $u$, i.e.

$$\kappa = \min \left\{ k \in \{1,\ldots,\omega-x+1\} : \sum_{k=1}^{x-1} q_k > u \right\}.$$

From $\kappa$ for each insured, realizations $b_{\kappa}$ of the benefits $B_{\kappa}$ can be calculated for the three model companies $[\cdot] = A, B$ and $C$.

To derive the empirical distribution of future profits in a portfolio we carry out $m$ simulations $j = 1,\ldots,m$. In each simulation, the random numbers $(d_{i,j},\ldots,d_{n,j})$, $(\hat{d}_{i,j},\ldots,\hat{d}_{n,j})$ and $(u_{i,j},\ldots,u_{n,j})$ are generated and the annuity amount, the duration of the annuity payment and thus the insurer’s profit can be calculated per policy.

and cumulated over all insured persons \( i = 1, \ldots, n \). We denote the realizations of the insurer’s profit for policy \( i \) in simulation run \( j \) by

\[
\pi_{[i], [j]} = P - b_{[i], [j]}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m; \quad [\cdot] = A, B, C,
\]

and the cumulated profit over all policies in simulation run \( j \) by

\[
\pi_\cdot_j = \sum_{i=1}^{n} \pi_{[i], [j]}, \quad j = 1, \ldots, m \quad \text{for} \quad [\cdot] = A, B, C \text{ and } A_{\text{sel}}.
\]

The Monte Carlo estimate for the distribution of \( \Pi(\cdot) \) is then given by

\[
\hat{F}_{\Pi(\cdot)}(\nu) = \hat{P}(\Pi(\cdot) \leq \nu) \approx \frac{1}{m} \sum_{j=1}^{m} I_{\{\pi(\cdot_j, \nu)\}},
\]

where \( I \) is the indicator function. In the following section, we have a closer look at this distribution and at certain functions of it in order to analyze the effect of enhanced annuities on the risk profile of an insurer.

### 3.2 Results

First, we examine the empirical distribution of the insurance companies’ profit/loss and particularly consider the expected value, the standard deviation and certain quantiles of it. This allows us to draw conclusions about the impact of enhanced annuities on the profit/loss situation and the risk structure of an insurer. In particular, we can analyze the effect of the quality of the underwriting. We also compare an insurer with enhanced annuities to a standard insurer without individual underwriting. Finally, adverse selection effects are investigated.

We “calibrate” our model such that allowing for heterogeneity in the insurance portfolio implied by the Gamma-distribution for the frailty factors introduced above, the expected profit for company A is zero.\(^{24}\) We then calculate the expected profit for company B as a function of the correlation coefficient \( \rho_{D, \hat{D}} \).

The result is displayed in Figure 1. Here, the expected profit of company A and C, respectively, which does not depend on \( \rho_{D, \hat{D}} \), is displayed as a straight line.

\(^{24}\) Cf. Appendix B.
FIGURE 1: Expected value of profit as a percentage of premium volume by correlation coefficient

Even for a rather poor quality of the underwriting, i.e. a correlation coefficient of zero, the expected profit of company B clearly exceeds the expected profit of the standard insurer A. This effect results only from the consideration of the mortality distribution in the population. With increasing quality of the underwriting, the expected profit almost doubles from slightly more than 0.8% of the premium volume (for $\rho_{D,\hat{D}} = 0$) to 1.6% (for $\rho_{D,\hat{D}} = 1$) which by definition coincides with the value for insurer C. In other words, yet under the quite weak assumption that the insurer is right on average in the sense described above (i.e. $D \sim \hat{D}$), individual underwriting has a beneficial effect on the insurer’s expected profit. The positive effect and thereby the profit increases with the quality of the underwriting.

We now analyze whether or not this increase of the expected profit has to be “paid” with an increase in the insurer’s risk. We find that for low values of $\rho_{D,\hat{D}}$, this is indeed the case (cf. Figure 2): the standard deviation of the profit for company B starts at about 2.40% of the premium volume for $\rho_{D,\hat{D}} = 0$ exceeding the value of company A (2.31%; black line). However, for increasing $\rho_{D,\hat{D}}$ the standard deviation decreases. At a correlation of slightly less than 0.5, the standard deviation of B falls below that of A approaching the level of company C.
(at about 1.90%) as \( \rho_{D,d} \) approaches 1. Hence, with increasing quality of the individual underwriting, the expected profit increases and the volatility decreases. For a correlation of 0.5 or higher, company B has both, a higher expected profit and a lower volatility of the profit than company A.

**Figure 2:** Standard deviation of profit by correlation coefficient

![Graph showing standard deviation of profit by correlation coefficient]

Figure 3 displays the empirical distribution of the insurer’s profit loss corresponding to the results displayed above. We can see that – as we move from no underwriting (company A) to imperfect underwriting (company B, here with \( \rho_{D,d} = 0.4 \)) to perfect underwriting (company C) – the distributions move to the right (i.e. expected profit increases) and become denser (i.e. volatility decreases).

The probability for a positive profit increases from 50% for company A \(^{25}\) over 63% for company B (\( \rho_{D,d} = 0.4 \)) to 69% for company C; already with a correlation coefficient of zero, the probability of a positive profit for company B is increased to 59%. With a probability of 5%, the loss of company A exceeds 5.4% of the premium volume. For company B with \( \rho_{D,d} = 0.4 \) this 5%-quantile is reduced to 4.3%, and for company C it only amounts to 3.5% of the total premium income.

\(^{25}\) These values would be significantly higher for all three companies if we considered safety loadings of any kind.
FIGURE 3: Probability density function of profit for 100 insured persons

We performed sensitivity analyses with respect to the portfolio size: If we quadruple the amount of insureds (i.e. 400 instead of 100 insured persons), the risk of random fluctuations is reduced by diversification effects. Figure 4 shows the resulting distribution. In comparison with Figure 3, we can see that - as one would expect - the range of random fluctuations is roughly cut in half by quadrupling the portfolio size.

FIGURE 4: Probability density function of profit for 400 insured persons
For company C the probability of a positive profit is increased from 69% to 85% for the larger portfolio. It is enhanced from 63% to 76% for company B if \( \rho_{D,B} = 0.4 \); from 59% to 69% if \( \rho_{D,B} = 0 \). For company A the 5% quantile described above is cut in half (to 2.7%), for company B it is reduced from 4.3% to 1.6%, and for company C it even goes down to 0.9% of the premium volume (compared to 3.5% in the portfolio with 100 insureds).

Finally, in Figures 5 and 6 we display results including selection effects as described in Section 2.4. We assume that \( s \% \) of all insured persons with a mortality multiplier exceeding a threshold of \( d^* = 1.5 \) prefer an insurer with individual underwriting over company A.\(^{26}\) Under our distribution assumption for the frailty factor in the population, about 9% of the general population have a mortality multiplier above this selection barrier.

**FIGURE 5:** Expected value of profit by selection intensity \( s \)

![Graph showing expected value of profit by selection intensity](image)

From simple linearity arguments, it follows that the expected profit is a linear function in \( s \). Figure 5 shows the expected profit of company A as a function of the selection intensity \( s \). The expected profit decreases as \( s \) increases up to an expected loss of about 1.7% for \( s = 1 \). The effect can also be seen in Figure 6.

\(^{26}\) We describe in Appendix A how this is considered in our Monte Carlo algorithm.
where the empirical distribution function of the insurer’s profit is shown for a selection intensity of 30% and 70%.

**FIGURE 6:** Probability density function of profit

Our analyses show that significant disadvantages will emerge for a standard insurer if competitors start offering enhanced annuities, even if only a portion of insureds knows of the new product.

4. **SUMMARY**

In this paper, we analyzed the impact of enhanced annuities on the risk profile of an insurer. A heterogeneous portfolio of enhanced annuities (with regard to the insured’s mortality) was considered using a frailty model. We modeled the result of the insurer’s underwriting by means of a distribution for pricing mortality rates that correlates with the distribution of individual mortalities. Applying Monte Carlo simulation techniques, we determined future cash-flows of an annuity portfolio and compared the results for insurers without and with individual underwriting of different qualities. Additionally, selection effects were taken into account.
Our results show that introducing enhanced annuities would be beneficial for both, the insurer and the insured. If the insurer can correctly assess the average distribution of excess and lower mortality in the population, we showed that individual underwriting will always increase the company’s profitability. The effect is the stronger, the higher the quality of the underwriting. Moreover, the volatility of future profits is decreasing in the quality of the underwriting and the number of insureds. We quantified the negative effect of adverse selection on insurers not offering enhanced annuities in a market where competitors do. The impact is significant, even if only a portion of impaired prefers enhanced annuities to standard annuities. Thus, we can conclude that offering enhanced annuities would be beneficial for the risk profile of an insurer.

On the other hand, enhanced annuities will pay significantly increased benefits to impaired persons. Consequently, the value for money is the same for all insureds and, unlike traditional annuity products, enhanced annuities are therefore also attractive for persons with a below average life expectancy. Hence, enhanced annuities would increase the acceptance of annuities in the general population, which can be shown to be beneficial for the annuitant under certain assumptions\textsuperscript{27}.

\textsuperscript{27} Cf. Brown et al. (2005).
A. GENERATION OF THE DISTRIBUTION OF $\Pi(A_{Sel})$

As a consequence of the selection effect described in 2.4, the distribution of the frailty factors in insurer A’s portfolio is no longer a Gamma-distribution. This is considered using the following algorithm to generate a random number as a realization of the mortality multiplier of an insured person:

i. Generate an $F_{\phi}$-distributed random number $\tilde{d}$.

ii. If $\tilde{d} \leq d^*$, accept $\tilde{d}$. (Interpretation: Persons with lower mortality still go to company A.)

iii. Otherwise, generate an additional $U(0,1)$-distributed random number $z$.

iv. If $z \leq 1-s$ accept $\tilde{d}$. (Interpretation: $1-s\%$ of those with increased mortality still go to company A);

v. Otherwise, do not accept $\tilde{d}$ and go back to i. (Interpretation: $s\%$ of the insureds choose an insurer with a product based on individual underwriting and therefore will not be included in company A’s portfolio.)

By producing a random frailty factor for each insured as described above, we receive a portfolio where the distribution of the mortality multipliers reflects the selection effects mentioned in the paper. The rest of the analysis is similar to the case without selection.

B. ADJUSTMENT OF PROFIT FOR COMPANY A

As a basis for comparison, we calibrate our model such that the expected value of future profits of company A is zero. Before this calibration, the expected gain of company A is given by

$$\Pi(A) = P - A_A \sum_{k=0}^{\omega-x} v_k^i p_s = P - B_A$$
with expected value

\[
E\left(\Pi(A)\right) = P - E(B_A) = P - E\left(\frac{P}{\sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - q'_{x+k})} \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot q'_{x+l})\right)
\]

\[
= \frac{P}{f(1)} \cdot E\left(f(D)\right) \leq 0
\]

This value is \(\leq 0\) since \(f(d) = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - d \cdot q'_{x+l})\), \(d \geq 0\) is a convex function as \((1 - d \cdot q'_{x+l}) \geq 0 \; \forall d, l\). Thus, Jensen’s inequality\(^{28}\) yields \(E\left(f(D)\right) \geq f\left(E(D)\right) = f(1)\).

Therefore, we determine a factor \(\mu\) to modify the pricing mortality rates such that

\[
E\left(\Pi(A)\right) = P - E\left(\frac{P}{\sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - \mu \cdot q'_{x+k})} \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot q'_{x+l})\right) = 0.
\]

To provide for comparability we assume that companies B and C use the modified rates in their annuity calculation as well, i.e.

\[
A_B = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - \dot{D} \cdot \mu \cdot q'_{x+k}) \quad \text{and} \quad A_C = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot \mu \cdot q'_{x+k}).
\]

REFERENCES


