Optimal Insurance and Reinsurance in Market Equilibrium:
Insights from the Cournot Paradigm

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Abstract

We employ the Cournot market-game paradigm with risk-averse buyers and sellers to show that, contrary to the spirit of Raviv (1979) and much subsequent literature, optimal insurance contracts in market equilibrium are characterized by policy limits, whereas deductibles and/or partial-insurance arrangements must be justified on other grounds. In the case of reinsurance markets, we find that the same principle applies. The flexibility of the Cournot approach, and its ability to accommodate a number of realistic market variations, are then discussed.

Keywords: Cournot equilibrium, market game, optimal insurance, optimal reinsurance, policy limit, deductible, retention.

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1. Introduction

1.1. Optimal Insurance

It is well known that in an insurance transaction between a single risk-averse buyer and a single risk-neutral seller, the buyer will purchase full coverage unless the transaction involves frictional costs (see Mossin, 1968). When transaction costs are introduced, the buyer will prefer to purchase insurance with a deductible (see Arrow, 1971). More general results have been given by Raviv (1979) – as emended by Blazenko (1985) – who considered a single-buyer, single-seller scenario in which both parties are risk averse.

Raviv’s (1979) basic theorem states that there exists a class of Pareto-optimal solutions in which the buyer and seller agree on policies in which either: (1) there is a deductible (i.e., no coverage) up to some specified dollar amount $x$, above which the seller provides partial coverage; or (2) there is an interval of full coverage up to a specified point $y$, above which the seller provides only partial coverage. Raviv (1979) also showed that the optimal insurance contract is characterized by a deductible if and only if the seller’s settlement-cost function has a positive first derivative everywhere.

Interestingly, Raviv’s (1979) core results are not consistent with the behavior of real-world markets, as recognized by the author himself. Whereas the results provide a theoretical justification for the existence of the insurance deductible, they fail to show why the insurance policy limit – a contract provision that may well be more common in practice than the deductible – should exist. Raviv explained this discrepancy by arguing that risk-averse sellers operate in markets subject to substantial price regulation in which premiums, even if actuarially based, are insufficient to cover the sellers’ full economic costs. In such cases, the Pareto-optimal insurance contract possesses a policy limit without any deductible. A major problem with this explanation,
of course, is that not all insurance markets are closely regulated, and many unregulated lines, such as commercial general liability insurance, invariably are characterized by policy limits.

Subsequent research on optimal insurance has been extensive, with notable contributions by Schlesinger (1981), Turnbull (1983), Briys and Loubergé (1985), Lee and Pinches (1988), Gollier and Schlesinger (1996), Meyer and Ormiston (1999), and Cummins and Mahul (2004). In general, this literature has stressed the optimality of deductibles in different contexts and under a variety of modeling assumptions. However, in most cases, the sellers of insurance – whether primary insurers or reinsurers – are treated as risk-neutral decision makers.

In the present article, we employ a Cournot market-game model to solve for optimal insurance and reinsurance contracts in market equilibrium. Taking both insurers and reinsurers to be risk averse, we find the optimal policies involve full insurance up to a given policy limit, with no deductible/retention or partial-coverage characteristics.

1.2. Cournot Market Games

The Cournot equilibrium represents the earliest application of a non-cooperative game-theoretic solution concept to the analysis of economic markets. Originally proposed by Cournot (1838) to model the behavior of duopolies making competitive quantity offers, the equilibrium requires that each player select the strategy that is the “best response” to the equilibrium strategies of the other player. In this sense, the Cournot equilibrium forms an important special case of the strategic equilibrium concept developed formally by Nash (1951) over a century later.

The Cournot approach may be extended to markets with arbitrary numbers of both buyers and sellers, in which the buyers’ strategies consist of price bids, and the sellers’ strategies consist of quantity offers. (See Dubey, 1982 and Dubey and Shubik, 1978a, 1978b, and 1980.)
These models were first introduced into the insurance literature by Powers and Shubik (1998) and Powers et al. (1998), who used the Cournot paradigm to study issues of oligopoly power and market configuration. For a market with risk-averse buyers making insurance-premium bids and risk-averse (or risk-neutral) sellers making insurance-indemnity offers, these authors quantified the tradeoff – as the number of sellers increases – between the benefits of increasing competition and the drawbacks of decreasing insurer solvency. The authors also noted that even when insolvency risk is completely eliminated (by full guaranty-fund backing), the benefits of decreasing premiums are still offset by the sellers’ constriction of coverage (because of the decreasing premiums). More recently, Powers and Shubik (2001, 2006) applied the Cournot market-game approach to the study of reinsurance and retrocession markets, showing that the price per unit of coverage in the primary insurance market is minimized when market capital is allocated between the primary and reinsurance sectors in such a way that the number of reinsurers is approximately equal to the square-root of the number of reinsurers.

The present work is the first application of the Cournot model to the problem of optimal insurance/reinsurance contracts.

2. Primary Market with Premium Bids and Indemnity Offers

2.1. Formulation

Consider a primary insurance market with \( m \) homogeneous risk-averse buyers, \( i = 1, 2, \ldots, m \), and \( n \) homogeneous risk-averse sellers, \( j = 1, 2, \ldots, n \). Initially, each buyer \( i \) possesses net wealth \( W_B^i \), and each seller \( j \) possesses net wealth \( W_S^j \). For a given policy period, buyer \( i \)’s assets are subject to a nonnegative random loss amount \( X_i \sim \text{i.i.d.} \ F_x(x) \), and \( i \) has the option of purchasing insurance from seller \( j \) by making a strategic insurance-premium bid,
\(\pi_i \geq 0\), that represents the amount that \(i\) is willing to pay for insurance. Simultaneously, each seller \(j\) has the option of selling insurance by making a strategic insurance-indemnity offer, 
\(I_j(X) \in [0, X]\), that represents the loss payment that \(j\) is willing to make to each buyer that chooses to purchase insurance.

To complete the model, we posit an invisible market mechanism\(^2\) that:

- randomly assigns each buyer \(i\) to a seller \(j;i\) so that each seller \(j\) ends up with a set \(M_j\) of exactly \(\mu\) buyers (where \(\mu = m/n\) is assumed to be an integer);

- collects all premium bids, \(\pi_i\), and distributes them to the \(n\) sellers in proportion to the sellers’ respective indemnity offers, \(I_j(X)\) (i.e., seller \(j\) receives a total premium amount of

\[
\approx \frac{\sum_{i \in M_j} E[X_i I_j(X_i)]}{\sum_{i \in M_j} E[X_i I_j(X_i)] + \sum_{f \in M_j, f \neq j} E[X_f I_f(X_f)]} \sum_{i=1}^n \pi_i;\]

- collects all indemnity payments, \(I_j(X)\), and remits them to the \(\mu\) buyers associated with seller \(j\) in proportion to the buyers’ respective premium bids, \(\pi_i\) (i.e., buyer \(i \in M_j\) receives a total indemnity amount of

\[
\mu \pi_i \left(\pi_i + \sum_{b \in M_j, b \neq i} \pi_b\right) I_j(X_i).
\]

We also assume that each seller \(j\) incurs transaction costs, \(c I_j(X)\), associated with its total losses, and that sellers remain able to pay all loss obligations, regardless of how unfavorable is the actual loss experience of their associated buyers (i.e., insurer insolvency is not possible).

\(^2\) Although the described mechanism is purely abstract, its assumption is necessary to permit the construction of a well-defined noncooperative game outside of equilibrium.
2.2. Equilibrium

We now seek the Cournot-Nash equilibrium solution \((\pi^*, I^*(x))\) such that \(\pi_i = \pi^*\) maximizes

\[
E_{X_i} \left[ u_B \left( W_B - \pi_i + \mu \pi_i \left( \pi_i + \sum_{h \neq i} \pi_h \right) \right) \right]_{\pi_i = \pi^*, I_j(x) = I^*(x)}
\]

(1)

over \(\pi_i \geq 0\) and \(I_j(x) = I^*(x)\) maximizes

\[
E_{X_i, i \in M_j} \left[ u_S \left( W_S + \sum_{i \in M_j} I_j(X_i) + \sum_{i \in M_j} E_{X_i} \left[ I_j(X_i) \right] + \sum_{i \in M_j} \sum_{i' \neq i, j} I_j(X_i) - a \sum_{i \in M_j} I_j(X_i) \right) \right]_{\pi_i = \pi^*, I_j(x) = I^*(x)}
\]

(2)

over \(I_j(x) \in [0, x]\). This is accomplished in several steps.

From (1) we find

\[
\frac{\partial}{\partial \pi_i} E_{X_i} \left[ u_B \left( W_B - \pi_i + \mu \pi_i \left( \pi_i + \sum_{h \neq i} \pi_h \right) \right) \right]_{\pi_i = \pi^*, I_j(x) = I^*(x)}
\]
\begin{align*}
&= E_X \left[ u'_B \left( W_B - \pi_i + \frac{\mu \pi_i}{\pi_i + \sum_{h \neq i \in M_j} \pi_h} I_j(X_i) - X_i \right) \right] \\
&\times \left[ -1 + \left( \mu \pi_i + \sum_{h \neq i \in M_j} \pi_h \right) \right] \\
&= E_X \left[ u'_B \left( W_B - \pi_i + I^* (X_i) - X_i \right) \right] - E_X \left[ u'_B \left( W_B - \pi_i + I^* (X_i) - X_i \right) \right] = 0. \quad (3)
\end{align*}

Then, in (2) we seek to maximize

\[
\int \left( u_S W_z + \sum_{i \in M_j} I_j(x_i) \right) \left( \sum_{i \in M_j} \prod_{f \neq j}^\mu f(x_i) \right) \prod_{i \in M_j} f(x_i) \right|^{I^*_j = I^*, I^*_j = I^*}_x
\]

subject to \( I_j(x_h) \leq x_h \) for \( h = 1, 2, \ldots, \mu \). This is a multivariable case of the isoperimetric optimization problem from the calculus of variations (see, e.g., Weinstock, 1974, p. 133).

Letting

\[
\varphi \left( Y = \sum_{h=1}^\mu I_j(x_h) \right) = I'_j(x_1), \ldots, I'_j(x_\mu) = I'_j(x_1, x_2, \ldots, x_\mu)
\]
\[
\begin{aligned}
&= u_S \left\{ W_S + \frac{\sum_{h=1}^{\mu} I_j(x_h)}{\sum_{h=1}^{\mu} I_j(x_h) + \mu \sum_{f' = 1, f' \neq j}^{\mu} I_f} \right. \\
&\quad \left. \sum_{f' = 1}^{m} \pi'_{f'} - \sum_{h=1}^{\mu} I_j(x_h) - c \left( \sum_{h=1}^{\mu} I_j(x_h) \right) \prod_{h=1}^{\mu} f_X(x_h) \right\},
\end{aligned}
\]

where \( \overline{I}_j = E_X [I_j(X_j)] \), it follows that

\[
\frac{\partial \varphi}{\partial Y} \sum_{h=1}^{\mu} \frac{d}{dx_h} \left( \frac{\partial \varphi}{\partial Y'(x_h)} \right) = 0,
\]

which implies

\[
\frac{\partial \varphi}{\partial Y} = 0
\]

(since \( \frac{\partial \varphi}{\partial Y'(x_h)} \equiv 0 \)). Then

\[
\begin{aligned}
&= u_S' \left\{ W_S + \frac{\sum_{h=1}^{\mu} I_j(x_h)}{\sum_{h=1}^{\mu} I_j(x_h) + \mu \sum_{f' = 1, f' \neq j}^{\mu} I_f} \right. \\
&\quad \left. \sum_{f' = 1}^{m} \pi'_{f'} - \sum_{h=1}^{\mu} I_j(x_h) - c \left( \sum_{h=1}^{\mu} I_j(x_h) \right) \prod_{h=1}^{\mu} f_X(x_h) \right\} \\
&\times \left[ \sum_{f' = 1, f' \neq j}^{\mu} \frac{\sum_{f' = 1, f' \neq j}^{\mu} I_f}{\sum_{f' = 1, f' \neq j}^{\mu} I_f} \right] ^{\sum_{f' = 1, f' \neq j}^{\mu} \pi'_{f'} - 1} - c \left( \sum_{h=1}^{\mu} I_j(x_h) \right) \prod_{h=1}^{\mu} f_X(x_h),
\end{aligned}
\]
\[
W_S = \left[ \sum_{h=1}^{\mu} I^* (x_h) \right] \left[ \frac{\sum_{h=1}^{\mu} I^* (x_h) + \mu (n-1) \bar{T}^* - c \left( \sum_{h=1}^{\mu} I^* (x_h) \right)}{\sum_{h=1}^{\mu} I^* (x_h) + \mu (n-1) \bar{T}^* - c \left( \sum_{h=1}^{\mu} I^* (x_h) \right)} \right]^{\frac{1}{2}}
\]

which yields

\[
\mu (n-1) \bar{T}^* m \bar{\pi}^* \left[ 1 + c \left( \sum_{h=1}^{\mu} I^* (x_h) \right) \right] \left( \sum_{h=1}^{\mu} I^* (x_h) + \mu (n-1) \bar{T}^* \right) = 0.
\] (4)

Now let \( c(I) = \gamma + \delta \). For pure internal solutions \( I^* (x_h) \) (i.e., solutions such that \( I^* (x_h) \leq x_h \) for \( h = 1, 2, \ldots, \mu \)), condition (4) implies

\[
\mu (n-1) \bar{T}^* m \bar{\pi}^* - \gamma \left( \sum_{h=1}^{\mu} I^* (x_h) + \mu (n-1) \bar{T}^* \right) = 0,
\]

and so \( I^* (x_h) \) must equal the constant

\[
I^* = \frac{(n-1) \pi^* \bar{T}^*}{1 + \gamma} - (n-1) \bar{T}^*.
\]

However, boundary solutions \( I^* (x_h) = x_h \) will occur if

\[
\mu (n-1) \bar{T}^* m \bar{\pi}^* - \gamma \left( \sum_{h=1}^{\mu} I^* (x_h) + \mu (n-1) \bar{T}^* \right) > 0,
\]

and this inequality must hold for any particular \( x_h' < I^* \) because
Thus,

\[
I^*(x) = \begin{cases} 
    x, & \text{for } x < \sqrt{\frac{(n-1)n\pi^* I^*}{1+\gamma} - (n-1)I^*} \\
    \sqrt{\frac{(n-1)n\pi^* I^*}{1+\gamma} - (n-1)I^*}, & \text{otherwise} 
\end{cases}
\]  
(5)

where

\[
I^* = E_X \left[ I^* (X) \right] = E_X \left[ \min \left\{ X, \sqrt{\frac{(n-1)n\pi^* I^*}{1+\gamma} - (n-1)I^*} \right\} \right],
\]  
(6)

and the solution \( (\pi^*, I^*(x)) \) is found by solving equations (3) and (6) simultaneously.

Clearly, the form taken by the equilibrium contract \( I^*(X) \) in equation (5) is that of insurance subject to a policy limit. In the following subsection, we illustrate how the equilibrium policy limit, \( I^* \), and premium, \( \pi^* \), vary over certain parameters of interest.

2.3. Example

Consider a hypothetical example of the market game described above in which:

\[
\mu = 500, \ n = 200 \quad \text{(unless } n \text{ is the parameter being varied)},
\]
\[ W_B = 2, W_S = 800, \]
\[ u_B(z) = -(z - W_B)^2, \quad u_S(z) = -(z - W_S - 1000)^2, \]
\[ X \sim \text{i.i.d. Gamma}(\alpha = 1/2, \beta = 1), \]
\[ \gamma = 0.2 \quad \text{(unless } \gamma \text{ is the parameter being varied), and } \delta = 0. \]

Figures 1 - 4 reveal how both \( I^g \) and \( \pi^* \) vary over \( \gamma \) (the transaction-cost coefficient) and \( n \) (the number of sellers), respectively. Notably, all of the plots are consistent with intuition. We find that decreasing transaction costs lead to both more favorable policy limits and more favorable premiums for the buyers, and that increasing competition among the sellers is similarly beneficial.

**Figure 1. Equilibrium Policy Limit vs. Transaction-Cost Coefficient**  
*(Primary Market with Premium Bids, Indemnity Offers)*

\[ [3] \text{In the interest of realism, all dollar values are taken to be in (} \$ \text{) millions.} \]
Figure 2. Equilibrium Premium vs. Transaction-Cost Coefficient (Primary Market with Premium Bids, Indemnity Offers)

Figure 3. Equilibrium Policy Limit vs. Number of Sellers (Primary Market with Premium Bids, Indemnity Offers)
Although not shown in any figures, one can see immediately from equations (3) and (6) that both \( I^b \) and \( \pi^* \) are largely insensitive to \( \mu \) (the number of buyers associated with each seller). This is not too surprising since the sellers cannot become insolvent, and so the law of large numbers plays no role with regard to the financial stability of the sellers.

3. Primary Market with “Quantity” Bids and Indemnity Offers

3.1. Formulation

One compelling attraction of the Cournot paradigm is its ability to capture a variety of different market characteristics and institutional settings. Although more will be said about this in Section 5, for the moment we simply offer an alternative formulation of the primary insurance market game described in Subsection 2.1 that allows us to question whether the optimal policy described in equation (5) is dependent on the asymmetry, in terms of complexity, of the buyers’
and sellers’ strategies. In short, we wish to investigate whether the optimality of the policy limit is merely an artifact of the buyers’ restriction to simple (scalar) premium bids, while the sellers are able to make more complicated (functional) indemnity bids.

To address this point, we let the premium amount \( \pi^* \) be fixed exogenously, and let each buyer \( i \) make a “quantity” bid, \( K_i(X_i) \), representing the amount of insurance coverage \( i \) would like to purchase at the fixed price per unit of \( \frac{\pi^*}{E_X[X_i]} \). Correspondingly, we assume that the “market mechanism” collects all premium bids, \( \frac{E_X[K_i(X_i)]}{E_X[X_i]} \pi^* \), and distributes them to the \( n \) sellers in proportion to the sellers’ respective indemnity offers, \( I_j(X_j) \) (i.e., seller \( j \) receives a total premium amount of approximately

\[
\frac{\sum_{i \in M_j} E_X[K_i(X_i)]}{\sum_{i \in M_j} E_X[I_j(X_i)] + \sum_{j \neq i} \sum_{c=1}^m E_X[K_c(X_c)]} \left( \sum_{j \in M_j} I_j(X_j) \right) \pi^*.
\]

Similarly, the mechanism collects all indemnity payments, \( I_j(X_j) \), and remits them to the \( \mu \) buyers associated with seller \( j \) in proportion to the buyers’ respective premium bids,

\[
\frac{K_i(X_i)}{E_X[X_i]} \pi^* \text{ (i.e., buyer } i \in M_j \text{ receives a total indemnity amount of approximately}
\]

\[
\left[ \mu E_X[K_i(X_i)] \right] \left( E_X[K_i(X_i)] + \sum_{h \neq i} E_X[K_h(X_h)] \right) I_j(X_j).
\]

3.2. Equilibrium

We now seek the Cournot-Nash equilibrium solution \( (K^*(x), I^*(x)) \) such that \( K_i(x) = K^*(x) \) maximizes
\begin{equation}
E_X \left[ u_B \left( W_B - \frac{K_i(X_i)}{E_X[X_i]} \pi^* + \left[ \frac{\mu K_i(X_i)}{K_i(X_i) + \sum_{h \in M_{i(\cdot)}} E_X[K_h(X_h)]} \right] I_{i(0)}(X_i) - X_i \right) \right]_{K_i = K_*^i, I_{i(0)} = I_*^i}
\end{equation}

over $K_i(x) \in [0, x]$ and $I_j(x) = I^*(x)$ maximizes

\begin{equation}
E_{X_i, i \in M_j} \left[ W_S \left( \sum_{i \in M_j} I_j(X_i) + \sum_{i \in M_j, i \neq j} E_X[I_j(X_{i'})] \right) \sum_{i \in M_j} \frac{E_X[K_j(X_{i'})]}{E_X[X_{i'}]} \pi^* - \sum_{i \in M_j} I_j(X_i) - c \left( \sum_{i \in M_j} I_j(X_i) \right) \right]_{I_{j(1)} = I_*^j, I_{j(0)} = I^*_{j(0)}}
\end{equation}

over $I_j(x) \in [0, x]$.

Using calculus-of-variants methods analogous to those employed in problem (2) above, and assuming $c(I) = \gamma I + \delta$, we find that (6) and (7) yield the respective solutions

\[ K^*(x) = \begin{cases} x, & \text{for } x < K^# \\ K^#, & \text{otherwise} \end{cases} \]

and

\[ I^*(x) = \begin{cases} x, & \text{for } x < I^# \\ I^#, & \text{otherwise} \end{cases} \]

where $K^#$ and $I^#$ are given by the system

\begin{equation}
\mu(\mu - 1)K^# I^# - \frac{\pi^*}{E_X[X]} \left[ K^# + (\mu - 1)K^* \right] = 0,
\end{equation}

\begin{equation}
\frac{n(n - 1)(I^* K^*) \pi^*}{E_X[X]} - (1 + \gamma) \left[ I^# + (n - 1)I^* \right] = 0,
\end{equation}

\[ K^* = E_X[K^*(X)], \]

and
\[ T^* = E_x \left[ I^* (X) \right] \]  

(11)

As in Section 2, the optimal insurance policy is again characterized by a policy limit, despite the change in the buyers’ strategy space.

### 3.3. Example

Consider the market game just described, but with the same parameter values and utility functions as those used in the example of Subsection 2.3 and setting \( \pi^* = 0.674 \) (the equilibrium premium from the same prior example). Figures 5 and 6 show how \( I^* \) varies over \( \gamma \) and \( n \), respectively. As in the earlier example, both plots are consistent with intuition: decreasing transaction costs and increasing competition among the sellers both lead to more favorable policy limits for the buyers. Instructively, we also find that \( K^* \) – the upper limit of the buyer’s “quantity” bid – diverges to positive infinity in all cases considered (although this result would not hold if \( \pi^* \) were set substantially higher). Finally, as in Subsection 2.3, \( I^* \) is largely insensitive to \( \mu \), for the reasons previously indicated.
Figure 5. Equilibrium Policy Limit vs. Transaction-Cost Coefficient
(Primary Market with “Quantity” Bids, Indemnity Offers)

Figure 6. Equilibrium Policy Limit vs. Number of Sellers
(Primary Market with “Quantity” Bids, Indemnity Offers)
4. The Case of Reinsurance

In modeling a reinsurance market, the Cournot approach offers several options. Naturally, the simplest formulation would be to ignore the primary market entirely, and simply apply one of the models from Section 2 or 3 to the transaction between primary insurers and reinsurers. At another extreme, one could model both the primary and reinsurance markets simultaneously in a single game with three players – buyers, sellers, and reinsurers. However, given the institutional nature of the global reinsurance market, we prefer to chart a middle course in which both markets are modeled in some detail, but operate in two stages, with the primary-market transactions occurring first, and the reinsurance ones second. Specifically, we apply the premium-bid/indemnity-offer model of Section 2 to the primary market, and the “quantity”-bid/indemnity-offer model of Section 3 to the reinsurance market.

The rationale for these modeling choices is simple. In primary markets, buyers tend to focus on price (premiums) and sellers on quantity (indemnity amounts), whereas in reinsurance markets, the primary insurers tend to focus on quantities of coverage ceded, and the reinsurers on the quantity of coverage assumes. In addition, the model of Section 3 neatly captures an important institutional reality of reinsurance – that prior to the reinsurance transaction, an exogenous price per unit of coverage (denoted here by $\frac{\pi^*}{E_x [X_i]}$) is established in the primary insurance market.

Naturally, the application of the “quantity”-bid/indemnity-offer model to reinsurance requires a few notational changes. Most fundamentally, “buyers” must be called “primary insurers,” and “sellers” must be called “reinsurers”. In addition, we define the following quantities:

- $\mu_r$, the number of primary insurers per reinsurer;
$n_R$, the total number of reinsurers;

$W_R$, the reinsurer’s initial net wealth;

$u_R(z)$, the reinsurer’s utility function;

$X_{R,j}$, primary insurer $j$’s total loss amount ($\sim$ i.i.d. $F_{X_R}(x)$);

$\gamma_R$ and $\delta_R$, the reinsurance transaction-cost parameters;

$C_j(x)$, primary insurer $j$’s “quantity” bid;\footnote{This corresponds to $K_i(x)$ in the primary insurance market game.} and

$A_k(x)$, reinsurer $k$’s indemnity offer.\footnote{This corresponds to $I_j(x)$ in the primary insurance market game.}

To illustrate the two-stage model, we employ the same parameter values for the primary market used in the example of Subsection 2.3, and use $\pi^* = 0.674$ and $I^* = 6.24$, the outputs from this market, in the reinsurance market. Further assumptions are:

$\mu_R = 20$ (unless $\mu_R$ is the parameter being varied),

$n_R = 10$ (unless $n_R$ is the parameter being varied),

$W_R = 80,000$,

$u_R(z) = -(z - W_R - 100,000)$,

$X_R = \sum_{i=1}^{\mu} \text{Min}\{X_i, I^* = 6.24\}$ where $X_i \sim$ i.i.d. Gamma($\alpha = 1/2, \beta = 1$),

$\gamma_R = 0.10$ (unless $\gamma_R$ is the parameter being varied), and $\delta_R = 0$.

Figures 7 and 8 show how the optimal reinsurance policy limit, $A^*$, changes over $\gamma_R$ and $n_R$, respectively. As expected, this behavior is completely analogous to that of $I^*$ with respect to $\gamma$ and $n$, respectively, in Subsection 3.3. We also note that $C^*$ – the upper limit of the
primary insurer’s “quantity” bid – diverges to positive infinity in all cases considered, just as did $K^*$ in the previous example. Finally, $A^*$ is largely insensitive to $\mu_R$, as was $I^*$ to $\mu$.

Figure 7. Equilibrium Policy Limit vs. Transaction-Cost Coefficient (Reinsurance Market)

Figure 8. Equilibrium Policy Limit vs. Number of Reinsurers (Reinsurance Market)
5. Discussion and Conclusions

5.1. Cournot Versatility

We have commented briefly, both directly and indirectly, on the general flexibility of the Cournot paradigm. As an illustration, we showed how a primary insurance market could be modeled using either a simple premium-bid/indemnity-offer game, or a more sophisticated “quantity”-bid/indemnity-offer game. We also mentioned that the Cournot approach can accommodate institutional realities such as the exogeneity of $\pi^*$ in reinsurance markets. We now wish to cite a few additional potential extensions that enhance the overall applicability of Cournot models.

To begin, we point out a natural generalization of the models of Sections 2 and 3: rather than having the buyer make either a premium bid or a “quantity” bid, it is possible to have the buyer do both – i.e., to select both a premium, $\pi_i$, and a “quantity”, $K_i(x)$, simultaneously. This extension essentially endogenizes the $\pi^*$ of the Section 3 model. Another way to endogenize $\pi^*$ in the “quantity”-bid/indemnity-offer game would be to have the seller select a premium offer in addition to its indemnity offer, in a manner reminiscent of Rothschild and Stiglitz (1976).[6]

Other generalizations, although somewhat technical, are interesting nonetheless.

Consider, for example, the market mechanism employed in the premium-bid/indemnity-offer game of Section 2. According to our assumptions, this mechanism collects all indemnity payments, $I_j(X_i)$, remitting them to the $\mu$ buyers associated with seller $j$ in proportion to the buyers’ respective premium bids, $\pi_i$, so that buyer $i \in M_j$ receives a total indemnity amount of

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[6] We are grateful to David Cummins for suggesting this possibility.
\[
\left[ \frac{\mu \pi_i}{\pi_i + \sum_{h \in M_j, h \neq i} \pi_h} \right] I_j(X_i) \]

However, this is not the only way in which the indemnity allocation could be done. One simple but significant alternative would be for the mechanism to distribute the indemnity payments, \( I_j(X_i) \), to the \( m \) buyers in the market in proportion to the buyers’ respective premium bids, \( \pi_i \), so that buyer \( i \) receives a total indemnity amount of

\[
\left[ \frac{m \pi_i}{\pi_i + \sum_{j' \in M} \pi_{j'}} \right] I_j(X_i) \]

This would have the effect of forcing the buyers to compete with one another across the entire market, rather than just within a given seller.

Another alteration would be to pool the indemnity amounts themselves – either across each seller or across the entire market, depending on the institutional context – yielding

allocations of

\[
\left[ \frac{\pi_i}{\pi_i + \sum_{h \in M_j, h \neq i} \pi_h} \right] \sum I_j(X_h) \text{ or } \left[ \frac{\pi_i}{\pi_i + \sum_{j' \in M} \pi_{j'}} \right] \sum I_j(X_{j'}) \]

respectively.

This approach would be especially useful in applications involving the study of seller and/or market financial capacity.

5.2. Whither Deductibles?

The principal contribution of the present research is to offer a simple economic explanation for the proliferation of policy limits in various lines of insurance. Without disputing the existence of alternative explanations – such as Raviv’s (1979) regulatory rationale, or the need of sellers to truncate losses characterized by unbounded first or second moments – we would argue that our analysis covers a substantially broader range of cases, including many in which other explanations fail.
We hasten to note that the policy-limit contract, while optimal in market equilibrium, remains dominated by the Pareto-optimal – but possibly unachievable – deductible contract. This point is illustrated in Figure 9, based upon the premium-bid/indemnity-offer model of Section 2.[7] Here the optimal policy in equilibrium is represented by the single point 
\[ \left( E_X \left[ U_y; \pi^*, I^* \right], E_X \left[ U_z; \pi^*, I^* \right] \right) = (-0.00949, -992,600) \]. Fixing \( \pi^* \), while allowing the policy limit to take on various values \( \ell \) above and below \( I^* \), yields the entire “policy-limit frontier” 
\[ \left( E_X \left[ U_y; \pi^*, \ell \right], E_X \left[ U_z; \pi^*, \ell \right] \right) \], indicated by the dashed curve. The Pareto-optimal “deductible frontier,” 
\[ \left( E_X \left[ U_y; \pi^*, d \right], E_X \left[ U_z; \pi^*, d \right] \right) \], is found by similarly fixing \( \pi^* \), but replacing \( I^* \) with deductible coverage for various deductibles \( d \).

Figure 9. Optimal Insurance in Market Equilibrium
Compared to Pareto-Optimal (Deductible) Frontier

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[7] The present example uses the same functional and parametric assumptions of Subsection 2.3, except that now \( n = 75 \) and \( X \sim \text{i.i.d. Pareto}(\alpha = 8, \theta = 0.5) \). These changes are made to increase the tail probability, \( 1 - F_X \left( I^* \right) \), so that \( I^* \) can be distinguished sufficiently well from full insurance.
Although the optimal policy contract in market equilibrium is generally not the same thing as the optimal policy under a criterion of Pareto optimality, we believe that it may be possible to develop an appropriate model or generalization capable of bridging these two approaches. In this way, one might be able to identify market conditions – short of requiring sellers, by regulatory fiat, to offer only deductible policies – sufficient to ensure Pareto optimality. Given the versatility of the Cournot approach, we have attempted to construct market-game models whose equilibrium solutions justify insurance deductibles. While results so far have been limited and insufficiently realistic, this remains an area of continued research interest.

Naturally, we are confident that insurance contracts will continue to be written with deductibles (as well as pro-rata arrangements) for very sound economic and institutional reasons apart from the types of “optimality” discussed herein. Even in insurance markets with risk-averse buyers, risk-neutral sellers, and minimal transaction costs, we suspect that the presence of deductibles is more easily explained by the sellers’ desire to reduce moral hazard, and the buyers’ desire to save on transaction inefficiencies, than by any tangible consideration of “optimal” insurance.

In the case of reinsurance, of course, the presence of a primary market covering a buyer’s losses up to a specified retention does nothing to reduce the buyer’s moral hazard. In a sense, then, our analysis suggests that, from the perspective of reinsurers, the primary insurance market may be viewed as existing simply for the purpose of eliminating the transaction inefficiencies associated with the small claims covered by primary insurers (which generally require a larger transaction-cost coefficient than reinsurance claims).
References


