A Cautious Note on Natural Hedging of Longevity Risk

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Illinois State University

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Georgia State University
1. Introduction

2. Mortality Forecasting Models

3. Economic Capital for a Stylized Insurer

4. Natural Hedging of Longevity Risk

5. Conclusion
Introduction

Mortality Forecasting Models

Economic Capital for a Stylized Insurer

Natural Hedging of Longevity Risk

Conclusion
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Background & Literature Review

Longevity risk

\[ \Downarrow \]

Policyholders’ future realized mortality rates

\[ \Downarrow \]

Life insurers’ liabilities

Approaches to protecting against longevity risk:
- **Stochastic** mortality forecasting models
- Externally \[ \rightarrow \] Mortality-linked securities
- Internally \[ \rightarrow \] natural hedging
  - life insurances \[ \leftrightarrow \] annuities

Literature:
- Cox and Lin (2007): Companies selling both life and annuity products charge cheaper prices ⇒ evidence of natural hedging
- Wetzel and Zwiesler (2008): Portfolio composition significantly impacts longevity exposure
- Tsai et al. (2010): Optimal product mix to minimize CVaR
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Underlying mortality forecasting models:

- **Existing literature:**
  - (Low-dimensional) factor models: Lee-Carter model (Lee and Carter (1992)), CBD model (Cairns et al. (2006))
  - Error term $\sigma_t$ affects time-$t$ mortality rates at different ages simultaneously
  - Cannot capture disparate shifts in mortality rates at different ages
  - Life insurances (working class) $\Leftrightarrow$ annuities (retirees)
  - *Positive* conclusions of natural hedging

- **This paper:**
  - Parametric factor model & non-parametric mortality model (Zhu and Bauer (2012))
  - *Natural* way to test natural hedging

**Main findings:**

- Using factor models helps to create a **perfect hedge** for mortality risk by utilizing natural hedging
- **BUT:** Different result from non-parametric mortality model
  - Natural hedging might not be as effective as we think
Contributions

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Mortality Forecasting Models

Non-Parametric Model

Forward survival probabilities:

\[
\tau p_x(t) 1_{\{\tau_{x-t} > t\}} = \mathbb{E}^{P} \left[ 1_{\{\gamma_{x-t} > t+\tau\}} \mid \mathcal{F}_t \vee \{\gamma_{x-t} > t\} \right], \quad 0 \leq T \leq t \leq T + \tau
\]

Generational survival data \(\tau p_x(t_j)\): \(j = 1, \ldots, N\)

\[
F(t_j, t_{j+1}, (\tau, x)) = -\log \left\{ \frac{\tau+1 p_x(t_{j+1})}{\tau+1+t_{j+1}-t_j p_x(t_{j+1}+t_j(t_j))} \right\}
\]

\[
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\[
\bar{F}(t_j, t_{j+1}) = (F(t_j, t_{j+1}, (\tau, x)))_{(\tau, x) \in \mathcal{C}}, \quad j = 1, 2, \ldots, N - 1
\]

\[
\Rightarrow \quad \bar{F}(t_j, t_{j+1}) \text{ are i.i.d. Gaussian distributed (Prop. 2.1, Zhu and Bauer (2012))}
\]

\[
\Rightarrow \quad \text{Simulate } \bar{F}(t_N, t_{N+1}) \text{ based on sample mean and covariance matrix from } F(t_j, t_{j+1}, (\tau, x)), \quad j = 1, \ldots, N - 1
\]

\[
\Rightarrow \quad \tau p_x(t_{N+1})
\]
Mortality Forecasting Models

Parametric Factor Model

Forward force of mortality (easier to model/work with than $\tau p_x(t, T + \tau)$):

$$\mu_t(\tau, x) = -\frac{\partial}{\partial \tau} \log \{\tau p_x(t, t + \tau)\}$$

Consider **time-homogenous diffusion-driven** models (cf. Bauer et al. (2012))

$$d\mu_t = (A \mu_t + \alpha) \, dt + \sigma \, dW_t$$

- **Drift condition** (Cairns et al. (2006, ASTIN)): With $W_t$ Brownian motion under $\mathbb{P}$,

  $$\alpha(\tau, x) = \sigma(\tau, x) \times \int_0^\tau \sigma'(s, x) \, ds$$

- **Bauer et al. (2012):** $\mu_t$ allows for a Gaussian finite-dimensional realization (FDR) iff

  $$\sigma(\tau, x) = C(x + \tau) \times \exp\{M\tau\} \times N$$

- **Zhu and Bauer (2012):**

  $$\sigma(\tau, x) = k \frac{\exp\{c(x + \tau) + d\}}{1 + \exp\{c(x + \tau) + d\}} (a + \tau) e^{-b\tau},$$
Economic Capital for a Stylized Insurer

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Economic Capital for a Stylized Insurer

Economic Capital Calculation

- Newly founded life insurer selling traditional products (term-life, endowment, annuity); Equivalence Principle; risk-neutral w.r.t. mortality risk

- Available Capital at time zero: \( AC_0 = E \)

- Available Capital at time one: \( AC_1 = E_Q^{Assets|F_1} - E_Q^{Liabilities|F_1} \)

- One-year mark-to-market approach for calculating Economic Capital:

\[
EC = \rho \left( \frac{AC_0 - AC_1}{p(0, 1)} \right)
\]

- \( \rho \): monetary risk measure \((L^2(\Omega, \mathcal{F}, \mathbb{P}) \rightarrow \mathbb{R})\)

  - Solvency Capital Requirement (Solvency II):

\[
EC = SCR = \text{VaR}_\alpha(L) = \arg \min_x \{P(L > x) \leq 1 - \alpha\}
\]

  - Conditional Tail Expectation (used within SST):

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EC = \text{CTE}_\alpha = \mathbb{E}[L|L \geq \text{VaR}_\alpha(L)]
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- U.S. female data (Human Mortality Database), year 1933-2007
- 46 generational life tables: 1963-2008, age: 0-95 \( \tau p_x(t_j), j = 1, \ldots, 46 \)
- Calibrate and forecast under:
  0 Deterministic mortality (Lee-Carter)
  1 Non-parametric model
  2 Parametric factor model

Financial market estimation:
- Financial portfolio: stock, 1-year, 3-year, 5-year, and 10-year gov. bond
- Financial market model: Extended Black-Scholes model with stochastic interest rates (Vasicek model)
- Calibrated to U.S. data from 01-1982 to 07-2012 using Kalman filter

50,000 simulations of \( A_1 \) and \( V_1 \) \( \Rightarrow \) \( AC_1 \) \( \Rightarrow \) \( EC \)
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Equal weights in financial portfolio; $E = $20,000,000

<table>
<thead>
<tr>
<th>Term Life</th>
<th>$x$</th>
<th>$i$</th>
<th>$n_{x,i}^{\text{term/end/ann}}$</th>
<th>$B_{x,i}^{\text{term/end/ann}}$</th>
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Economic Capital for a Stylized Insurer

Base Case

Equal weights in financial portfolio; $E = 20,000,000$

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Nan Zhu
Natural Hedging Examination
Optimal static hedge:

- Minimizing economic capital by changing weights in bonds/stock

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Optimal static hedge:

- Exposure in annuity/endowment ⇒ fixed
- Adjust exposure in term-life insurance $n^{\text{term}}$:
  - Minimize capital with optimizing financial risk
- Three cases: deterministic mortality vs. factor mortality model vs. non-parametric model
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Natural Hedging of Longevity Risk

Observations

- Without systematic mortality, EC increases in $n^{\text{term}}$
- With factor mortality model, EC convex of $n^{\text{term}}$ ($n^{\text{term}*} = 16,500$)
- **BUT** With non-parametric forecasting model, EC again is strictly increasing in $n^{\text{term}}$
  - Only a very mild effect of natural hedging observed

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  - (Almost) perfect hedge of mortality risk with natural hedging
- Using the non-parametric mortality model, adding mortality risk increases the optimal economic capital a lot (64%)
  - Natural hedging does not work as well as we expect
  - Factor models too simplified
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Economic capital: ($n^{\text{term}} = 16,500$)

<table>
<thead>
<tr>
<th>Deterministic Mortality</th>
<th>Factor Model</th>
<th>Non-parametric Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9,909,874$</td>
<td>$9,954,353$</td>
<td>$16,259,641$</td>
</tr>
</tbody>
</table>

- Using the factor mortality model, adding mortality risk increases the optimal economic capital slightly (0.44%)
  - (Almost) perfect hedge of mortality risk with natural hedging
- Using the non-parametric mortality model, adding mortality risk increases the optimal economic capital a lot (64%)
  - Natural hedging does not work as well as we expect
  - Factor models too simplified
Conclusion

1 Introduction

2 Mortality Forecasting Models

3 Economic Capital for a Stylized Insurer

4 Natural Hedging of Longevity Risk

5 Conclusion
Conclusion

Natural hedging proposed to handle longevity risk

- Positive results from existing literature
  - Use factor mortality models
  - Neglect disparate mortality evolutions under different ages
  - Entail potential biases

- We compare results derived from both parametric factor and non-parametric stochastic mortality model
  - Concur the existing literature when the factor model used
  - With non-parametric model, natural hedging much less effective

How much should we trust model-based results?

- Advantages: simple, easy to use, etc.
- CAVEAT: important features might be stripped
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