Heideggerian Temporality and the Demand for Insurance

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with assistance from Evan Magnusson
Martin Heidegger, *Being and Time* (1927)

• ‘DaSein’ (anti-Cartesian ‘person’) situated in time and space, have to account for:
  – Pre-reflective periods (engagement w/ world)
  – Reflective periods (“carpentry” to differentiate)
Problem:
Find an estimator for the “best” amount of insurance for next period, $\tau$.

One source of information for best coverage comes from maximizing expected utility. Call this $\alpha$.

But another source of information comes past engagements as DaSein (generically, this information source is denoted $\Sigma$). Let $d(\Sigma)$ be the index that summarizes all prior ‘experience in the world, or events from engagement’ related to insurance demand that the insured in the reflective period chooses to consider in the insurance purchase decision.

Let’s assume that $\alpha$ yields an unbiased estimator for insurance demand, $E(\alpha) = \tau$. Why consider $d(\Sigma)$?
\[ E[(wd(\Sigma) + (1-w)\alpha - \tau)^2] \]
\[ E[((wd - w\alpha) + (\alpha - \tau))^2] \]
\[ E[((wd - w\alpha)^2 + (\alpha - \tau)^2 + 2(wd - w\alpha)(\alpha - \tau)) \]
\[ (1 - 2w + w^2)V(\alpha) + 2(w - w^2)Cov(d, \alpha) + w^2(\tau^2 + E(d^2) - 2\tau Ed) \]

**FOC:**
\[ (-2 + 2w)V(\alpha) + 2(1 - 2w)Cov(d, \alpha) + 2w(\tau^2 + E(d^2) - 2\tau Ed) = 0 \]

**SOC:**
\[ V(\alpha) + (-2)Cov(d, \alpha) + (\tau^2 + E(d^2) - 2\tau Ed) \]
\[ w = \frac{V(\alpha) - Cov(d, \alpha)}{V(\alpha) - 2Cov(d, \alpha) + E[(d - \tau)^2]} \]

i.e., Heidegger, the first behavioral economist.
Integrated past experience means:

• Lab experiments (which ignore \(d(\Sigma)\)) won’t reflect real world decisions very well

• \(d(\Sigma)\) in developed market countries may include experience which increases individuals intuition about the allocative role of prices and information in price signals

• \(d(\Sigma)\) may include past information revealed from close associates’ behavior was valuable, suggesting a rationale for claims contagion

• Historsis effects: I buy policy X because my uncle bought policy X
Three approaches to $d(\Sigma)$:

• Indices of lottery structures as data summary for lotteries just as mean and standard deviation are data summary for random vars

• Estimate empirical indices ($d(\Sigma)$) based on cohort-specific experience with various health conditions (i.e., health, disability cond. specific)

• Or the combo deal: use empirical indices to check against lottery choices based on deductibles, co-payments, OOP max, etc (i.e., health plan specific empirics)
3 Versions of Lottery Structure
(to help form ‘lottery certainty’ intuition)

- Mean preserving spread lotteries
- General spread/mean-variance lotteries
- Changed wealth lotteries

\[
\alpha s_1 W + (1 - \alpha)s_2 W = W
\]

**Mean preserving spread lotteries:**

\[
\alpha s_1 W + (1 - \alpha)s_2 W = W
\]

**General spread lotteries (or mean-variance lotteries):**

\[
s_1 > 1; s_2 < 1 \text{ or the other way around}
\]

**Wealth changing lotteries:**

\[
s_1 \geq 1; s_2 \geq 1 \text{ with at least one scaling factor strictly greater than one, OR } s_1 \leq 1; s_2 \leq 1 \text{ with one scaling factor strictly less than one.}
\]
North, South, East, and West

- Horizontal Reductions allowed (by all)
- Vertical Reductions (not necessarily allowed)
  - More on these later
Mean Preserving Lotteries

• Branch multiplication
  – Preserve expected wealth
  – Sensible that individuals prefer certain wealth to a lottery
  – Intuitive that a lottery built up from other lotteries is less preferred
L₀: W with probability 1

L₀ X L₁: W

\[ \alpha \]
\[ W \left(1 + \frac{L}{W\alpha}\right) \]
\[ 1 - \alpha \]
\[ W \left(1 - \frac{L}{W(1-\alpha)}\right) \]

L₀ X L₁ X L₂: W

\[ \alpha \]
\[ W \left(1 + \frac{L}{W\alpha}\right) \]
\[ \alpha (1-\beta) \]
\[ W \left(1 + \frac{L}{W\alpha}\right) \]
\[ 1 - \frac{M}{\beta W(1 + L/(W\alpha))} \]
\[ W \left(1 - \frac{L}{W(1-\alpha)}\right) \]

It is straightforward to show that the expected values of both lotteries is W, but that the variance of the L₀ X L₁ X L₃ lottery is greater than the variance of the L₀ X L₁ lottery.

\[ L₀ > L₀ \otimes L₁ > L₀ \otimes L₁ \otimes L₂ \]
Lottery Structure

• Logarithmic (since $h(s_1 s_2) = h(s_1) + h(s_2)$)
  – Easily allows horizontal reduction

  $\int_{p_i W_i}^{p_j W_j} \frac{1}{\psi} d\psi = \ln(p_j W_j) - \ln(p_i W_i) = \ln\left(\frac{p_j W_j}{p_i W_i}\right)$

• Lottery Certainty $= \sum p_i \ln(p_i W_i) = \sum p_i \ln(p_i) + \sum p_i \ln(W_i)$

  ▪ Sum of 2 terms
    ▪ Log of geometric mean of wealth
      ▪ Discounts outlying levels of wealth
      ▪ concave
    ▪ Negative of Shannon’s information entropy equation
Shannon Information Theory

• Previous Applications: Only Yang and Qiu (2005) have previously applied entropy to lottery choices (but as an ad hoc add-on)
  – Theil—information theory and covariance structure (*Theory and Measurement of Consumer Demand*)
  – Garner and Hake (1951)
  – Norwich (1993)
Shannon Information Theory

- \( h(p) = - \ln(p) \)
  - Unexpected events are more informative, i.e., choose \( d(\Sigma) \), such that \( \text{Cov}(\alpha, d(\Sigma)) \leq 0 \)
  - Equation decreases at a decreasing rate
- \( p=\) subj. prob. before outcomes, \( h(p) \) is what information there is in an event occurring
  - The greater the value of \( p \), the less information is conveyed
- - Entropy = probability certainty = \( -\Sigma p \ h(p) = \Sigma p \ln(p) \)
Merits of Shannon’s Equation

• Accommodates Knightian uncertainty
  – an entirely unanticipated event, $p=0$, is an infinite surprise with infinite information content

• Only equation to satisfy all four of the axioms presented by Theil (1972)
  – Information depends only on $p$
  – continuous, monotonically declining in $p$
  – $h(1)=0$
  – Information from independent events is additive
Merits, cont.

• Lottery entropy decreases in prizes (i.e., as prizes increase in value, the consumer is happier in a lottery entropy sense), holding probabilities constant
The Model

• General Lottery Certainty = \( \sum p_i \ln(p_i) + \lambda \sum p_i \ln(W_i) \)
  • Assume \( \lambda = 1 \)

• Shannon’s model suggests that of all past experiences in life, the kinds of information that ends up in the \( d(\Sigma) \) function will tend to be the most informative events: those that are most surprising given the subjective probabilities of a loss occurring in the classical model
Why we prefer simpler lotteries

• Easier to comprehend
• Developed as a life intuition
  – We experience life in simple lotteries
    • Agriculture
    • Storms
LC as a generalization of EU theory when utility is logarithmic

• There are vertical reductions (if we allow them) to make the LC into the EU model

• LC doesn’t use vertical reductions
  – Implies ‘consequentialism’, or non-separability of preferences, that we take the past into account (Machina 1989)
  – Model fits behavior better without them
An Example of Vertical Reductions

\[ \ln(100) > \alpha \ln(100) + \alpha \ln(\alpha) + (1-\alpha) \ln(105) + (1-\alpha) \ln(1-\alpha) \]

Without Vertical reductions:

With Vertical Reductions:

\[ \alpha \ln(100) + (1-\alpha) \ln(100) + \alpha \ln(\alpha) + (1-\alpha) \ln(1-\alpha) < \alpha \ln(100) + \alpha \ln(\alpha) + (1-\alpha) \ln(105) + (1-\alpha) \ln(1-\alpha) \]

\[ \ln(100) < \ln(105) \]

Thus, with vertical reductions, the consumer always prefers the wealth improving lottery.
Consumers Maximize LC

• Consumers maximize LC in their reflective periods when they are summarizing past information about risk

• The way the lottery is presented matters
Maximizing LC over Lotteries with the Same Expected Value

- Implies risk aversion (symmetric and asymmetric outcomes)
- Implies ambiguity aversion (Ellsberg paradox)
- Implies prudence
- Implies temperance
- Implies the pseudocertainty effect
Risk Aversion: Symmetric Outcomes

\[ \ln(W) > 0.5 \ln(W-L) + 0.5 \ln(W+L) + \ln(0.5) \]

\[ 0 > 0.5 \ln\left(1 - \frac{L^2}{W^2}\right) + \ln(0.5) \]
Risk Aversion: Asymmetric Outcomes

\[
\begin{align*}
0 & > p \ln \left( 1 + \frac{L}{pW} \right) + (1-p) \ln \left( 1 - \frac{L}{(1-p)W} \right) + p \ln(p) + (1-p) \ln(1-p) \\
& \quad \ln \left( 1 - \left( \frac{L}{W} \right)^2 \right) < 0
\end{align*}
\]
Ambiguity Aversion

Lottery A has more certainty than B if

\[ \frac{1}{2} \ln(W) + \ln(0.5) + \frac{1}{2} \ln(W-L) + \ln(0.5) > \frac{1}{2} [\alpha \ln(W) + \ln(0.5\alpha)] + (1-\alpha) \ln(W-L) + \ln(0.5(1-\alpha))] \]

\[ + \alpha \ln(W-L) + \ln(0.5\alpha)] + (1-\alpha) \ln(W) + \ln(0.5(1-\alpha))] \]

\[ 0 > \alpha \ln(\alpha) + (1-\alpha)\ln(1-\alpha) \]
Prudence

Lottery C is certainty maximizing relative to lottery D if:

\[ 0.5 \, E_\varepsilon \{ \ln(W-k) + \ln(0.5) + \ln(W+\varepsilon) + \ln(0.5) \} > 0.5 \, E_\varepsilon \{ \ln(W) + \ln(0.5) + \ln(W-k+\varepsilon) + \ln(0.5) \} \]

\[ E_\varepsilon \ln \gamma + E_\varepsilon \ln \left( 1 + \frac{-k\varepsilon}{\gamma} \right) > E_\varepsilon \ln \gamma, \text{ or if } E_\varepsilon \ln \left( 1 + \frac{-k\varepsilon}{\gamma} \right) > 0 \]

General Proof:

\[ E_\varepsilon \ln \left( 1 + \frac{-k\varepsilon}{W^2-kW+\varepsilon W} \right) = \int_{-\infty}^{0} \ln \left( 1 + \frac{-k\varepsilon}{W^2-kW+\varepsilon W} \right) \varphi(\varepsilon) \, d\varepsilon + \int_{0}^{\infty} \ln \left( 1 + \frac{-k\varepsilon}{W^2-kW+\varepsilon W} \right) \varphi(\varepsilon) \, d\varepsilon \]

\[ \ln \left( 1 + \frac{\varepsilon^2 k (2W-k)}{(W^2-kW)^2 - (\varepsilon W)^2} \right) > 0 \]
Lottery E is certainty maximizing relative to lottery F if:

\[ .5 E_{\varepsilon_1} E_{\varepsilon_2} \{\ln(W+\varepsilon_1) + \ln(.5) + \ln(W+\varepsilon_2) + \ln(.5)\} > .5 E_{\varepsilon_1} E_{\varepsilon_2} \{\ln(W) + \ln(.5) + \ln(W+\varepsilon_1+\varepsilon_2) + \ln(.5)\} \]

\[ E_{\varepsilon_1} E_{\varepsilon_2} \ln(1 + \frac{\varepsilon_1 \varepsilon_2}{\gamma}) > 0 \]
Temperance, cont.

General Proof:

\[ E_{\varepsilon_1} E_{\varepsilon_2} \ln(1 + \frac{\varepsilon_1 \varepsilon_2}{W^2 + \varepsilon_1 W + \varepsilon_2 W}) = \]

\[ \int_0^\infty \int_0^\infty \ln \left[ \left( 1 + \frac{(-\varepsilon_1)(-\varepsilon_2)}{W^2 - \varepsilon_1 W - \varepsilon_2 W} \right) \left( 1 + \frac{\varepsilon_1 (-\varepsilon_2)}{W^2 + \varepsilon_1 W - \varepsilon_2 W} \right) \left( 1 + \frac{-\varepsilon_1 \varepsilon_2}{W^2 - \varepsilon_1 W + \varepsilon_2 W} \right) \left( 1 + \frac{\varepsilon_1 \varepsilon_2}{W^2 + \varepsilon_1 W + \varepsilon_2 W} \right) \right] \varphi(\varepsilon_1) d\varepsilon_1 \varphi(\varepsilon_2) d\varepsilon_2 \]

\[ \ln \left[ 1 + \frac{\varepsilon_1^4 \varepsilon_2^4 + 2W^2 \varepsilon_1^2 \varepsilon_2^2 (3W^2 - (\varepsilon_1^2 + \varepsilon_2^2))}{ABCD} \right] > 0 \]

Where:

\[ W^2 - W(\varepsilon_1 + \varepsilon_2) = A \]
\[ W^2 - W(\varepsilon_2 - \varepsilon_1) = B \]
\[ W^2 + W(\varepsilon_2 - \varepsilon_1) = C \]
\[ W^2 + W(\varepsilon_1 + \varepsilon_2) = D \]
An individual maximizing lottery certainty would be expected to choose lottery D. Tversky and Kahneman found that 58% of participants did so.

An individual maximizing lottery certainty would be expected to choose lottery E. Tversky and Kahneman found that 74% of participants did so.
Maximizing Lottery Certainty without same expected Means

- Implies that individuals partially insure at actuarially unfair rates of insurance
- Implies the Allais paradox
- Implies Samuelson’s 1963 conjecture
Classical expected utility model to solving the demand for insurance problem:

\[
\begin{align*}
\max \quad & p_{t+1} U(V_t - L_{t+1} - q_t \alpha \theta_t + \alpha) + (1 - p_{t+1}) U(V_t - q_t \alpha) \\
\alpha_t &= \alpha(\theta_t) \\
p U'(V - L - q \alpha + \alpha)(1 - q) - (1 - p)U'(V - q \alpha)q &= 0 \\
U'(V - L - q \alpha + \alpha) &= U'(V - q \alpha) \\
\frac{U'(V - L - q \alpha + \alpha)}{U'(V - q \alpha)} &= \frac{q(1 - p)}{p(1 - q)} \\
\alpha_t &= f(V, q \mid \theta_t)
\end{align*}
\]
Insurance At Actuarially Unfair Rates

Full insurance lottery

\[
\begin{align*}
 p & \quad W - qL - L + L \\
 1-p & \quad W - qL
\end{align*}
\]

\[
\ln(1 - \frac{qL}{W}) < p \ln(1 - \frac{qL* + L - L^*}{W}) + (1 - p) \ln(1 - \frac{qL^*}{W})
\]

\[
\ln \left(1 - \frac{q(L - L^*)}{W(1 - \frac{qL}{W})}\right) < p \ln \left(1 - \frac{L - L^*}{W(1 - \frac{qL}{W})}\right)
\]

No-Full insurance lottery, \( L^* < L \)

\[
\begin{align*}
 p & \quad W - qL^* - L + L' \\
 1-p & \quad W - qL'
\end{align*}
\]

\[
- \frac{q(L - L^*)}{W(1 - \frac{qL}{W})} < - \frac{p(L - L^*)}{W(1 - \frac{qL}{W})}
\]

True as long as \( q > p \)
Allais Paradox

A1: 1.00 chance of $1,000,000  
(LC=13.81)
Or
A2: .10 chance of $5,000,000  
(0.89 chance of $1,000,000  
0.01 chance of $1

AND

A3: .10 chance of $5,000,000  
(LC=1.22)
Or
A4: .11 chance of $1,000,000  
(LC=1.17)  
0.89 chance of $1
A1 is preferred to A2 when:

\[ \frac{X}{Y^{\alpha \beta} 1^{\alpha (1-\beta)}} > \frac{(\alpha \beta)^{\alpha \beta} (\alpha (1 - \beta))^{\alpha (1-\beta)} (1 - \alpha)^{1-\alpha}}{1} \]

Or,

\[ \frac{X^\alpha}{Y^{\alpha \beta}} > \alpha^\alpha \beta^{\alpha \beta} (1 - \beta)^{\alpha (1-\beta)} (1 - \alpha)^{1-\alpha} \]

A3 is preferred to A4 when:

\[ \frac{Y^{\alpha \beta} 1^{\alpha (1-\beta)}}{X^{\alpha} 1^{1-\alpha}} > \frac{\alpha^\alpha (1 - \alpha)^{(1-\alpha)}}{(\alpha \beta)^{\alpha \beta} (1 - \alpha \beta)^{(1-\alpha \beta)}} \]

Or,

\[ \frac{X^\alpha}{Y^{\alpha \beta}} < \frac{(\alpha \beta)^{\alpha \beta} (1 - \alpha \beta)^{(1-\alpha \beta)}}{\alpha^\alpha (1 - \alpha)^{(1-\alpha)}} \]

So that the generalized conditions for a1, and a3 to be simultaneously preferred:

\[ \alpha^\alpha \beta^{\alpha \beta} (1 - \beta)^{\alpha (1-\beta)} (1 - \alpha)^{1-\alpha} < \frac{X^\alpha}{Y^{\alpha \beta}} < \frac{(\alpha \beta)^{\alpha \beta} (1 - \alpha \beta)^{(1-\alpha \beta)}}{\alpha^\alpha (1 - \alpha)^{(1-\alpha)}} \]
Samuelson’s 1963 conjecture

\[
\beta \left[ \alpha \left( \ln \left( \frac{W + X}{\beta} \right) + \ln(\alpha) \right) + (1 - \alpha) \left[ \ln \left( \frac{W - L}{\beta} \right) + \ln(1 - \alpha) \right] \right] > \alpha \left[ \ln(W + X) + \ln(\alpha) \right] + (1 - \alpha) \left[ \ln(W - L) + \ln(1 - \alpha) \right]
\]

\[
W > \frac{\beta}{\beta^{\beta - 1}}
\]