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  → Penalty against misrepresentation/concealment in insurance market
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- The introduction of post-loss testing (claim adjustment) to self-selection model

- The relationship between post-loss testing and ex-ante testing (underwriting)
Testing after entering a contract

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- Kessler, Lulfesmann, and Schmitz (2005) and Kofman and Lawarree (1993): The impact of limited liability on the impact of ex-post testing
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Q. Does a penalty against misrepresentation affect equilibrium in insurance market?
Adverse Selection: Rothschild and Stiglitz (1976)

- Low-risk $\Rightarrow$ Partial coverage
- High-risk $\Rightarrow$ Full coverage
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Risk Classification

Contribution
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- The market outcome approximates the first-best if a total loss of wealth causes an infinite utility penalty.

- In the presence of a post-loss test, underwriting may contribute sustaining a separating equilibrium especially when rebate payment is costly.
Competitive Insurance Market
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Public information:
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- The proportion of the risk type
- The cost of post-loss test and its accuracy
Competitive Insurance Market

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- Public information:
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  - The cost of post-loss test and its accuracy

- Asymmetric post-loss test (claim adjustment):
  - Low-risk policyholders are always identified as a low-risk
  - High-risk policyholders may be misclassified into a low-risk
Competitive Insurance Market

The Structure of Competitive Insurance Market
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The Structure of Competitive Insurance Market

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- Insurers are allowed to pay policyholders a premium rebate without cost if there is any surplus.
Notations and Assumptions

- The proportion of HR and LR: $\lambda$ and $(1 - \lambda)$
- Individual’s initial wealth: $W > 0$
- The probability of loss: $0 < \pi_L < \pi_H < 1$
- Fixed loss amount: $D > 0$
- Utility function: $u$ where $u' > 0$ and $u'' < 0$
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- Policy: $C \equiv (t, l, \pi)$ where a per policy testing fee, $t$
Policy with Post-loss Testing

- High-risk
  - Policy without post-loss test
    - 1 - \( \pi_H \) No loss
    - \( \pi_H \) Loss
    - Claim paid
  - Policy with post-loss test
    - 1 - \( \pi_H \) No loss
    - \( \pi_H \) Loss
    - 1 - \( p \) Claim paid
    - \( p \) Policy voided
Post-loss Test

\[ U^H(C_L) = \pi_H (1 - p(e)) u(W_{L1}) + \pi_H p(e) u(W_{L2}) + (1 - \pi_H) u(W_{NL}) \]

where we use shorthand notations for terminal wealth in the states:

- \( W_{L1} = W_0 + (1 - \pi_L) l - t - D \)
- \( W_{L2} = W_0 - D \)
- \( W_{NL} = W_0 - \pi_L l - t \)

We look for \( p \) such that the self-selection constraint satisfies:

\[ U^H(C_H) \geq U^H(C_L) \]
A competitive Nash equilibrium exists if $e^* \pi_L \leq \mu$, and offers two full-coverage separating contracts: $C_H = (0, D, \pi_H)$ and $C_L^* = (e^* \pi_M, D, \pi_L)$. Since the contract $C_L^*$ involves rebate $\Pi$ at the end, the net test fee is $e^* \pi_L$ where

$$p(e^*) = \frac{\pi_H - \pi_L}{\pi_H(1 - \pi_L)}.$$ 

and

$$\Pi = e^* \lambda (\pi_H - \pi_L).$$
Proposition (Post-loss testing with infinite utility penalty)

Under $D = W_0$ and $u(0) = -\infty$, a competitive Nash equilibrium always exists and approximates the first-best outcome. Two separating policies are offered: one offers $C_H = (0, D, \pi_H)$ without test; the other offers $C_L^* = (\varepsilon, D, \pi_L)$ with arbitrarily small test fee $\varepsilon$. 
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Underwriting becomes relevant when $\tau < 1$, because underwriting potentially reduces the test fee charged by an insurer up front.
Policy with Underwriting and Post-loss Testing

- **High-risk**
  - **Policy without post-loss test**
    - \( p_1 \)
    - \( 1-p_1 \)
  - **Policy with post-loss test**
  - **Underwriting**
    - \( \pi_H \)
    - \( 1-\pi_H \)
    - **No loss**
    - **Loss**
      - **Claim paid**
      - **Policy voided**

Kamiya and Schmit

Misrepresentation
Policy with Underwriting and Post-loss Testing

Underwriting characteristics:
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- Underwriting is defined by insurer’s expenditure on the test and the probability that a high-risk individual who applies for $C_L$ is correctly identified as a high-risk.
- Assume low-risk individuals are always correctly classified as low-risk.
- Assume an underwriting fee is charged to an applicant only when an applicant purchases a policy based on an underwriting test.
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Notations:
- Insurer’s per application expenditure on underwriting and per claim expenditure on a post-loss test are denoted by $e_1$ and $e_2$.
- The per policy fee on an underwriting test and that on a post-loss test are $t_1$ and $t_2$.
- The accuracy of the tests are denoted by $p_1(e_1)$ and $p_2(e_2)$.
- Assume $p(0) = 0$, $p'(e_i) > 0$ and $p''(e_i) < 0$, $p(e_i) < 1$ for $i = 1, 2$. 
Policy with Underwriting and Post-loss Testing

Cost and benefit of underwriting:
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- An underwriting test brings the average loss probability down and decreases the charged post-loss test fee $t_2$ to:

\[
e_2^* \pi'_M \equiv e_2^* \left[ \lambda (1 - p_1) \pi_H + (1 - \lambda (1 - p_1)) \pi_L \right]
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$$e_2^* \pi'_M \equiv e_2^* [\lambda(1 - p_1) \pi_H + (1 - \lambda(1 - p_1)) \pi_L]$$

- The per policy fee on underwriting is defined by:

$$t_1 = \frac{e_1}{1 - \lambda}$$
Policy with Underwriting and Post-loss Testing

Condition for using underwriting test, $t_1 + t_2 - \tau \Pi_2 < t^* - \tau \Pi$, can be reduced to:

$t_1 < p_1 \Pi (1 - \tau)$. 
Policy with Underwriting and Post-loss Testing

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t_1 < p_1 \Pi (1 - \tau).
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Proposition (Existence of Separating Policies)

*There exists a competitive Nash equilibrium where both an underwriting and a post-loss test are employed in a full-coverage contract, \( C_L^* \), if both \( t_1 + t_2 - \tau \Pi_2 \leq \mu \) and \( t_1 < p_1 \Pi (1 - \tau) \) are satisfied.*

The optimal underwriting is unique at: \( t_1' = p_1' \Pi (1 - \tau) \) where the per policy marginal fee of underwriting equals the marginal benefit of reducing unpaid rebate.
Implication of Costly Rebate Payment
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Introducing an underwriting test in the presence of a post-loss test is Pareto improving.
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- Welfare effect of underwriting must be discussed with the implementation of claim management.
Implication of Costly Rebate Payment

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- Competition of the development of underwriting techniques is a driving force of reducing premiums.
Summary

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- With finite utility penalty, the equilibrium contract for low-risk individuals earns strictly positive rebate payment, which is necessary to send a signal.
- The market outcome approximates the first-best outcome when a contract cancellation causes a negative infinite utility.
- Underwriting may contribute to attaining separating equilibrium by reducing the cost of the post-loss test.
Thank You