“Optimal Insurance Contracts with Insurer’s Background Risk”
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Arrow (1971, 1974): If the premium rate set by a risk-neutral insurer, then the optimal policy for a risk-averse expected utility maximizer is 100% coverage above a deductible.
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This paper adds insurer’s background risk to Raviv’s model.
Summary

Model Setting

Insured’s terminal wealth is:

\[ W_1 - x - P - I(x) \]

Insurer’s terminal wealth with its background risk is:

\[ W_2 - y(x) + P - I(x) - c(I(x)) \]

In the range where \( 0 < I^*(x) < x \), the marginal coverage satisfies:

\[ I^*(x) = \frac{R_U(A^*) - y'(x)R_V(B^*)}{R_U(A^*) + R_V(B^*)(1+c^*) + c^{**}/(1+c^*)} \]

where \( R(.) \) denotes the coefficient of absolute risk aversion for insured or insurer and \( A \) and \( B \) are the terminal wealth of insured and insurer.
Pereto Optimal Contracts with CARA Utilities

\[ y (\text{insurer's background loss}) \]

\[ y'' > 0 \]

\[ y'' < 0 \]

\[ y' = \frac{a}{b} \]

\[ x (\text{insured loss}) \]
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The insured loss $x$ tends to be insurable by the insurer. $y(x)$ includes both insurable and uninsurable risk. Not too much as background risk?

Section 3 defines insurer’s background loss as

$$y = \sum_{i \neq j} [I_i(x_i) + c(I_i(x_i))]$$
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“the negative effect of the insured’s loss on the insurer’s background wealth cannot exceed the amount of itself:

$$y'(x) \geq -1$$

No basis risk for hedging.
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Great work!