ROSS MEETS BELL:  
Linex utility and riskier background risk

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**Q**: Do increases in an independent BGR make one buy more insurance?  
**A**: Only if we have Ross DARA & DAP $\iff$ only if we have Linex utility
Some Background on Background Risk

Doherty & Schlesinger (1983)
Even independent BGR alters insurance decisions

Kimball & Eeckhoudt (1992)
Condition on utility: SRA $\rightarrow$ buy less insurance

Gollier & Pratt (1996)
Necessary & sufficient conditions for less insurance

VERYY COMPLICATED

Eeckhoudt, Gollier & Schlesinger (1996)
What if the BGR gets “riskier?” (necessary & sufficient conditions)

VERYY VERYY COMPLICATED*

* Georges Dionne (my ARIA discussant back then): “I don’t like this paper because the conditions are too complicated.”
RISK AVERSION: Pratt (1964) vs. Ross (1981)

**Pratt**: Willingness to pay to remove an undesirable risk.

**Ross**: Willingness to pay to reduce an undesirable risk.

Reduce via a Rothschild & Stiglitz (1970) reduction in risk

Notion of ‘decreasing absolute risk aversion” (DARA)

*WTP decreases with wealth if ...*

Pratt:

\[-\frac{u''(y)}{u''(y)} > -\frac{u''(y)}{u'(y)} \quad \forall y\]

Ross

\[-\frac{u''(y)}{u''(y)} > -\frac{u''(\hat{y})}{u'/(\hat{y})} \quad \forall y, \hat{y} > 0\]

(“stronger measure”)
Eeckhoudt, Gollier & Schlesinger (1996)

An SSD increase in BGR will cause insurance demand to increase if utility is Ross DARA & Ross DAP (at least one strictly decreasing)

But what utility functions are both?

**Note:** Many examples can be constructed, such as in Keenan & Snow (2012), that are valid over a restricted range of wealth. But can we find preferences that always satisfy these criteria?

**Before answering that question, let me make one switch.**

*It’s an inside joke .... You’ll see.*
**One Switch Utility (Bell 1988)**

“AND NOW FOR SOMETHING COMPLETELY DIFFERENT …” Monty Python

Suppose that I have $W$ and I get to add one wealth lottery, $\tilde{X}$ or $\tilde{Y}$.

Suppose that I like $\tilde{X}$ better.

As I get richer … maybe at some point I will switch to $\tilde{Y}$?

And … if I get richer still, maybe I will switch back to $\tilde{X}$?

Bell asked the following question:

*What preferences guarantee no more than one switch?*

If we also want DARA there is only one answer: **Linex Utility**
**Linex = Linear plus exponential**

\[ u(y) \equiv ly - ce^{-\gamma y} \]

where \( l \geq 0, \ c \geq 0 \) (one strict) and \( \gamma > 0 \)

If \( c = 0 \), we have risk-neutral preferences

If \( l = 0 \), we have well-known CARA preferences

Although Linex is well known in management science, it is rarely used in economics and finance.
ROSS (finally) meets BELL

Okay ... so this is where our new results start ...

If I want a general utility function that satisfies both Ross DARA and Ross DAP (at least one strict) what would it look like?

Using Ross (1981) and solving a few differential equations:

Linex and only Linex has these properties

Interesting artifact: Ross (1981) actually uses a Linex utility (not yet so named) in one of his examples!
Application to Insurance Demand

Rather than use EGS (1996), let’s use Linex directly and see what we get:

$$\text{Max } V(a) \equiv Eu(w + \varepsilon - P(a) - \bar{x} + a\bar{x}),$$

where $$P(a) \equiv (1 + \lambda) aE\bar{x}, \quad \lambda > 0$$

Remark: Always $$a^* = 1$$ if fair price, $$\lambda = 0$$

With Linex utility, this yields the FOC:

$$V'(a) \equiv -l\lambda E\bar{x} + Ef(\varepsilon)\gamma cE[e^{\gamma(P(a) - (1-a)\bar{x})} \cdot (\bar{x} - (1 + \lambda)E\bar{x})] = 0$$

where $$f(\varepsilon) \equiv e^{-\gamma(w+\varepsilon)}$$
Note: $f(\varepsilon)$ is positive, decreasing and convex ($\varepsilon$ terms separable in utility)

\[
V'(a) \equiv -l\lambda E\tilde{x} + Ef(\tilde{\varepsilon})\gamma cE[e^{\gamma (P(a)-(1-a)\tilde{x})} \cdot (\tilde{x} - (1 + \lambda)E\tilde{x})] = 0
\]

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Some observations:

1. If $l = 0$ (CARA), then BGR has no effect.

2. If $c = 0$ (risk neutral), then $V'(a) < 0$ (buy zero insurance)

3. If $l$ and $c$ both positive, we get negative + positive

   *We find the insurance level $a^*$ that make us happiest.*

4. If $l$ increases (more risk neutral), we buy less insurance
5. If $c$ increases (more risk averse). We buy more insurance

$$V'(a) \equiv -l \lambda E \tilde{x} + Ef(\tilde{\epsilon}) \gamma c E[e^{\gamma (P(a) - (1-a) \tilde{x})} \cdot (\tilde{x} - (1 + \lambda) E \tilde{x})] = 0$$

6. We know that $f(\epsilon) \equiv e^{-\gamma (w + \epsilon)}$ is decreasing and convex.

From Rothschild & Stiglitz (1970), we know that any increase in risk for $\tilde{\epsilon}$ implies that $Ef(\tilde{\epsilon})$ increases.

$\Rightarrow$ More weight on positive term
$\Rightarrow$ $V'(a) > 0$ at the old optimum $a^*$

**Hence: Riskier BGR $\Rightarrow$ More insurance**

*(as we already knew from EGS 1996 in their very complicated model)*
10-SECOND SUMMARY

Ross (1981) develops metric for reduction in risk

EGS (1996) used much math to show Ross DARA & DAP means a higher BGR leads to more insurance.

Bell (1988) examines one switch utility

We introduce: ROSS meets BELL

We show Ross DARA & DAP only happens for Bell’s Linex utility

Using Linex, it was (relatively) simple to show:

Higher BGR \rightarrow Buy more insurance

Maybe surprising: result only guaranteed (for all wealth and risks) with Linex utility
Thank you!

David Bell

Steve Ross

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