

IMPLICIT CLIENT SORTING AND BONUS MALUS CONTRACTS

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Abstract

In an earlier paper, we showed that, in a multi-period setting (but with no multi-period commitment) with two client types (adverse selection) and a competitive insurer environment, a comparison between periodic repetitions of the classic Rothschild-Stiglitz separating equilibrium contract menu and a second contract based on Bayesian learning (which we identify as a bonus-malus contract) yields situations in which either one or the other will be used, depending on the parameter which is used to discount future wealth. The fact that the two contract formats are mutually exclusive (i.e. cannot co-exist) depends critically on all clients having the same discount factor. In the current paper, we relax the assumption of common discount factors, and we find that situations exist in which both contract formats may co-exist.

Keywords: Insurance, bonus-malus, Bayesian learning

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1. Introduction

In a paper presented to the European Group of Risk and Insurance Economists, Watt and Vazquez (1996) considered the comparison of the traditional Rothschild-Stiglitz separating equilibrium (Rothschild and Stiglitz (1976)) and a bonus-malus contract involving full coverage and a premium that is updated according to Bayes' rule over time as insured accidents are observed. In a model based on the traditional competitive assumption of zero expected profits, Watt and Vazquez show that there exist conditions under which all individuals in the economy (both high risk and low risk) will prefer the Bayesian updating contract (BUC) to the Rothschild-Stiglitz separating contract menu (RS). In Watt and Vazquez's model the only difference between clients is the probability of accident, and it turns out that the Pareto dominance of the BUC depends on low risk individuals having a sufficiently high discount factor for future consumption, and on the insurance company offering a sufficient number of contract renewals.

The principal results of Watt and Vazquez can be set out as follows:

1. All high risk individuals prefer the BUC to RS, since, in their case, the BUC converges to the expensive full-coverage contract of RS from above (i.e. the premium begins cheaper and gets progressively more expensive, while coverage is always complete).
2. Low risk individuals will earn less periodic expected utility under the BUC than under the cheaper partial-coverage contract of RS for the initial periods (under the assumption that RS exists), but as their real type is gradually revealed (by suffering relatively less accidents), the periodic expected utility of the BUC overtakes that of

RS. Hence the overall intertemporal sum of expected utilities will only be greater for the BUC than for RS if there are sufficient periods during which the single period utility of BUC is greater than that of RS (that is to say, the insurer offers sufficient contract renewals), and the client is sufficiently patient over time (i.e. has a high enough intertemporal discount factor).

A principal ingredient of the Watt-Vazquez Pareto dominance theorem is that there be only one discount factor for all individuals in the economy (at least, for all low risk individuals). It is interesting, and realistic, to consider what would happen when discount factors over individuals are allowed to differ ¹. The current paper explores this idea.

Furthermore, the Watt and Vazquez paper would imply that in any one economy there only exists either the RS contract menu (when discount rates are low), or the BUC (when discount rates are high). Hence no insurance company would offer a bonus-malus contract as well as a voluntary excess contract (partial coverage at a low constant premium). Since it is frequent to observe both contracts on offer, it would seem that there is implicit client sorting within insurance companies, according to the terms of the BUC that are offered. While it remains clear that all high risk individuals will prefer the BUC to the high premium-full coverage contract of RS (and hence this contract would not be offered), intuition suggests that the more patient low risk clients will be attracted to the BUC, and the more impatient low risk clients will be attracted to the RS partial coverage contract.

2. The Model.

We take the same principal assumptions of Watt and Vazquez, namely:

i) that insurance companies act in a competitive environment (and hence must earn zero expected profits per period)

ii) that there are only two types of individual with respect to accident probability (one high risk type and the other low risk)

iii) there is at most one accident per contract period.

iv) that there are no moral hazard problems (most importantly, all accidents are observable by all participants)

v) that the only two contract formats under consideration are periodic repetition of the RS separating equilibrium and a bonus-malus (no claims bonus) contract based on full coverage and a premium that is updated according to Bayes' rule.

vi) that there is assumed to exist a sufficient proportion of low risk individuals in the economy for the RS separating equilibrium to exist.

Our only change is that the discount factor for low risk individuals is now distributed according to some probability density function $f(\gamma)$ ². We denote the accumulative distribution by $F(\gamma)$, so that the value $1 - F(\gamma)$ represents the probability that any particular individual will have a discount factor greater than γ .

We face the following problem; it is known (see Watt and Vazquez (1996)) that there is a minimum requirement on γ which depends on z , the number of contract renewals offered under the BUC, represented by $\gamma_0(z)$, such that all low risk clients with a discount factor greater than or equal to $\gamma_0(z)$ will be attracted to the BUC as opposed to RS. Assume that the probability of accident for a high risk individual is p_a and that of a low risk individual is p_b .

Now consider an insurer who calculates the initial period premium corresponding to the BUC (with z renewals) as $\tilde{\lambda} p_b + (1 - \tilde{\lambda}) p_a$, where $\tilde{\lambda}$ is the probability that any particular individual selected randomly from the entire market is low risk. This initial premium offers zero expected profits if the client mix that ends up preferring the BUC pool is exactly the same as the proportions existing in the entire market. Since all high risk individuals will purchase the BUC (independent of their discount factors) this requires that all low risk individuals do likewise³. However, it is now possible that some low risk individuals have discount factors below $\gamma_0(z)$, and so will not purchase the BUC. Hence the client mix at the BUC would be more heavily weighted towards high risk, and the initial premium must be revised upwards in order to avoid negative initial expected profits.

Raising the initial premium has the effect of either raising the required number of contract renewals in order that low risk individuals still prefer the BUC to RS, or raising the minimum discount factor for the BUC to be preferred. Assume (for the time being) that z is constant, hence raising z is out of the question, in which case the minimum discount factor is raised, which can only have the effect of eliminating further low risk individuals from the BUC pool in favor of RS. Hence a further increase in the initial premium is required, and so on⁴.

One wonders if this process drives the BUC down to the full-coverage high premium contract of RS, at which there are only high risk clients, and all low risk clients accumulate at their corresponding RS contract.

Really we are searching for a fixed point in that the initial weights on the two probabilities for the first period premium determine the minimum discount rate for the BUC to be preferred, but then this minimum discount rate in turn determines the appropriate initial weights on the probabilities.

Watt and Vazquez (1996, theorem 3 and corolary) proved the following theorem:

THEOREM 1 (WV): For any given proportion of low risk individuals in the economy, l , there exists a strictly finite number of contract renewals, z_0 such that, for all $z > z_0$, there exists a range of discount factors $\gamma_0(z) \leq \gamma \leq 1$ for which low risk individuals prefer the BUC to RS.

Now consider the following. Suppose that instead of all low risk individuals entering the BUC pool, only a given proportion, q , of them do ($0 \leq q \leq 1$). We can state the following:

THEOREM 2: For all q such that $0 < q \leq 1$, there exists a strictly finite number of contract renewals, $z_0(q)$, such that for all $z > z_0(q)$, there exists a range of discount factors $\gamma_0(z, q) \leq \gamma \leq 1$ for which low risk individuals prefer the BUC pool to RS.

Proof. Theorem 1 is robust to the exact value of the proportion of low risk clients in the economy. Hence, any weighting of high to low risk individuals at the BUC, λ ,

will end up defining some γ_0 as the minimum discount factor for which the BUC pool is preferred to RS.

When only a proportion q of all possible low risk clients actually do purchase the BUC, then the initial average probability of accident within the BUC pool is:

$$p = \frac{1 - \tilde{\lambda}}{1 - \tilde{\lambda} + q\tilde{\lambda}} p_a + \frac{q\tilde{\lambda}}{1 - \tilde{\lambda} + q\tilde{\lambda}} p_b$$

Defining:

$$\lambda \equiv \frac{1 - \tilde{\lambda}}{1 - \tilde{\lambda} + q\tilde{\lambda}} \quad (1)$$

and then following the proof of theorem 3 and corolary in Watt and Vazquez gives the result.

Q.E.D.

Theorem 2 establishes the result that, so long as some low risk individuals prefer the BUC pool to the RS separating contract, then there exists a range of discount factors for which the BUC is preferred by low risk clients. If $q=0$, that is to say, no low risk clients are purchasing the BUC, then the BUC itself collapses onto the full insurance high premium contract of the RS separating equilibrium, and the premium does not change over time, no matter how the client's accident history turns out. Hence in this case, the BUC replicates the full coverage high premium contract of the RS separating equilibrium. All low risk clients would purchase the cheaper partial coverage contract, and all high risk clients would purchase the more expensive full coverage contract (which is just the BUC in this case) .

What theorem 2 does not guarantee is that the q initially used to calculate the first period premium of the BUC is exactly the same probability of being a low risk client for which this particular BUC would be preferred to the RS separating contract menu. If this were not the case, we would not have a stable equilibrium situation since the proportion of low risk clients that prefer the BUC pool is not the same as the proportion of low risk clients assumed to prefer it (i.e. those that are assumed to purchase BUC) when the BUC was initially offered. Hence we must consider the fixed point argument alluded to above.

For simplicity of exposition, we shall define $\lambda = \lambda(q)$, that is to say, for any particular q there is associated a value λ that would define the initial period premium of the BUC, as in equation (1).

Following Watt and Vazquez (1996), let the difference (after z contract renewals) between the expected utility of the BUC with initial premium defined by λ , and the expected utility of z repetitions of the RS partial coverage contract, for a low risk client whose discount factor is γ , be expressed by the function $H(\gamma, z, \lambda)$. Hence $H(\cdot) > 0$ indicates that the BUC with z renewals is preferred to z repetitions of RS. The lowest discount factor for which the BUC is preferred is that which solves $H[\gamma_0(z, \lambda), z, \lambda] = 0$. Watt and Vazquez showed that, so long as z is sufficiently high, $\gamma_0(z, \lambda = 1) < 1$.

Consider how a change in λ affects $\gamma_0(z, \lambda)$. Firstly, since we are assuming (for the time being) that z is constant, it shall not be explicitly written into the equations. An increase in λ corresponds to a decrease in the initial period premium, and, given our assumption on z , a decrease in the minimum discount factor required for a low risk client to prefer the BUC pool to RS. Hence $\gamma_0(\lambda) < 0$. While $\gamma_0(\lambda=0)$ does not exist, it is true that as $\lambda \rightarrow 0$, $\gamma_0(\lambda) \rightarrow 1$. This is true given theorem 2 (i.e. $\gamma_0(\lambda) \leq 1 \quad \forall \lambda > 0$) and the fact that $\gamma_0(\lambda)$ is a continuous monotone relationship.

Now consider the density of discount factors, $f(\gamma)$. As we defined above, for any particular discount factor γ , the probability that any given low risk client has a discount factor greater than γ is:

$$\int_{\gamma}^1 f(s) ds = 1 - F(\gamma)$$

Note that $1 - F[\gamma_0(\lambda)]$ just defines the proportion of low risk individuals that find the BUC pool to be preferred to the RS separating menu, when the first period BUC premium is calculated using λ .

What is required for the BUC pool to be consistent is that the proportion of low risk individuals that enter it be the same proportion of them that in fact find it to be preferred, i.e. that there exist some value λ^* such that:

$$K(\lambda^*) \equiv \frac{\tilde{\lambda}\{1 - F[\gamma_0(\lambda^*)]\}}{1 - \tilde{\lambda} + \tilde{\lambda}\{1 - F[\gamma_0(\lambda^*)]\}} = \lambda^*$$

Hence we are searching for a fixed point of the function $K(\lambda)$.

Note that this fixed point equation implies that:

$$G(\lambda^*) \equiv 1 - F[\gamma_0(\lambda^*)] = \frac{\left(\frac{\lambda^*}{1 - \tilde{\lambda}}\right)}{\left(\frac{\tilde{\lambda}}{1 - \tilde{\lambda}}\right)} \equiv J(\lambda^*)$$

i.e. a fixed point implies that the probability that a low risk individual will find the BUC pool to be preferred must be equal to the ratio formed by the proportion of low risk to high risk clients entering the BUC pool, and the same proportion for the entire economy.

In figure 1 we represent the graph of $J(\lambda)$, and in figure 2 we represent the construction of the graph of $G(\lambda)$, for the case of a typical bell shaped density function. Superimposing the upper left quadrant of figure 2 onto figure 1 indicates how the fixed point can be found (see figure 3).

3. Existence of a Fixed Point

Note that, from figure 3, the existence of a fixed point λ^* (the intersection of the two graphs) can be guaranteed if $G'(\lambda) > J'(\lambda)$ for $\lambda \rightarrow 0$.

It is easy to see that:

$$J'(\lambda)|_{\lambda \rightarrow 0} = \frac{1 - \tilde{\lambda}}{\tilde{\lambda}}$$

while:

$$G'(\lambda) = -F'[\gamma_0(\lambda)]\gamma_0'(\lambda) = -f[\gamma_0(\lambda)]\gamma_0'(\lambda)$$

Since $\gamma_0'(\lambda) < 0$, we have $G'(\lambda) > 0$. However, $\gamma_0(\lambda=0)$ does not exist, so we must consider the slope of G as λ approaches 0. We can state:

THEOREM 3: A sufficient condition for the existence of a fixed point is:

$$\lim_{\lambda \rightarrow 0} \{-f[\gamma_0(\lambda)]\gamma_0'(\lambda)\} > \frac{1 - \tilde{\lambda}}{\tilde{\lambda}}$$

It is easy to see that it is more likely that the sufficient condition is fulfilled:

1. The larger is $\tilde{\lambda}$, the probability of any particular client being low risk
2. The larger is $f(\gamma)$, as $\gamma \rightarrow 1$. That is to say, the greater is the probability weight on high discount factors.
3. The steeper is the function $\gamma_0(\lambda)$ for small values of λ .

Since both $J(\lambda)$ and $G(\lambda)$ are monotone increasing, and since $J(\lambda)$ is strictly convex, then strict concavity of $G(\lambda)$ is sufficient for a unique fixed point, if it exists.

If we relax the assumption that z is constant, an increase in z would imply a vertical shift to a new $\gamma_o(\lambda)$ function that would be located below the original function. In turn, this would imply a vertical shift that would displace $G(\lambda)$ upward. Given that $J(\lambda)$ is unaffected, the only effect would be to increase the value of λ^* (assuming it exists), and hence to decrease the limit value of the discount factor, $\gamma_o(\lambda^*)$. Hence, when an insurance company decides its policy regarding exactly how many contract renewals to offer in bonus-malus contracts, it is implicitly sorting its eventual client characteristics. That is to say, those companies that offer less contract renewals, are implicitly sending low risk individuals with lower discount factors (older people, people with a higher propensity to emigrate, etc.) to the RS contract rather than to the BUC.

The existence problem can also be studied from the perspective illustrated in figure 4. Choose some starting point λ , and then consider the graph of $H(\gamma, z=\infty, \lambda)$, for all values for which $H(\cdot) \leq 0$. Next, in the upper quadrant represent the graph of $1-F(\gamma)$. If it turns out that $1-F[\gamma_o(\infty, \lambda)] > \lambda$ where $\gamma_o(\infty, \lambda)$ is the point where $H(\gamma, z=\infty, \lambda)=0$, then a fixed point can be guaranteed by simply reducing z to something finite. To find the exact number of repetitions, z , we need only set $1-F(\gamma)=\lambda$, calculate the implied discount factor, and call it $\gamma_o(z, \lambda)$, and finally calculate

the value of z that solves $H[\gamma_0(z,\lambda),z,\lambda]=0$. It is worth noting that there may not be an exact solution, since the contract repetitions z are necessarily discrete, and the solution to $H[\gamma_0(z,\lambda),z,\lambda]=0$ may very well not turn out to be a round number (in this case we should just take the closest round number solution). It is also worth noting that if $1-F[\gamma_0(\infty,\lambda)]<\lambda$ then that particular λ can never give a fixed point. Hence, non-existence of a fixed point for a given density $f(\gamma)$ would require that $1-F[\gamma_0(\infty,\lambda)]<\lambda$ for all possible λ .

Secondly, we may be able to solve a non-existence problem by simply altering our “scale”. Note that we have defined z as being one period of given duration, during which at the most one accident per client can occur. If the insurer were prepared to reduce the duration of the period z , then (assuming accidents themselves are instantaneous) there would still be never more than one accident per period, there would still be only two types of client, one risky and the other less risky (although their respective probabilities of accident will both have been reduced), and more importantly, since the time horizon has been reduced, everyone’s discount factor will increase. This will have the effect of shifting the density $f(\gamma)$ to the right, and the function $1-F(\gamma)$ will be displaced upwards and to the right. Hence, for any given initial value of λ , a sufficiently small duration of contracts is always sufficient to guarantee the existence of a fixed point. This implies that, if the time horizon implied by z can be chosen by the insurer, then he is always able to choose a horizon such that he can attract any client mix desired. Of course, the smaller is the time horizon

chosen, the greater would be the contracting costs, and so there would seem to be a limit to this type of client sorting.

4. Conclusions.

We have re-considered the Watt-Vazquez model of bonus-malus contracts versus repetitions of the Rothschild-Stiglitz separating equilibrium contract menu, under the assumption that discount rates are not constant over clients. This does not introduce further adverse selection problems, since the client's discount rate is of no real interest to the insurer, however, it does imply that both bonus-malus contracts and the Rothschild-Stiglitz contract menu can co-exist, as is frequently observed in real world insurance. If no co-existence is possible, then the bonus-malus contract itself collapses onto the full coverage high premium contract of the Rothschild-Stiglitz menu anyway.

We have pointed out that the characteristics of the eventual clients that are attracted to the bonus-malus contract are dependent on the insurer's policies regarding the parameters implied by this contract. Most importantly, the choice of the maximum number of possible renewals of the contract (and possibly the time horizon between contract renewals) implicitly sort the insurer's clients between the bonus-malus contract and the partial coverage contract of Rothschild and Stiglitz. In particular, for any given parameters, an equilibrium would imply that all high risk clients and the most patient low risk clients prefer the BUC while impatient low risk clients prefer the RS partial coverage contract.

FIGURES

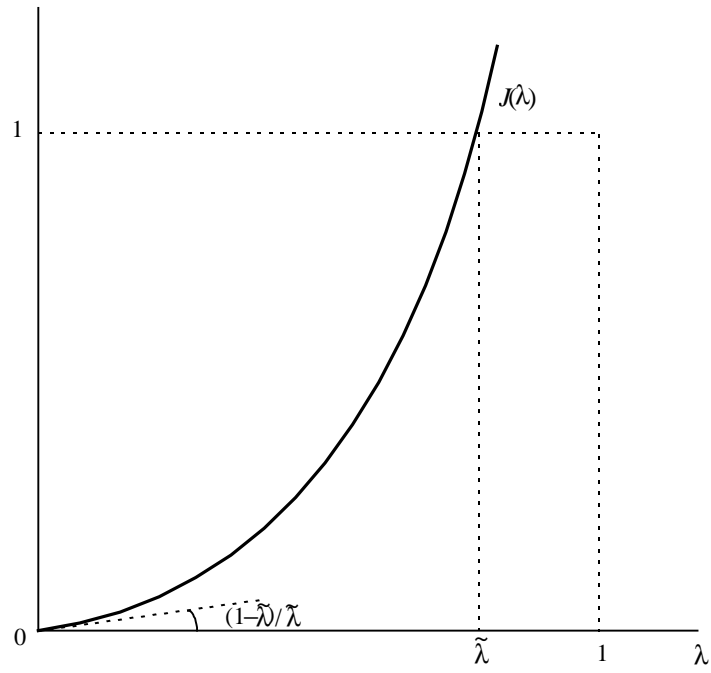


Figure 1

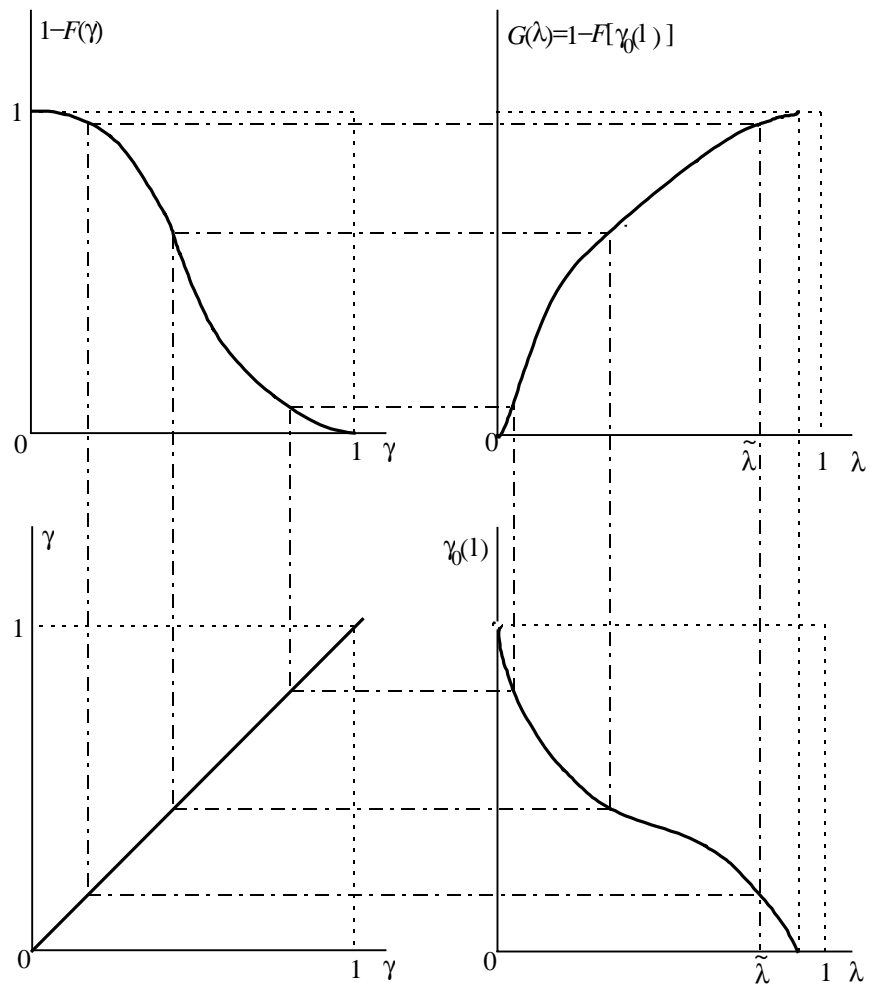


Figure 2

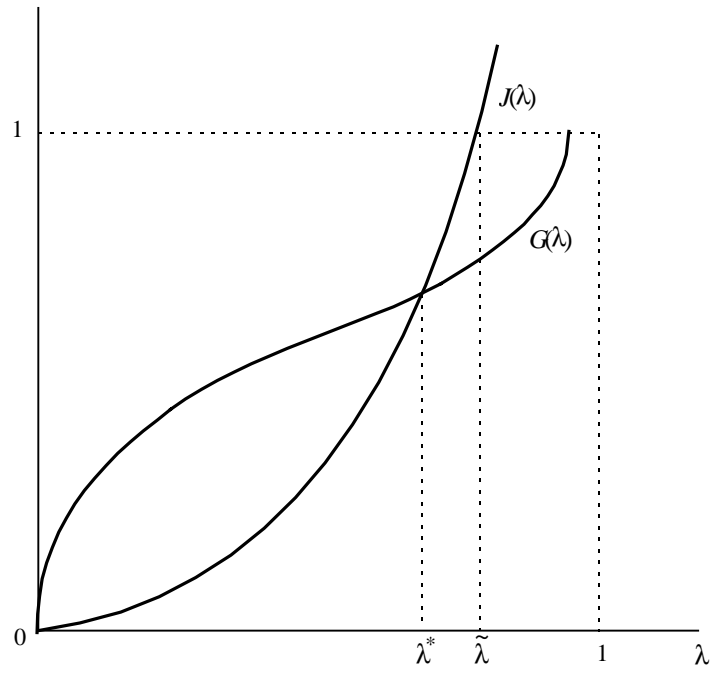


Figure 3

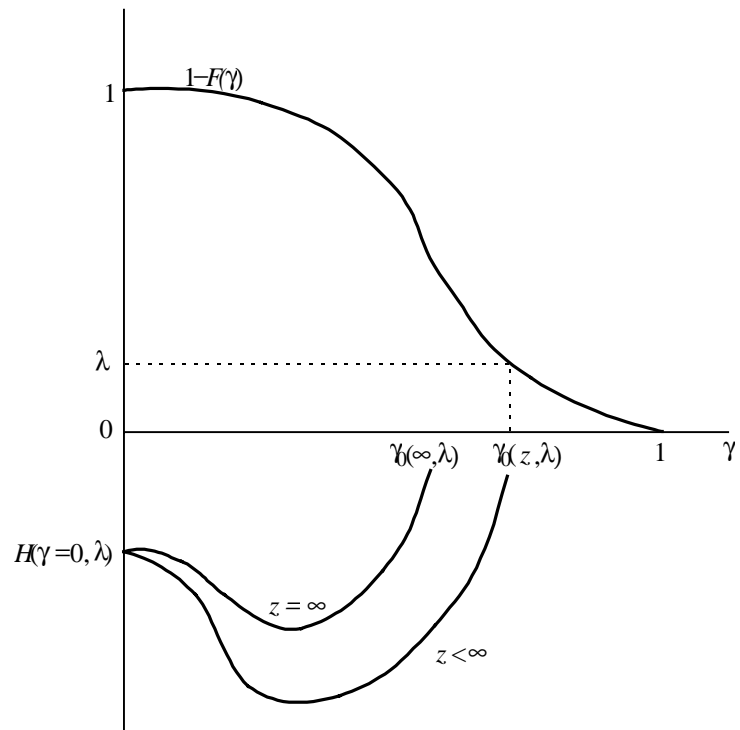


Figure 4

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NOTES

- ¹ We are indebted to Art Snow for suggesting this.
- ² Exactly how the discount factor of high risk individuals is distributed is irrelevant, since they all prefer the BUC independent of their discount factor.
- ³ Hence, the Watt and Vazquez Pareto dominance result holds if and only if $\gamma \geq \gamma_0(z)$.
- ⁴ It is important to note that we are not describing a dynamic convergence, but rather the insurer is assumed to be recalculating the initial conditions that he plans to offer to the market, before he actually offers anything.