Exclusions and Demand for Insurance: Risk Theory Seminar 1999

Rod Garratt and John M. Marshall[1]
Department of Economics
University of California, Santa Barbara 93106
marshall@econ.ucsb.edu
garratt@econ.ucsb.edu

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1 Introduction

Exclusions are part of any property insurance. A prototypically simple contract covers losses from a single peril but excludes losses from all other perils and, typically, from the insured peril in some instances. Alternatively, a complex contract like any of the forms of homeowners insurance covers a wide range of perils. The exclusions are fewer but more acutely recognized, typically catastrophe risks such as flood and earthquake.

Demand for insurance is impacted by excluded risks. For instance, the lack of earthquake coverage can be a significant factor in the demand for homeowners insurance, and limitations on coverage of automobile liability can influence the decision to purchase coverage for collision. There are two contrary tendencies here: Exclusions might increase demand by heightening risk aversion, or they might have the opposite effect by lowering the quality of insurance. The balance between these tendencies depends upon the exact form of the insurance contract, as this paper shows.

Historically, the growth of coverages often consists of adding new perils to contracts for previously covered perils. Thus current homeowners policies arise from coverage first of fire and then from successive accumulation of theft coverage, liability coverage, and so on. There are numerous incentives for this type of accumulation, and the interesting possibility examined here is that the addition of a new coverage might increase demand for the old one.

2 Precursors

The classic theory of demand for insurance ignores exclusions because it assumes that risks to wealth are insurable. The theory of background risk amends the theory by recognizing an imperfect ability to insure[1]. In a typical application of background risk, wealth consists of risky real property, which is insurable, and a risky financial or human capital portfolio, which is not, and the presence of the uninsurable risk influences the demand for insurance of the insurable one.

The assumptions of the classic and background-risk models are summarized in the table below. Risky wealth is w in the classic case, it is insurable, and consumers select a vector of parameters θ such as premium, deductibles and coinsurance, leading to insurance $I(w;\theta)$. The components of wealth in the case of background risk are random variables x and y. The risks are additive and insurance is available only for the x component.

[1] See Doherty and Schlesinger (1983), Smith and Mayers (1983), Kimball (1990), Kimball and Eeckhoudt (1993), and Garratt and Marshall (1999).

	utility	insurance
classic	$U(w)$	$I(w;\theta)$
background risk	$U(x+y)$	$I(x;\theta)$

The effects of background risk upon optimum insurance should be expected to depend upon the model of insurance $I(x;\theta)$. Nevertheless, the existing results are quite similar, and typically have this form: The starting assumption is that y is insurable at fair prices, is therefore fully insured, and hence there is no background risk. Removing insurance of y has no effect when insurance of x is fairly priced because then x is fully insured regardless of the status of y. The interesting case is that insurance of x is unfairly priced and therefore, even in the base case, partial. Now the effect of background risk is dependent upon the correlation between x and y. Positive correlation, zero correlation, or a small negative correlation all imply that insurance of x will increase. Sufficient negative correlation leads to less market insurance of x because the consumer possesses in y a kind of self-insurance. The problem in specific instances is to determine the degree of correlation that is sufficient to reduce demand for insurance of x, and that can be a hard problem to solve.

The prevalence of this type of result is explained in part by chance and in part by a tendency to model insurance as the choice of a coinsurance fraction. In Mayers and Smith, Doherty and Schlesinger, and some of the results in Kimball and Eeckhoudt, the model is coinsurance and the results are dependent upon degrees of correlation between the insurable and uninsurable risks. Other results of Kimball and Eeckhoudt concern the deductible, but correlations are still central. Garratt and Marshall (1999) examine a different model of insurance – the same one used below – in the presence of a particular type of background risk, and again find that correlations are critical.

The concentration of attention on the coinsurance model is not inappropriate. The model is inherited from the classic theory, in which risks to wealth are insured and as a consequence there are no immediate barriers to full insurance of wealth. In that context the coinsurance model is as good as any other, and it is more convenient than most. These justifications are weaker in the context of limited insurance. Instead of insuring wealth or components of wealth, limited insurance contracts insure damages, liabilities, and losses. Translating damage into reduction of wealth is, as illustrated in Garratt and Marshall (1996, 1999), and below, not completely automatic. Moreover, coinsurance policies are uncommon in important instances of limited insurance such as property insurance, which is the main interest here. While coinsurance policies are rare in property insurance, policies with upper limits are prevalent. This paper examines background risk in a model of insurance that centers on choice of the upper limit.

3 Exclusions

The background risk we study arises from two types of exclusions in coverage of damage to property. One is exclusion by peril – a homeowner's policy may cover fire but exclude rising waters. The structure of exclusion by peril is naturally described as follows: The consumer has some nonrandom components of wealth that are ignored. In addition, she has a property with undamaged value $\bar{v}$ that is subject to two perils. Damages from the separate perils are represented by the random variables t and r, and total damage is f which satisfies $f \geq \max[t, r]$. Insurance is available for t only.

The second type of exclusion is by event – the policy excludes losses from the insured perils if the cause of loss is, for instance, an act of war or a civil insurrection. Event exclusions are different. Damage from the peril is denoted by t, and the event – the war or civil insurrection – is represented by a set D of values of a second random variable e. Damage is excluded from the insurance contract when e belongs to D. As before, choice of insurance is choice of parameters θ.

	utility	insurance
exclusion by peril	$u(\bar{v} - f)$	$I(t; \theta)$
exclusion by event	$u(\bar{v} - t)$	$I(t \mid e \notin D; \theta)$

Exclusion by event is parallel to ideas studied by Chen (1997) in his work on insolvency of insurance companies.

The table defines exclusion risks in a natural way, but it is not immediate that these exclusion risks have the necessary additive feature of background risk. They must be recast formally as examples of background risk. The risk of exclusion by peril can be rewritten by observing that

$$\bar{v} - f = \bar{v} - t + (t - f)$$

Thus wealth is the sum of an insurable $\bar{v} - t$, and an uninsurable $t - f$. Similarly, wealth in exclusion by event can be decomposed into two random variables. They are $x = \bar{v} - t$ if $e \notin D$ and $x = 0$ otherwise; and $y = \bar{v} - t$ if $e \in D$ and $y = 0$ otherwise. Then wealth is $x + y$ and the x component is insurable while the y component is not. These formulations are useful for classifying the two types of exclusion as background risks, but the additive form is suppressed in the analysis below.

4 Property insurance

Exclusion is particularly interesting in models of property insurance for consumers. Characteristically, the deductible is insignificant, there is no coinsurance, and the choice variable is the upper limit. The insurance contract is written in terms of damage to property. The concept of damage is important: it is the cost to restore the property to its pre-event condition. Practical application of damage may deviate somewhat from the definition, as discussed in Garratt and Marshall (1999), but the deviations are slight and do not materially affect the analysis. The important characteristic of real property is that the present building might be razed and the site converted to a different use. Typically the current building on a site is in some measure depreciated and obsolete, and it may be entirely different from the building that would have the highest value on the site by today's standards.

As a consequence of the option to convert, loss of wealth from damage is bounded. After slight damage the building is restored, but when damage passes a critical level the building is razed and replaced by something different. Once damage passes the critical level, further damage does not reduce wealth. In that way, the amount of damage that is barely sufficient to induce conversion is the bound on loss of wealth from damage[2].

Notation is needed. Look at the situation that exists when an event has caused damage. The realization of the random variable for damage is f. The property in undamaged condition would have a value of $\bar{v}$. When the owner of damaged property restores it, he attains wealth $\bar{v} - f$. On the other hand, there is some best option for converting the property. The conversion chosen by the property owner is the one that maximizes net value. Let the value of the land and improvements in the highest valued use be v^*, and let the cost of building the best improvements be c. Among the conversion options, the greatest attainable net value is $v^* - c$, which is also the value of the land. The decision whether to restore or convert is the decision to select the greater of $\bar{v} - f$ and $v^* - c$. At the critical level of damage, $\bar{q} = \bar{v} - (v^* - c)$, the options are equally valuable. After some rewriting, the option to convert becomes the option to possess $\max[\bar{v} - f, \bar{v} - \bar{q}]$ or, equivalently, $\bar{v} - \min[f, \bar{q}]$. Thus, $\bar{v} - \bar{q}$ is a floor beneath which wealth cannot fall, no matter the extent of damage. The second expression is convenient and equally intuitive. It says that the loss of wealth due to damage is no more than $\bar{q}$. The risk of f (conditional probability density function $g(f|t)$) may or may not be dependent on the risk of t (probability density function $h(t)$).

In choosing an insurance contract, the consumer selects the upper limit, which is denoted by b and thought of as "bound." There is no coinsurance or deductible. Under these conditions, the insurance payment is $\min[b, t]$, and its expected value is

[2] Garratt and Marshall, 1996, 1999 elaborate the point.

the fair premium

$$P(b) = \int_0^b th(t)dt + b\int_b^\infty h(t)dt. \tag{1}$$

Combining the pre-insurance wealth with the insurance variables, insured wealth is

$$\bar{v} - \min[\bar{q}, f] + \min[b, t] - P(b). \tag{2}$$

The expression captures two key features of property insurance: the option to convert appears in the second term, and the indemnity with upper limit is in the third term. The exclusion is seen in the difference between the actual damage f and the insurable damage t. Other features are standard.

5 Exclusion by peril

Three types of "damage" are involved. Insurable damage t is the cost to restore losses caused by one peril, uninsurable damage r is the cost to restore losses caused by the other, and joint damage f is the whole cost to restore the property to its pre-damage condition. The three damages are jointly distributed, with the following restrictions: When $t = 0$, $f = r$, and when $r = 0$, $f = t$. In addition, the total cost can be as little as $\max[t, r]$. For instance, if a house is shaken to pieces by an earthquake and also burned to the ground, the whole cost to restore the property is the cost of rebuilding the house. In that case $t = r = f$. At least that is true when differences in demolition costs are ignored, as they are here. On the other hand, the joint damage can exceed the sum of the insurable and uninsurable damages, as when a flood takes away the drive, a fire destroys the house, and the lack of a drive makes restoration of the house more expensive by delaying it.

The consumer of insurance is a risk averse maximizer of expected utility with utility function u. The problem of the consumer is to choose the upper limit on insurance, b, so as to maximize

$$E_{t,f}u(\bar{v} - \min[f\ , \bar{q}] + \min[b, t] - P) =$$
$$\int_{t=0}^{\infty}\left[\int_{f=t}^{\infty} u(\bar{v} - \min[f\ , \bar{q}] + \min[b, t] - P)g(f|t)df\right]h(t)dt \tag{3}$$

It is convenient that the damage from the uninsured peril does not appear explicitly. Less convenient is the fact that the function being optimized is not globally concave in b.[3]

[3] G&M (1999) describe this difficulty in detail.

The interesting situations are ones in which the whole loss f sometimes exceeds the insurable loss t, and sometimes exceeds the cap $\bar{q}$ on damage placed by options to convert. Otherwise the situation reduces trivially to a single insurable loss. The following result holds true if either of the non-triviality conditions does.

Proposition 1 *Suppose that* u *is risk averse and that there is positive probability mass on at least one of the sets,* $\{(t,f)|t \in (0,\bar{q}),\quad f \in (t,\bar{q})\}$ *and* $\{(t,f)|t \in (0,\bar{q}),\quad f \in (\bar{q},\infty)\}$*. Insurance is fairly priced. Then the optimum upper bound on insurance of insurable risk* t *is* $b < \bar{q}$*.*

Proof. Since concavity is not global, the proof consists of checking that for all $b \geq \bar{q}$, the derivative of the objective is negative. Rather than proving differentiability at $b = \bar{q}$, it is more convenient to show that both its right-hand and left-hand derivatives are negative at that point. Elsewhere differentiability is obvious. Let the objective in equation (3) be denoted by $K(b)$. Look at the situation in which $0 < b < \bar{q}$. In that range, expected utility is written:

$$
\begin{aligned}
K(b) = & \int_{t=0}^{b} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u(\bar{v} - \bar{q} + t - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u(\bar{v} - f + t - P)] g(f|t) df \end{array} \right] h(t) dt & (4) \\
& + \int_{t=b}^{\bar{q}} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u(\bar{v} - \bar{q} + b - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u(\bar{v} - f + b - P)] g(f|t) df \end{array} \right] h(t) dt & (5) \\
& + \int_{t=\bar{q}}^{\infty} \left[\int_{f=t}^{\infty} [u(\bar{v} - \bar{q} + b - P)] g(f|t) df \right] h(t) dt & (6)
\end{aligned}
$$

The derivative with respect to b is

$$K'(b) = -P'(b) \int\limits_{t=0}^{b} \left[\begin{array}{c} \int\limits_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + t - P)]g(f|t)df \\ + \int\limits_{f=t}^{\bar{q}} [u'(\bar{v} - f + t - P)]g(f|t)df \end{array} \right] h(t)dt$$

$$+(1 - P'(b)) \int\limits_{t=b}^{\bar{q}} \left[\begin{array}{c} \int\limits_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + b - P)]g(f|t)df \\ + \int\limits_{f=t}^{\bar{q}} [u'(\bar{v} - f + b - P)]g(f|t)df \end{array} \right] h(t)dt$$

$$+(1 - P'(b)) \int\limits_{t=\bar{q}}^{\infty} \left[\int\limits_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + b - P)]g(f|t)df \right] h(t)dt \tag{7}$$

The Leibnitz rule terms cancel.

Use the definition of the premium function in equation (1) to derive

$$P'(b) = \int_{b}^{\infty} h(t)dt. \tag{8}$$

Now evaluate at $b = \bar{q}$,

$$K'(\bar{q}) = u'(\bar{v} - P) \left[\int_{t=0}^{\bar{q}} h(t)dt \right] \int\limits_{t=\bar{q}}^{\infty} h(t)dt$$

$$- \left[\int\limits_{t=\bar{q}}^{\infty} h(t)dt \right] \int\limits_{t=0}^{\bar{q}} \left[\begin{array}{c} \int\limits_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + t - P)]g(f|t)df \\ + \int\limits_{f=t}^{\bar{q}} [u'(\bar{v} - f + t - P)]g(f|t)df \end{array} \right] h(t)dt \tag{9}$$

Finally

$$K'(\bar{q}) =$$

$$\left[\int\limits_{t=\bar{q}}^{\infty} h(t)dt \right] \int\limits_{t=0}^{\bar{q}} \left[\begin{array}{c} \int\limits_{f=\bar{q}}^{\infty} [u'(\bar{v} - P) - u'(\bar{v} - \bar{q} + t - P)]g(f|t)df \\ + \int\limits_{f=t}^{\bar{q}} [u'(\bar{v} - P) - u'(\bar{v} - f + t - P)]g(f|t)df \end{array} \right] h(t)dt \tag{10}$$

The integrands in the inner-most brackets are negative on sets having nonzero probability and non positive everywhere. Therefore, when approaching from the left,

$$K'(\bar{q}) < 0 \tag{11}$$

To complete the proof, look at the situation in which $0 < \bar{q} < b$. In that range, expected utility is written:

$$\begin{aligned}
K(b) = &\int_{t=0}^{\bar{q}} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u(\bar{v} - \bar{q} + t - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u(\bar{v} - f + t - P)] g(f|t) df \end{array} \right] h(t) dt \\
&+ \int_{t=\bar{q}}^{b} \left[\int_{f=t}^{\infty} [u(\bar{v} - \bar{q} + t - P)] g(f|t) df \right] h(t) dt \\
&+ \int_{t=b}^{\infty} \left[\int_{f=t}^{\infty} [u(\bar{v} - \bar{q} + b - P)] g(f|t) df \right] h(t) dt
\end{aligned} \tag{12}$$

The derivative with respect to b is

$$\begin{aligned}
K'(b) = -P'(b) &\int_{t=0}^{\bar{q}} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + t - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u'(\bar{v} - f + t - P)] g(f|t) df \end{array} \right] h(t) dt \\
-P'(b) &\int_{t=\bar{q}}^{b} \left[\int_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + t - P)] g(f|t) df \right] h(t) dt \\
+(1 - P'(b)) &\int_{t=b}^{\infty} \left[\int_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + b - P)] g(f|t) df \right] h(t) dt
\end{aligned} \tag{13}$$

Again, the Leibnitz rule terms cancel.

Use

$$1 - P'(b) = \int_0^b h(t) dt.$$

and get

$$K'(b) = -P'(b) \int_{t=0}^{\bar{q}} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + t - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u'(\bar{v} - f + t - P)] g(f|t) df \end{array} \right] h(t) dt$$
$$-P'(b) \int_{t=\bar{q}}^{b} \left[\int_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + t - P)] g(f|t) df \right] h(t) dt$$
$$+ \int_{t=b}^{\infty} h(t) dt \int_{t=0}^{b} \left[\int_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + b - P)] g(f|t) df \right] h(t) dt \qquad (14)$$

Then

$$K'(b) = -P'(b) \int_{t=0}^{\bar{q}} \left[\begin{array}{c} \int_{f=\bar{q}}^{\infty} [u'(\bar{v} - \bar{q} + t - P) - u'(\bar{v} - \bar{q} + b - P)] g(f|t) df \\ + \int_{f=t}^{\bar{q}} [u'(\bar{v} - f + t - P) - u'(\bar{v} - \bar{q} + b - P)] g(f|t) df \end{array} \right] h(t) dt$$
$$-P'(b) \int_{t=\bar{q}}^{b} \left[\int_{f=t}^{\infty} [u'(\bar{v} - \bar{q} + t - P) - u'(\bar{v} - \bar{q} + b - P)] g(f|t) df \right] h(t) dt (15)$$

The expressions in square brackets are sometimes positive and always non negative. Therefore $K'(b) < 0$ for all $b > \bar{q}$.

The conclusion is that the optimum b satisfies $b < \bar{q}$. ■

It is obvious that in the absence of the uninsured peril, the optimum upper limit is $b = \bar{q}$. Thus the derivation shows that the uninsured peril makes the optimum upper limit less than it would be in the absence of the uninsured peril, and less than it would be in an all-peril policy that insured f.

The result does not rely on any particular dependence. Even if f is perfectly correlated with t, there is nevertheless a reduced demand for insurance of t. Think of the case that $f = 2t$, perfect correlation. Start with small t. The total loss is $2t$, indemnity is t, and the uninsured loss is t. Now consider progressively larger realizations of t. The uninsured loss grows until $t = .5\bar{q}$. At that point $f = \bar{q}$ and further damage does not further diminish wealth. As t grows further, the uninsured loss declines and reaches zero when $t = \bar{q}$, assuming that b is large enough. Now consider the possibility that $b = \bar{q}$ might be the optimum upper limit. Large losses are fully insured, so a larger b makes no sense, and there is, in fact, a reason to lower b. Doing so raises wealth in the high marginal utility states like $t = .5\bar{q}$ while lowering it in some states with low marginal utility, like $t = \bar{q}$. Therefore the optimum upper limit is less than $b = \bar{q}$, as the proof above shows.

The result has an interesting application. Consider the insurance company offering home-owners insurance. It can raise the demand for its product by offering an earthquake rider. Even if the earthquake premia are ceded in such a way that they do not raise profit directly, they raise it indirectly through stimulating demand for the other insurance product.

6 Exclusion by event

The other type of exclusion is triggered by an event such as an act of war that negates all liabilities. Consider an all-peril homeowner insurance contract with an exclusion for all damage in the case of the event. Let t be damage. In determining the conversion option and the bound on loss of wealth, t has the same role as f had in the previous section.

The event happens with probability π. In that case, the probability distribution function of damage is $g(t)$. With probability $1 - \pi$ there is no earthquake, but the house suffers damage t according to the probability density function $h(t)$. The fair premium is P given by

$$P(b) = (1 - \pi) \left[\int_0^b t h(t) dt + b \int_b^\infty h(t) dt \right] \tag{16}$$

The goal of the consumer is to choose b to maximize

$$J(b) = (1 - \pi) \int_0^\infty u(\bar{v} - \min[t,\ \bar{q}] + \min[b, t] - P) h(t) dt$$
$$+\pi \int_0^\infty u(\bar{v} - \min[t,\ \bar{q}] - P) g(t) dt$$

Exclusion by event unambiguously lowers demand for fairly priced insurance. That is the burden of

Proposition 2 *When event exclusions are present in a fairly priced insurance contract, the risk averse consumer demands less than full coverage of the included risks.*

Proof. As in the previous proposition, the proof consists of checking that for all $b \geq \bar{q}$, the derivative of the objective is negative. It is again more convenient to show that both right-hand and left-hand derivatives are negative at $b = \bar{q}$. The first part of the proof shows that the left-hand derivative of the objective function at the point $b = \bar{q}$ is negative. The second part shows that for $b > \bar{q}$, the derivative is always negative. Begin, then, with $b < \bar{q}$ and b is approaching $\bar{q}$ as a limit. In this range

$$J(b) = (1-\pi)\left[\int_0^b u(\bar{v} - P)h(t)dt + \int_b^{\bar{q}} u(\bar{v} - t + b - P)h(t)dt + \int_{\bar{q}}^{\infty} u(\bar{v} - \bar{q} + b - P)h(t)dt\right] + \pi \int_0^{\infty} u(\bar{v} - \min[t, \bar{q}] - P)g(t)dt \tag{17}$$

Write the derivative with respect to b.

$$J'(b) = (1-\pi)\left(-P'(b)\right)\left[\begin{array}{c} \int_0^b u'(\bar{v} - P)h(t)dt + \\ \int_b^{\bar{q}} u'(\bar{v} - t + b - P)h(t)dt \\ + \int_{\bar{q}}^{\infty} u'(\bar{v} - \bar{q} + b - P)h(t)dt \end{array}\right] (1-\pi)\left[\begin{array}{c} \int_b^{\bar{q}} u'(\bar{v} - t + b - P)h(t)dt \\ + \int_{\bar{q}}^{\infty} u'(\bar{v} - \bar{q} + b - P)h(t)dt \end{array}\right] + \pi\left[-P'(b)\int_0^{\infty} u'(\bar{v} - \min[t, \bar{q}] - P)g(t)dt\right] \tag{18}$$

Evaluating at $b = \bar{q}$,

$$J'(\bar{q}) = (1-\pi)(-P'(\bar{q}))\left[\int_0^{\bar{q}} u'(\bar{v} - P)h(t)dt + \int_{\bar{q}}^{\infty} u'(\bar{v} - P)h(t)dt\right] + (1-\pi)\int_{\bar{q}}^{\infty} u'(\bar{v} - P)h(t)dt + \pi\left[(-P'(\bar{q}))\int_0^{\infty} u'(\bar{v} - \min[t, \bar{q}] - P)g(t)dt\right] \tag{19}$$

The first two terms cancel. Then

$$J'(\bar{q}) = \pi\left[(-P'(\bar{q}))\int_0^{\infty} u'(\bar{v} - \min[t, \bar{q}] - P)g(t)dt\right] \tag{20}$$

A negative quantity. Consequently $J'(\bar{q}) < 0$.

Consider now $b > \bar{q}$.

$$J(b) = (1-\pi)\left[\int_0^{\bar{q}} u(\bar{v} - P)h(t)dt + \int_{\bar{q}}^{b} u(\bar{v} - \bar{q} + t - P)h(t)dt + \int_b^{\infty} u(\bar{v} - \bar{q} + b - P)h(t)dt\right] + \pi \int_0^{\infty} u(\bar{v} - \min[t, \bar{q}] - P)g(t)dt \tag{21}$$

Write the derivative with respect to b, noting that the Leibnitz terms cancel.

$$J'(b) = (1-\pi)\left(-P'(b)\right)\begin{bmatrix} \int_0^{\bar{q}} u'(\bar{v} - P)h(t)dt + \\ \int_{\bar{q}}^{b} u'(\bar{v} - \bar{q} + t - P)h(t)dt \\ + \int_b^{\infty} u'(\bar{v} - \bar{q} + b - P)h(t)dt \end{bmatrix} (1-\pi)\left[\int_b^{\infty} u'(\bar{v} - \bar{q} + b - P)h(t)dt\right] + \pi\left[-P'(b)\int_0^{\infty} u'(\bar{v} - \min[t, \bar{q}] - P)g(t)dt\right] \tag{22}$$

Using equation (8) for the derivative of the premium function, the middle term is

$$(1-\pi)P'(b)u'(\bar{v} - \bar{q} + b - P) \tag{23}$$

Then

$$J'(b) = (1-\pi)\left(-P'(b)\right)\begin{bmatrix} \int_0^{\bar{q}}[u'(\bar{v} - P) - u'(\bar{v} - \bar{q} + b - P)]h(t)dt + \\ \int_{\bar{q}}^{b} [u'(\bar{v} - \bar{q} + t - P) - u'(\bar{v} - \bar{q} + b - P)]h(t)dt \end{bmatrix} + \pi\left[-P'(b)\int_0^{\infty} u'(\bar{v} - \min[t, \bar{q}] - P)g(t)dt\right] \tag{24}$$

In the first term, the square bracket is positive because in each of the inner square brackets, the value is sometimes positive and never negative. Thus the whole first term is negative. So is the second term. Therefore, for all $b > \bar{q}$, $J'(b) < 0$. Thus optimum b satisfies $b < \bar{q}$. ■

The result makes sense: instead of fully insuring for the included perils, the consumer accepts some risk from them in order to transfer wealth into states associated with the excluded loss. It is another case of degradation in the quality of insurance leading to reduced demand for it.

7 Concluding remarks

Previous results on background risk consider a coinsurance contract written for risks to wealth, but results derived in that environment do not generalize to all situations. In the upper-limit style contracts studied in this paper, the correlations that were central in the coinsurance model turn out to be unimportant, and the effect of background risk is unambiguously to reduce the demand for insurance of the insurable risk.

This insight is important for understanding consumer demand for insurance but it also sheds light on profit opportunities for insurers. The implication is that insurers might earn double profits from extending coverages, a primary profit from the new coverage, and a secondary profit from increased demand for existing coverage. In some cases the secondary effect can be bigger than the primary and can be, in fact, the whole motivation for extending coverages. Thus when coverage of theft joins with coverage of fire in a homeowners insurance policy, profits from the increased demand for fire coverage may be more important than the profits arising directly from theft insurance. Alternatively, earthquake coverage may be offered under a reinsurance pact that absorbs all profits from it, but the supplier of homeowners insurance can still gain because demand for the underlying coverage increases.

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