

Support Contingent Insurance: The Optimality of Supplementary Coverage

by Richard Watt^{*+} and Francisco J. Vázquez^{*}

Abstract

In this paper we reconsider the basic notion that an insurance contract premium must be independent of the outcome of the insured event. This is done by simply assuming that the parties to an insurance contract may agree that the contract becomes null should the value of the insurable loss fall outside a pre-established support, in which case any premiums corresponding to the contract that have been paid are returned. Using this concept of insurance, we show that the classic deductible contract format is no longer optimal, since the insured will always prefer to purchase supplementary insurance to cover the deductible.

Key words: Deductible insurance, optimal insurance contracts

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1. Introduction

Insurance is the exchange of one random variable for another. In effect, all insurance does is alter the probabilities that must be associated with each possible payoff, within the original support. For example, if an uninsured random variable is defined by the density $f(x)$, with distribution $F(x)$, on support (a, b) , with $a < b$, and an insurance contract calls for full coverage in exchange for a premium payment of p , where $a < p < b$, then the post-insurance distribution is simply defined as:

$$G(x) = \begin{cases} 0 & a \leq x < p \\ 1 & x = p \\ 0 & p < x \leq b \end{cases}$$

Hence, the study of insurance is equivalent to the study of preferences over probability densities.

In 1963, Ken Arrow proved a fundamental theorem for insurance economics, namely that conditional on the insurer charging a premium that is proportional to the expected value of the insured loss, then given the choice of any feasible indemnity function all risk averse insurance consumers prefer a contract that has full coverage above a deductible (see Arrow (1963) and Arrow (1971)). The intuition behind this result was provided by Gollier and Schlesinger (1994) who show that such a contract second order stochastic dominates all others. Hence, given an insurable random variable, \tilde{x} , with probability density function (here-in-after, pdf) $f(x)$, defined on support (a, b) , then the optimal contract (from the insured's point of view) would stipulate some number D , where $a \leq D \leq b$, such that the contracted indemnity function is:

$$I(x) = \begin{cases} 0 & x \leq D \\ x - D & x > D \end{cases}$$

and the premium is:

$$p(D, \lambda) = \lambda \int_D^b (x - D) f(x) dx \quad (1)$$

where $\lambda \geq 1$ is the loading factor used by the insurer. Under this contract, assuming the insured has an initial wealth of w , his expected utility is:

$$EU(D,\lambda)=\int_a^D u[w-x-p(D,\lambda)]f(x)dx+u[w-D-p(D,\lambda)]\int_D^b f(x)dx \quad (2)$$

Mossin (1968) analyzed this type of contract, and concluded that:

- i. if the first order condition for an optimal deductible is satisfied at a point on the support of $f(x)$, then at the same point the second order condition will also be satisfied,
- ii. $D^* > a$ if $\lambda > 1$, or in words, if the premium is actuarially unfair, then the insured will always retain some risk, and
- iii. the optimal deductible is increasing in wealth.

However, although it is true that for a general utility function it is impossible to solve for the first order condition algebraically, the general characteristics of the optimal deductible can be studied. Pashigian, Schdade and Menefee (1966) and Gould (1969) provide solutions for particular utility functions, while Moffet (1977) and Schlesinger (1981) analyze certain aspects of the optimal deductible.

One interesting feature of the classic deductible insurance contract is that the contract itself is not made contingent upon the support of the insured random variable. To illustrate this point, note that any insurable random variable of the sort described above can be split into many “partial” random variables along the initial support. For example, a random variable with density $f(x)$ on support (a, b) with $a < b$, can be split into two separate random variables, according to Table 1:

Table 1

random variable	Support	Conditional density
1	$a \leq x \leq c$	$f_1(x) = \frac{f(x)}{F(c)}$
2	$c \leq x \leq b$	$f_2(x) = \frac{f(x)}{1-F(c)}$

In this way, the original random variable is split into two separate variables, one covering the first part of the support and the other covering the second part of the support. Without loss of generality, the exact value c has been assumed to be included in both supports, which is possible since the probability that x takes exactly the value c is 0. The conditional densities give the probability of each possible value of each random variable, conditional on the random variable taking a value within the relevant support. Naturally, there are infinitely many splits that can be made, i.e. there is no need at all for the original random variable to be split into only two parts. If a random variable is split up in this way, there would seem to be no reason why separate insurance contracts cannot be written, each one conditional on each partial random variable that the client wishes to define.

We define a “support contingent” insurance contract in the following way:

Definition 1: A support contingent insurance contract stipulates wealth transfers only if x takes a value within an agreed support.

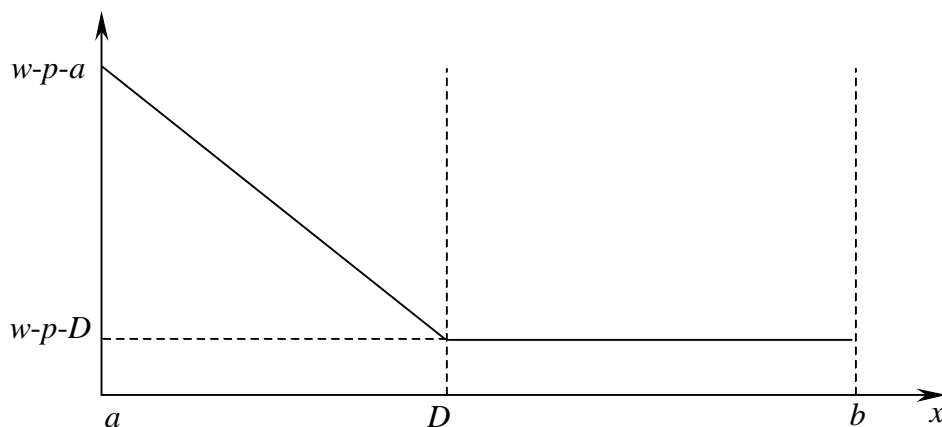
Hence, since traditional insurance contracts stipulate that at least a premium payment is transferred independently of the final value of x , such contracts are contingent upon the entire original support. It is within such a scenario that Arrow’s theorem of optimal contracts is cast. Therefore, the use of support contingent insurance may be able to provide improvements to the expected utility that can be achieved under Arrow’s classic deductible contract, since Arrow’s theorem can be seen to be a special case of support contingent insurance, when the original support is retained, i.e it is “split” into only one segment. What it will not alter, however, is the basic optimal nature of the deductible format for each individual contract, taken in isolation of the others.

While support contingent contracts are, as far as we are aware, not used for insurance, they are used elsewhere. For example, it is common in the contracts that professional athletes sign with their sponsors that their income depends upon the sporting results that they achieve, with better results being paid relatively more

handsomely. However, in most of these contracts, clauses are included to the effect that the contract will be re-negotiated in the event of certain extra-ordinary events occurring (perhaps becoming Olympic champion, or setting a world record). Hence, the contract effectively becomes null for certain contingencies.

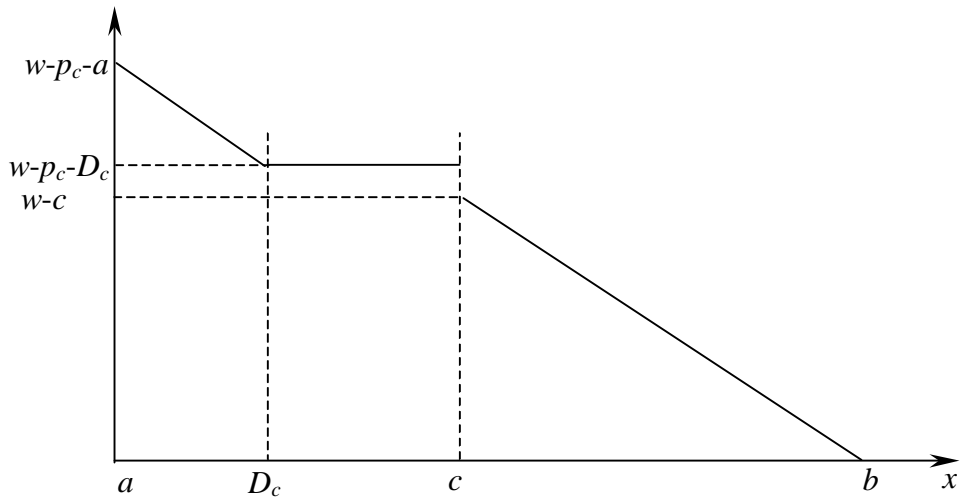
Note that support contingent insurance can lead to a final wealth distribution that is substantially different to what can be attained under a traditional contract. For example, if an initial random variable is insured under a traditional deductible contract, then the insured's final wealth in function of the value of the random variable is given by Figure 1. In Figure 1 it is assumed that the insured has wealth independent of the random variable of w , that the premium paid under the insurance contract is p , and that the deductible is D . This wealth distribution can be compared with the pre-insurance distribution, which would just be a line with slope -1 over the entire support of x , beginning at the point $w-a$, i.e. parallel to but above the first segment of figure 1.

Figure 1



On the other hand, suppose the insured split the initial random variable into two separate supports, as in Table 1, and a support contingent deductible contract was signed for the first of the resulting random variables, the wealth distribution would be given by Figure 2 (where p_c is the premium and D_c is the deductible contracted when the initial random variable is split at some intermediate value c).

Figure 2



The reason why a support contingent contract may dominate a usual deductible contract is simply that the premium that must be paid under the support contingent contract is only paid if the random variable takes a value within the relevant support. Hence, the insurance consumer would balance the effects of not obtaining coverage for a given part of the random variable but not paying the premium outside of the insured support either.

In order for insurance to make any sense at all, we assume throughout that the insurance purchaser is strictly wealth loving and strictly risk averse, i.e. $u'(\cdot) > 0, u''(\cdot) < 0$. Furthermore, it is assumed that $0 < F(x) < 1$ for all $a < x < b$.

2. How will the original support be divided?

In principle, support contingent insurance may provide expected utility improvements over the traditional contract, defined on the entire support. However, care must be taken, since as we will now show, the part of the original support used to define a support contingent contract is fundamental if the classic deductible format is to be dominated.

Theorem 1: It is never optimal to reduce the size of the support of the original density from below, and then to use only one support contingent contract.

Proof: This is proven by showing that, for any two values defining the a support $c < b$, and for any intermediate value D defining a contract deductible as in equation (2), with $c < D < b$, then contracting the same deductible, but defined on the entire support $[a, b]$ will always give greater expected utility. That is, for any given D , we will show that $EU(c) < EU(a)$ for all c greater than a and less than D .

Firstly, note that the density of the random variable defined on the smaller support (c, b) is:

$$f_c(x) = \frac{f(x)}{[1-F(c)]}$$

and, assuming the same loading factor as for the contract defined on the entire support, the premium corresponding to this contract is:

$$P(c) = \lambda \int_D^b (x-D) f_c(x) dx = \frac{1}{[1-F(c)]} P(a) > P(a)$$

The expected utility of the contract conditional on support (c, b) is:

$$\begin{aligned} EU(c) &= \int_a^c u(w-x) f(x) dx + [1-F(c)] \left\{ \int_c^D [u(w-x-P(c))] f_c(x) dx + u[w-D-P(c)] \int_D^b f_c(x) dx \right\} \\ &= \int_a^c u(w-x) f(x) dx + \int_c^D [u(w-x-P(c))] f(x) dx + u[w-D-P(c)] [1-F(D)] \end{aligned}$$

and the expected utility of the contract conditional on the entire support (a, b) is as given in (2):

$$EU(a) = \int_a^D [u(w-x-P(a))] f(x) dx + u[w-D-P(a)] [1-F(D)]$$

Hence, the expected utility difference between the two can be expressed as:

$$EU(a) - EU(c) = \int_a^c H_1(x) f(x) dx + \int_c^D H_2(x) f(x) dx + [u[w-D-P(a)] - u[w-D-P(c)]] [1-F(D)]$$

where:

$$H_1(x) \equiv u[w-x-P(a)] - u(w-x)$$

$$H_2(x) \equiv u[w-x-P(a)] - u[w-x-P(c)]$$

Since by the concavity of $u(\cdot)$, $H_1'(x) = -u'[w-x-P(a)] + u'(w-x) < 0$ for all values of x between a and c , and $H_2'(x) = -u'[w-x-P(a)] + u'[w-x-P(c)] > 0$ for all values of x between c and D , we can write:

$$\begin{aligned} EU(a) - EU(c) \geq & \{u[w-c-P(a)] - u(w-c)\}F(c) + \{u[w-c-P(a)] - u[w-c-P(c)]\}[F(D) - F(c)] + \\ & + \{u[w-D-P(a)] - u[w-D-P(c)]\}[1 - F(D)] \end{aligned}$$

Now, since the utility function is strictly concave, it is true that for any x and y :

$$u(x) - u(y) \leq u'(y)(x - y)$$

Hence:

$$\begin{aligned} u(w-c) - u[w-c-P(a)] & \leq u'[w-c-P(a)]P(a) \\ u[w-c-P(c)] - u[w-c-P(a)] & \leq u'[w-c-P(a)][P(a) - P(c)] \\ u[w-D-P(c)] - u[w-D-P(a)] & \leq u'[w-D-P(a)][P(a) - P(c)] \end{aligned}$$

Multiplying each of these by -1 and substituting into the equation for the difference in expected utilities gives:

$$\begin{aligned} EU(a) - EU(c) \geq & -u'[w-c-P(a)]\{F(c)P(a) - [F(D) - F(c)][P(c) - P(a)]\} + \\ & + u'[w-D-P(a)]\{[1 - F(D)][P(c) - P(a)]\} \end{aligned}$$

Finally, since $P(a) = [1 - F(c)]P(c)$, we have $P(c) - P(a) = P(c)F(c)$, and so:

$$\begin{aligned} EU(a) - EU(c) \geq & -u'[w-c-P(a)]\{F(c)[1 - F(c)]P(c) + [F(D) - F(c)]F(c)P(c)\} + \\ & + u'[w-D-P(a)]\{[1 - F(D)]F(c)P(c)\} \\ = & F(c)P(c)\{-u'[w-c-P(a)][1 - F(c) - F(D) + F(c)] + u'[w-D-P(a)][1 - F(D)]\} \\ = & F(c)P(c)[1 - F(D)]\{u'[w-D-P(a)] - u'[w-c-P(a)]\} \geq 0 \end{aligned}$$

where the final inequality is due to the concavity of the utility function and the fact that $D > c$.

Since this has been done for any general D , it holds for the optimal D value for the contract contingent on support (c,b) , i.e. for any c located between a and b , the optimal contingent deductible $D^*(c)$ gives less expected utility than purchasing the same deductible based on the entire support (a,b) . Since this in turn will be dominated by the optimal deductible based on the support (a,b) , $D^*(a)$, it is true that simply reducing the support from below is never optimal if only one contract is to be used.

Q.E.D.

Theorem 1 shows that, for any given random variable support (a,b) the insured cannot increase his expected utility by purchasing a single support contingent contract defined on a smaller support (c,b) where $c > a$. In some respects this may be surprising, since under both the support contingent contract defined by c , and the contract defined on the full support, no coverage is purchased for loss values on $a \leq x \leq D$, and full coverage is purchased beyond the deductible. Hence both contracts have exactly the same indemnity schedule. On the other hand, the support contingent contract implies that no premium is paid between a and c , while under the traditional contract, the premium is always paid. The intuition behind the result is that the support contingent contract has a higher premium (it is equal to the traditional contract premium divided by something less than one), and the expected premium over the entire support is equal under both contracts. Hence, the support contingent option gives the same indemnity function, and the same expected premium, but a greater premium variance. Hence it results in a wealth distribution that has the same expected value as the classic deductible format, but with a higher variance, which must reduce expected utility.

The reasoning behind why a single support contingent contract defined by reducing the support from below does not dominate a traditional deductible contract does not carry over to the case when more than one mutually exclusive support contingent contract is purchased, since although the expected premium is still constant a different indemnity schedule is achieved. It also does not carry over to the case when the original support is reduced from above, even though only one contract is purchased, since once again a different indemnity schedule is purchased, and also in this case the expected premium also varies.

From theorem 1 we can directly conclude:

Corollary 1: If an insurance consumer uses support contingent insurance, then he must either reduce the original support from above (leaving large loss values uncovered), or use at least two simultaneous contracts.

It is useful to consider support contingent insurance with a simple example, in which both premiums are paid for some of the support of the original random variable, and only one premium is paid for the rest of the support. The example demonstrates the potential value of support contingent insurance in current insurance markets.

3. Supplementary support contingent insurance for deductible contracts

If we assume the same general environment as Arrow, namely full and perfect information regarding the pdf, the fact that the optimal contract leaves a part of the risk uninsured cannot be thought of as some sort of incentive compatibility device. Indeed it is the result of the insured balancing out the premium cost against the cost of retaining risk. Hence the insurer should have no complaints if the insured decided to purchase a new contract in order to insure the deductible of the first contract. In this paper, we call the random variable that the insured retains with the deductible the “residual” random variable, which is defined on the support $[a, D]$, where D is the deductible purchased on the first contract (calculated on the entire original random variable). If the residual random variable is insured, we say that “supplementary” insurance has been purchased. Following Arrow, the insured may seek a second contract that involves a second deductible, R , to cover the residual risk. The insurable residual random variable is defined on the support $a \leq x \leq D$, and we will denote its (conditional) pdf by:

$$f_r(x) = \frac{f(x)}{F(D)}$$

The important point to note is that the second contract is support contingent on the support of the residual random variable. Hence, if supplementary insurance is purchased, the insured will now pay both the premium from the original contract and the premium from the supplementary contract if the final value of x is less than or equal to D , and will only pay the premium corresponding to the original contract if the final value of x is greater than D .

It is assumed that the option of supplementary support contingent insurance exists at the same loading factor as an initial insurance contract. Since the support of the insurable residual random variable depends on the deductible chosen on the first contract, we now consider the theory of optimal deductibles.

4. Aspects of the optimal deductible

Using the equation for the premium (1), it can easily be seen that:

$$p'(D) = -\lambda[1-F(D)]$$

where F is the cumulative distribution associated with f . Hence, the first derivative of the expected utility of the insured (2) is:

$$EU'(D) = [1-F(D)] \left\{ \lambda \int_a^D u'[y(x)] f(x) dx - u'(z) [1-\lambda[1-F(D)]] \right\} = [1-F(D)] S(D) \quad (3)$$

where we have used:

$$y(x) \equiv w - x - p(D) \quad z \equiv w - D - p(D)$$

The second derivative of expected utility can be seen to be:

$$EU''(D) = -f(D)S(D) + [1-F(D)] \left\{ \lambda^2 [1-F(D)] \int_a^D u''[y(x)] f(x) dx + u''(z) [1-\lambda[1-F(D)]]^2 \right\} \quad (4)$$

$$= -f(D)S(D) + [1-F(D)] S'(D)$$

Note that, for any value of D such that $S(D) \geq 0$, it is true that the second derivative is non-positive, since the concavity of the utility function implies directly that $S(D)$ is strictly decreasing, $S'(D) < 0$. Hence, expected utility is concave over all values of D for which expected utility is non-decreasing. In particular, it can be affirmed that at any value of D strictly less than b such that the first order condition for an optimum is satisfied, then that point must be unique, and at that same point the second order condition will also be satisfied, i.e. any internal point at which marginal expected utility is zero must be a global maximum. Given this, the optimal value of D for any insurable variable is given by either $D=b$, which sets $F(D)=1$, or, if such a point exists, $D:a \leq D < b$ such that $S(D)=0$. If an internal point exists such that the first order condition is satisfied, then the second solution, $D=b$, must be a local minimum of expected utility (the other local minimum is at $D=a$).

Given this, it is necessary to study the function $S(D)$ as defined in (3) in order to identify the situations in which a strictly internal optimum exists. This is done with the following series of lemmas, all of which are well known from past literature. It is explicitly brought to the reader's attention that $P(b)=0$.

Lemma 1: Independently of the existence or not of an internal optimum, it is true that:

- i. $S(a) = u'[w-a-p(a)](\lambda-1) > 0$
- ii. $EU'(b) = 0$

From lemma 1, it holds that, independently of the existence of an internal optimum:

$$EU'(a) = S(a) > 0$$

and so it is also true that:

$$EU''(a) = -f(a)S(a) + u''[w-a-p(a)](\lambda-1)^2 < 0$$

Hence, independently of the existence or not of an internal optimum, the graph of marginal expected utility begins at some strictly positive number and has initially strictly negative slope.

Lemma 2: If no internal optimum exists, then it must hold that:

$$S(D) > 0 \quad \forall D: a \leq D < b$$

and

$$EU''(b) = -f(b)S(b) \leq 0$$

Hence, we have:

Lemma 3: If an internal optimum exists, then it must hold that:

$$S(b) < 0 \quad \text{and} \quad EU''(b) = -f(b)S(b) \geq 0$$

Note that if an internal optimum exists, it is unique.

Lemma 4: A necessary and sufficient condition for the existence of an internal optimum at D is that $\lambda[1-F(D)] < 1$.

Indeed, if $S(D)$ is to be equal to zero, its second summand must be negative at that value of D , which will require the condition stated in lemma 4.

Lemma 5: A sufficient condition for the existence of an internal optimum at D is:

$$\lambda < \frac{u'(w-b)}{\int_a^b u'(w-x)f(x)dx}$$

Lemma 6: If an internal optimum exists at D , then:

$$\lambda \int_a^D [w-x-p(D)]f(x)dx = [1-\lambda[1-F(D)]]u'[w-D-p(D)]$$

Lemma 6 is just the condition that at an internal optimum, it must be true that $S(D)=0$.

5. How supplementary support contingent insurance increases expected utility

In this section we consider the following situation. An initial risk has been insured under a deductible contract, and the optimal deductible has been found to be strictly less than the greatest possible loss, b . The insurance consumer then considers the possibility of purchasing supplementary coverage for the residual random variable that the original deductible implies, but where the supplementary insurance will be support contingent.

Theorem 2: If support contingent insurance is available to insure the residual random variable, then the insured will always purchase supplementary support contingent insurance after purchasing an initial deductible.

Proof: If a supplementary deductible is purchased, expected utility becomes:

$$\begin{aligned}
 EU(D,R) &= F(D) \left\{ \int_a^R u[w-x-P_r(D,R)] f_r(x) dx + u[w-R-P_r(D,R)] \int_R^D f_r(x) dx \right\} + \\
 &\quad + u[w-D-P(D)] [1-F(D)] \\
 &= \int_a^R u[w-x-P_r(D,R)] f(x) dx + u[w-R-P_r(D,R)] [F(D)-F(R)] + u[w-D-P(D)] [1-F(D)]
 \end{aligned}$$

where:

$$P_r(D,R) = \lambda \int_R^D (x-R) f_r(x) dx + \lambda \int_D^b (x-D) f(x) dx = \left(\frac{\lambda}{F(D)} \right) \int_R^D (x-R) f(x) dx + P(D)$$

and $F(\cdot)$ is the cumulative distribution associated with $f(\cdot)$.

Since this problem is formally so similar to the problem of finding the original optimal deductible, D , from (3) the first derivative of expected utility with respect to R is given by:

$$EU'_R(D,R) = \frac{[F(D)-F(R)]}{F(R)} S_r(D,R)$$

where:

$$S_r(D,R) \equiv \lambda \int_a^R u'[w-x-P_r(D,R)] f(x) dx + \{-F(D) + \lambda[F(D) - F(R)]\} u'[w-R-P_r(D,R)]$$

Once again, it is easy to show that $S_r(D,a) > 0$, and that $S'_r(D,R) < 0$ so if an optimal R exists that is strictly less than D , then using lemma 3 it must hold that $S_r(D,D) < 0$. Once again, it is explicitly brought to the reader's attention that $P_r(D,D) = P(D)$, and so:

$$S_r(D,D) = \lambda \int_a^D u'[w-x-P(D)] f(x) dx - F(D) u'[w-D-P(D)] \quad (5)$$

However, since we are using the assumption that $D < b$, the equation in lemma 6 must hold, which together with the equation for the insurable residual random variable implies that (5) can be written as:

$$\begin{aligned} S_r(D,D) &= [1 - \lambda[1 - F(D)]] u'[w-D-P(D)] - F(D) u'[w-D-P(D)] \\ &= u'[w-D-P(D)] \{1 - \lambda[1 - F(D)] - F(D)\} \\ &= u'[w-D-P(D)] (1 - \lambda)[1 - F(D)] < 0 \end{aligned}$$

where the strict negativity follows since $\lambda > 1$.

Q.E.D.

Corollary 2: $P_r(D,R) - P(D) < D - R$

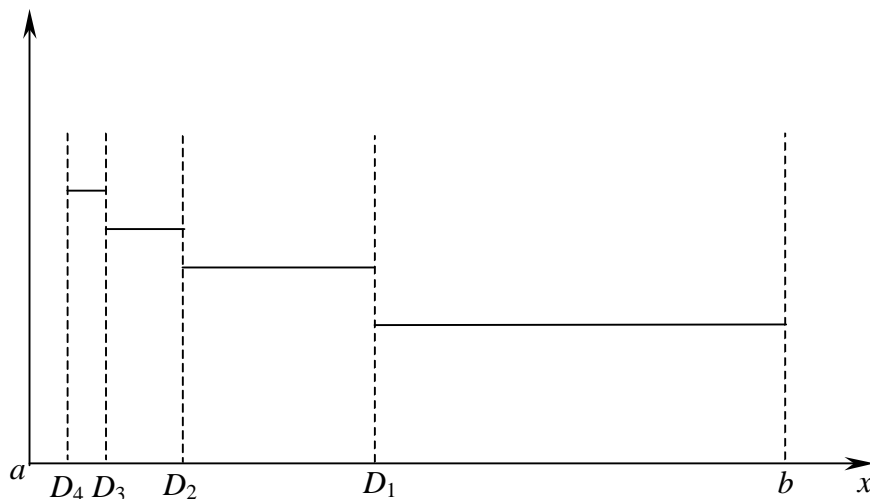
Corollary 2 is immediate from a graph of the distribution of wealth that the insured obtains when he purchases both contracts. If the corollary were not true, then the negatively sloped part of the wealth distribution obtained from the single deductible contract would be everywhere above the distribution obtained, over the same x values, with the supplementary insurance. This, of course, would contradict the optimality of supplementary support contingent coverage for the residual random variable. The corollary implies that the horizontal part of the wealth distribution obtained from the

supplementary contract is at a higher wealth level than the horizontal part of the wealth distribution from the first contract.

Theorem 2 defines the optimal insurance strategy for a myopic consumer. The consumer is myopic in the sense that he does not take into account the supplementary insurance option when setting the optimal deductible on the original support. However, for this type of consumer, the result of theorem 2 can always be reapplied to each successive residual random variable (defined by the deductible of each successive supplementary support contingent contract). Hence, we can state directly:

Corollary 3: The expected utility maximizing insurance strategy for a myopic consumer will be an infinite succession of support contingent contracts, each one providing supplementary coverage over the deductible of the preceding contract, such that the sequence of deductibles (D_1, D_2, \dots) converges to a (see figure 3).

Figure 3



Obviously, at this point no claim at all can be made to the effect that the situation described in corollary 3 in any way constitutes optimal behavior by the insurance consumer. In particular, optimal behavior would require that, in theorem 2, the option of supplementary support contingent insurance on the residual random variable left over be taken into account when the first contract (D) is established. In this way, a non-

myopic consumer would want to maximize his expected utility with respect to a vector of deductibles, one for each possible support contingent contract, under the restrictions that each deductible falls within the relevant support. This is a most difficult problem, since each support is defined by the previous deductible, and so each successive deductible depends on the preceding one. Hence, while this problem can be reduced to a single dimension, where the decision variable is the deductible on the original support, this variable appears in many (up to infinite) places in the objective function, with an increasingly complex effect.

In any case, since the myopic supplementary insurance scheme considered here does give greater expected utility than a simple traditional deductible contract, obviously the solution to the general problem will give an even greater improvement.

6. Conclusions

The fact that the supplementary support contingent insurance increases expected utility beyond what has been achieved with the first deductible contract is entirely due to the fact that the second premium is not paid if x falls beyond D . If the second premium were paid over the entire support of the initial random variable, then the solution to the supplementary insurance problems would be $R=D$, i.e. that no coverage would be purchased. Indeed it is relatively simple to show that, if the insured must pay a premium of $P_r(D,R)$ for all values of x , then the corresponding value of $S_r(D,D)$ is positive, which implies that no internal solution exists for the supplementary insurance problem. However, this result is obvious anyway, since it is covered by Arrow's optimal insurance indemnity theorem.

Theorem 2 shows that an optimal, even though myopic, supplementary support contingent insurance strategy is expected utility improving, however, we have yet to find the general optimal insurance strategy. The general solution to the problem may involve splitting the initial random variable into mutually exclusive parts, and

purchasing an independent optimal deductible contract on each of these parts. In general, the theoretical number of splits of the initial random variable is unlimited, which may lead to a new concept of insurance contract, where both the premium and the indemnity are a smooth function of the loss that occurs.

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