Optimal Accident Compensation Schemes

Abstract

This paper considers the way in which accident compensation is offered as insurance against personal injury due to accidents. We firstly set up a simple microeconomic model in which accident compensation schemes can be studied, and then we characterise the optimal accident compensation scheme, as that which maximizes the expected utility of the insured, for a given expected outlay of the scheme (that is, for a budget constraint for the insurer). We show that, in order for the optimal schedule of indemnities to be increasing (more severe accidents lead to greater compensation) then, contrary to what has been assumed in the literature, the marginal utility of wealth must be decreasing in health. Furthermore, any increasing optimal indenmity schedule cannot provide full compensation, in the sense that utility in each state is a constant. Finally, we show that, if the accident compensation scheme is financed by taxes on wealth, then changes in the total expected outlay (the budget) that the system can accommodate do not affect expected utility, and also the organizer of the system can use a budget size such that the optimal indemnity in the no-accident state is zero.

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1 Introduction

Typically, the economic theory of insurance decisions is set out in models in which utility only depends upon one attribute, namely wealth. In such models the insurable risk is thought of as being a given part of endowed wealth. When the asset at risk is not wealth, this model can only be used if one is willing to substitute the asset at risk for some sort of wealth equivalent. While this may make sence in some cases, for example a replaceable asset like a car or a house, it does not in others. In particular when the asset at risk is the individual's health status, which is not replaceable under many types of accident, no amount of wealth can restore the individual to his initial endownment point. In this sense, insurance of irreplaceable goods, in particular, health status, is an example of a situaction of incomplete markets, since health status cannot be effectively bought and sold.

The introduction of irreplaceable goods into a model of insurance has many interesting effects, some of which are unexpected. For example, Cook and Graham (1977) show that partial coverage may be optimal even though the premium charged is actuarially fair, something that does not occur when the only good considered is wealth. In this paper, we consider exactly how insurance coverage for health risks should be constructed. In particular, we attempt to characterize optimal monetary compensation functions as indemnities for health losses.

In particular to health insurance, many authors have argued that preferences should be modelled as being state dependent, that is, that the very utility function depends on the state of health (see, for example, Zeckhauser (1970, 1973), Arrow (1974) and Viscusi and Evans (1990)). The idea is basically that utility should be written as $u_h(w)$, where h represents the individual's health status. Hence, a different health status implies a different utility function. In reality, this assumption is only of use in that it means that the only argument of each possible utility function is monetary wealth, and so traditional insurance theory can be applied. In particular, if there exists a probability[1] , p_i, for each possible health status, h_i, then the individual's objective function is $U(w) = \sum_{i=0}^{n} u_{h_i}(w)p_i$, where there are n possible values for health.

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[1] We assume a discrete probability density. Taking a continuous one will not alter the problem significantly, but does imply a considerable increase in complexity later on when we study the problem of an optimal compensation schedule. Also, in all real world compensation schemes, accidents are defined discretely.

However, the state dependent utility function argument only really masks the problem somewhat, and seems to be used only to underline an assumption of a non-continuous set of possible health status. Naturally, writing $u(w, h)$ for the utility of a particular wealth level and a particular level of health, where of course $u(\cdot)$ is state independent, gives the same final objective function, $U(w) = \sum_{i=1}^{n} u(w, h_i)p_i$. This will be the method used in the present paper.

A very important point regarding the shape of the utility function is the sign of the second order cross derivative. Viscusi and Evans (1990) present evidence that suggests that the marginal utility of income is reduced by a loss in health. In the context of the utility function $u(w, h)$, such a conjecture can be stated quite simply as the second order cross derivative being non-negative:

$$\frac{\partial^2 u(w,h)}{\partial w \partial h} \geq 0$$

We shall refer to this as the Viscusi and Evans proposition.

In particular, if the second order cross derivative is non-negative, then we can be certain that the utility function is quasi-concave in its arguments, that is, its indifference curves are convex. However, for the case at hand, of h being an irreplaceable commodity, there is no need for quasi-concavity in order to fully contemplate compensation insurance. It turns out that eactly what sign for this cross derivative is assumed is critical to the problem of optimal accident compensation schemes. As we shall show, if the second order cross derivative is non-negative, then it will be the case that the optimal accident compensation scheme will imply an indemnity that is decreasing in accident sevrity, that is, more severe accidents lead to lower compensation. Needless to say, this is not a feature of any known accident compensation scheme, and so we are left with only two options to consider with regards real life accident compensation schemes. Either the Viscusi and Evans proposition is correct and accident compensation schemes are not optimal, or the Viscusi and Evans proposition is incorrect and accident compensation schemes may be optimal. In particular, we note that it is a necessary condition that the Viscusi and Evans proposition be incorrect for any accident compensation scheme that has indemnities that increase with accident severity to be optimal.

In any case, that marginal utility of wealth is increasing in health is in all reality a rather debateable assumption. It seems to be based on the idea that, with a loss in health comes a loss in the ability to enjoy spending in the same way as previous to the loss in health. What is surely true, however, is that after a loss in health (assuming that it is a significant loss), spending habits will change as some previously consumed goods will no longer be useful, and others become very useful. For example, consider the case of a normal, healthy man who enjoys bicycle riding, and then suffers an accident, the result of which is that he is confined to a wheelchair. He will now have no use what-so-ever for such things as bicycle helmets and isotonic beverages (whereas previously such things gave high utility), but will now have a high demand for medical aid, medicines and other special apparatus that previously had absolutely no utility for him. The question of exactly how his marginal utility for wealth is affected, however, is by no means clear. Indeed, surely a very good argument can be made for it having increased, as his most basic wants may now imply a greater wealth requirement.

2 The model and defining assumptions

We assume an individual with utility $u(w,h)$, where w is his monetary wealth and h is his physical wealth (i.e. his health). Both w and h are assumed to be continuous varibles, even though in what follows, h will only take descrete values. We assume that u is strictly increasing and concave in both arguments:

$$u_h(w,h) > 0,\ u_h(w,h) > 0,\ u_{ww}(w,h) < 0,\ u_{hh}(w,h) < 0 \tag{1}$$

It is tempting to add the assuption that $u(w,0) = 0$, that is, utility is only positive when there is some physical wealth, on the grounds that $h = 0$ can be associated with complete loss of life. However, we shall **not** make this assumption, since it belies a natural aspect of accident compensation, namely that when the accident leads to loss of life, a monetary compensation can be paid to the victim's family, and that this assurance will give satisfaction to the victim before any accident occurs, which, of course, is when the insurance must be taken out. That is, a man who, before suffering an accident, is confronted with the decision about how much compensation would need to be paid to his family should he be killed in an accident, will typically state a positive, strictly finite, amount. Hence it cannot hold that utility conditional on $h = 0$ is independent of wealth.

We assume that the individual has endowed monetary wealth of w_0 and endowed physical wealth of h_0. The individual lives in a society in which there exists a risk to his physical wealth in the form of an accident. Specifically, he suffers and accident of size x_i with probability p_i, and his post-accident physical wealth becomes $h_i = h(x_i) = h_0 - x_i$. Hence we assume that x is defined on the support $[0,\ h_0]$. We assume that the vector of accidents, $x = (x_0, x_1, ..., x_n)$ is ordered such that $0 = x_0 < x_1 < ... < x_n \leq h_0$, and so we have $h_i > h_{i+1}$ for all i. Finally, we assume that all parameters and variables (w_0, h_0, x) are public information. Given these assumptions, the individual has an endowed expected utility of:

$$Eu(w_0, h(x)) = \sum_{i=0}^{n} u(w_0, h_0 - x_i)p_i$$

Now suppose that the legal authorities in the society establish a monetary compensation for accident victims that depends on the severity of the accident suffered. Specifically, the accident compensation schedule stipulates that if an accident of size x_i is suffered, then the individual will receive a monetary compensation of $c_i = c(x_i)$, so that his post-accident wealth becomes $w_i = w(x_i) = w_0 + c(x_i)$. We assume that the compensation indemnity is net of any initial premium paid to take part in the compensation mutual. Given the accident compensation schedule, the individual has expected utility of:

$$Eu(w(x), h(x)) = \sum_{i=0}^{n} u(w_0 + c(x_i), h_0 - x_i)p_i$$

3 Full compensation

The exact objective that the accident compensation scheme should achieve is, obviously, crucial to its design. In Spain, the law stipulates that the accident compensation for automobile accidents must be sufficient to "repair, via monetary payment, the loss suffered in physical, emotional and monetary wealth". This objective explicitly covers contingencies under which the accident causes both physical and monetary losses (for example, the costs of hospitalization, losses in earnings, etc.). Here we shall assume that accidents only cause physical damages (although physical damages can include any emotional damage). Under such a criterion, it must be true that, for all possible accident sizes, the individual is fully compensated, in the sence that after the accident and the compensation, he is indifferent to having suffered the accident or not. We shall refer to such an accident compensation scheme as having *full compensation*, and we denote a compensation schedule that provides for such compensation by $c_f(x)$, and the corresponding wealth is denoted by $w_f(x) = w_0 + c_f(x)$. In terms of expected utility, an accident compensation scheme that provides for full compensation must satisfy:

$$u(w_0 + c_f(x_i), h_0 - x_i) \equiv v(x_i) = v(x_0) \text{ for all } x_i \tag{2}$$

Two points are noteable. Firatly, full compensation must be acceptable by the insured, in the sense that there is no question of participation. This is because, in absense of any compensation, it holds that $v(x_i) \leq v(x_0)$ with $<$ for all $i > 0$. Hence full compensation will always satisfy a participation constraint. Secondly, note that an accident compensation scheme that satisfies full compensation as defined by (2) is not fully defined, since it has $n + 1$ unknowns in the vector of compensations (here-in-after, the indemnity schedule), $c_f(x) = (c_f(x_0), c_f(x_1), ..., c_f(x_n))$, but only n equations. A further equation is required to fully define the scheme.

Two options for the additional equation are obvious. Firstly, we can set $c_f(x_0) = 0$, that is, we can define the system such that in the no-accident state, no compensation is given. In this case, we must be identifying the initial wealth, w_0, as being the individual's wealth once all taxes have been paid, in particular, once the premium to participate in the accident compensation scheme has been satisfied[2] . Under this type of system, the total cost of the scheme is an endogenous variable, and is necessarily positive if the indemnity schedule is increasing in accident size. Note that, under this type of scheme, in principle, the indemnity schedule is independent of the probability vector for accidents, p, and, for given parameters $u(\cdot)$, w_0 and h_0, the indemnity schedule is unique.

On the other hand, one could let the accident compensation for the no-accident state be defined by the equations (2) and add the equation $\sum_{i=0}^{n} c_f(x_i)p_i = B$, where B is some pre-defined budget. Naturally, $B = 0$ indicates a self-financing scheme. Under this type of scheme,

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[2] In most countries (in particular, in Spain) participation in the social security accident compensation scheme is compulsory, and the premium is a fixed proportion of the individual's income.

since different values of B will give different indemnity schedules, we get $c_f(x_i, B)$, and so the indemnity schedule now depends on the probability vector for accidents. Once again, it turns out that for given parameters $u(\cdot)$, w_0 and h_0, the corresponding indemnity schedule is unique. Also, it is clear that for non-positive values of B, if the indemnity schedule is increasing in accident size, then it must be true that $c_f(x_i) < 0$ for small values of i. In this sense, it is logical to assume that the indemnity schedule determines the indemnity gross of premium, that is, it is a non-negative indemnity that depends on accident size less a fixed premium. In this case, we should consider the initial wealth, w_0, to be wealth net of taxes but without having paid any premium to participate in the compensation scheme, and then consider that the premium to participate is simply $c_f(x_0)$.

Full compensation will certainly create moral hazard problems. To see why, note that if different individuals have different underlying parameters (preferences and endowed wealth levels), then a compensation scheme that provides full compensation to one individual will not, in general, do the same for another. If some individuals percieve more than full coverage, they may have an incentive to create accidents. Hence, just as one of the mechanisms under which moral hazard can be controlled in other insurance markets is partial coverage, so in the accident compensation scheme it may be necessary to offer partial coverage. However, although it is certainly interesting, this problem will not be tackled in the current paper.

As an example, take the case of $u(w, h) = Ln(w+1) + Ln(h+1)$, where we have summed 1 to each argument in order to avoid problems of negatively infinite utility. We take the option here of adding the equation $c_f(x_0) = 0$. In this case, full compensation requires:

$$Ln(w_0 + c_f(x_i) + 1) + Ln(h_0 - x_i + 1) = Ln(w_0 + 1) + Ln(h_0 + 1) \quad \text{for all } i$$

This can be reordered to give:

$$Ln(w_0 + c_f(x_i) + 1) = Ln\left(\frac{(w_0+1)(h_0+1)}{h_0 - x_i + 1}\right) \quad \text{for all } i$$

that is:

$$\begin{aligned} c_f(x_i) &= \frac{(w_0+1)(h_0+1)}{h_0 - x_i + 1} - (w_0 + 1) \\ &= (w_0 + 1)\left(\frac{x_i}{h_0 - x_i + 1}\right) \quad \text{for all } i \end{aligned}$$

Clearly, with this example, we have:

$$c_f'(x) = (w_0 + 1)\left(\frac{h_0 + 1}{(h_0 - x + 1)^2}\right) > 0$$

so that greater accidents lead to greater compensation.

Going back to the general formulation, our initial assumptions give us directly the following proposition:

Proposition 1 *Full accident compensation schemes are increasing in accident size in the sense that* $c_f(x_i) < c_f(x_{i+1})$ *for all* $i = 0, 1, 2, ..., n - 1$.

Proof. Since we have assumed that utility is strictly increasing in both arguments (monetary wealth and physical wealth), and given that by assumption $h_i > h_{i+1}$ for all i, an increase in the accident size (from x_{i-1} to x_i) will imply a greater loss in physical wealth, that can only be compensated by a greater increase in monetary wealth. ■

4 The optimal compensation schedule

We now go on to study the optimal compensation scheme. In order to do so, we now use the following notation: the vector of compensations (net of the initial premium for participation) is $c = (c_1, c_2, ..., c_n)$, state contingent wealth is $w_i = w_0 + c_i$ and state contingent health is $h_i = h_0 - x_i$. The problem is to maximize the expected utility of the insured, subject to participation and a budget constraint. In this problem, we no longer have the option of maximizing with respect to the vector c conditinal on $c_0 = 0$, since such a program would yield $c_i = \infty$ for $i = 1, 2, ..., n$. Hence, here we have no option but to use the budget constraint. For the time being, we take w_0 to be initial wealth, independent of the compensation scheme, although later on we shall alter this slightly. The problem can be set out as follows:

$$\max_c \sum_{i=0}^{n} u(w_i, h_i)p_i \text{ subject to } \sum_{i=0}^{n} u(w_i, h_i)p_i \geq \sum_{i=0}^{n} u(w_0, h_i)p_i \text{ and } \sum_{i=0}^{n} c_i p_i \leq B$$

where B is the budget that the legal authorities have to finance the compensation scheme. We shall refer to the first restriction as the participation contraint, and to the second as the budget constraint. In words, the problem is to find the compensation function that maximizes the ex-ante expected utility of the potential accident victim, subject to this expected utility being at least as high as the default system ($c_i = 0$ for all i), and subject to a budget constraint by the organizer of the compensation scheme. Note that, since we have assumed that utility is strictly concave in wealth, and since the restriction is linear, then by the Weierstrauss theorem, there exists a unique optimum to the problem. Note that we are assuming that there is only one individual in the economy, or that all individuals can be represented by a single social utility function.

An interesting case emerges when one sets $B = 0$, since such a system is a self-financing compensation scheme (expected indemintes are equal to the intial premium) or, in more familiar terms, the participation premium is actuarially fair. Of particular interest is whether or not the solution to this problem will ever turn out to be the full compensation scheme, since this is what the law typically attempts to provide for.

The Lagrangian for the problem is:

$$L(c,\lambda) = \sum_{i=0}^{n} u(w_i,h_i)p_i + \lambda_1 \left[\sum_{i=0}^{n} u(w_i,h_i)p_i - \sum_{i=0}^{n} u(w_0,h_i)p_i \right] + \lambda_2 \left[B - \sum_{i=0}^{n} c_i p_i \right]$$

If we denote the optimal compensation in state i by c_i^*, and the corresponding state contingent wealth by w_i^*, then the first order conditions for an optimal solution are:

$$\frac{\partial L(c^*,\lambda)}{\partial c_i} = 0 \;\; i = 1,2,...,n \quad \Longrightarrow \quad \frac{\partial u(w_i^*,h_i)}{\partial w} p_i(1+\lambda_1) - \lambda_2 p_i = 0 \;\; i = 1,2,...,n$$

But since $\lambda_i \geq 0$, we have:

$$\frac{\partial u(w_i^*,h_i)}{\partial w} = \frac{\lambda_2}{(1+\lambda_1)} \quad i = 1,2,...,n \tag{3}$$

On the other hand, the complementary slackness conditions are:

$$\lambda_1 \left[\sum_{i=0}^{n} u(w_i^*,h_i)p_i - \sum_{i=0}^{n} u(w_0,h_i)p_i \right] = 0 \tag{4}$$

and

$$\lambda_2 \left[B - \sum_{i=0}^{n} c_i^* p_i \right] = 0 \tag{5}$$

Clearly, since we have assumed that $\frac{\partial u(w_i,h_i)}{\partial w} > 0$ for all i, we have $\lambda_2 > 0$, and so from (5) the organizer of the scheme will always spend all the budget:

$$\sum_{i=0}^{n} c_i^* p_i = B$$

From this point, we can now note that the first restriction (the participation constraint) is really irrelevant to the problem. To see why, note that since in any optimum the budget constraint is saturated, we can write the optimal compensation vales as functions of B, that is, $c_i^*(B)$. However, the greater is B, the greater can be the compensation values (clearly, it is feasible to increase the compensation in each and every accident state) and so the new optimum compensation vector must increase expected utility[3] , that is $\frac{\partial Eu(c_i^*(B))}{\partial B} > 0$. Hence, there exists a miminum value for B, say B_0, for which the participation constraint saturates. In this case, any problem with a budget $B < B_0$ has no optimal solution that guarantees participation, while any problem with a budget $B > B_0$ will leave the participation constraint slack.

Proposition 2 $B_0 \leq 0$.

Proof. Clearly, $\sum_{i=0}^{n} c_i p_i = 0$ if $c_i = 0$ for all i. Furthermore, if $c_i = 0$ for all i, then the participation constraint saturates by definition. Hence, the optimal solution of the problem with $B = 0$ must fulfil the participation constraint, although it may not saturate it. ■

As we have already pointed out, $B = 0$ corresponds to a self financing compensation scheme, which would always seem to be feasible (abstracting from operational costs). Given this, we simplify the problem by eliminating the participation constraint and assuming that $B \geq 0$.

For the reformulated problem, the first order conditions become:

$$\frac{\partial L(c^*, \lambda)}{\partial c_i} = 0 \quad i = 1, 2, ..., n \quad \Longrightarrow \quad \frac{\partial u(w_i^*, h_i)}{\partial w} = \lambda \quad i = 1, 2, ..., n$$

and the complementary slackness condition is:

$$\lambda \left[B - \sum_{i=0}^{n} c_i^* p_i \right] = 0$$

Once again, since $\frac{\partial u(w_i^*, h_i)}{\partial w} > 0$, we get $\lambda > 0$, and so the constraint must saturate:

$$\sum_{i=0}^{n} c_i^* p_i = B$$

Note that the set of first order conditions can be expressed as:

$$\frac{\partial u(w_i^*, h_i)}{\partial w} = \frac{\partial u(w_{i+1}^*, h_{i+1})}{\partial w} \quad i = 0, 1, ..., n-1 \tag{6}$$

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[3] From elementary microeconomics, a marginal increase in B will increase expected utility by an amount equal to the relevant multiplyer, λ_2, which is positive.

We now state an important consequence of this system:

Proposition 3 *The optimal accident compensation schedule is increasing ($c_i^* < c_{i+1}^*$ for all i) if $\frac{\partial^2 u(w,h)}{\partial w \partial h} < 0$, decreasing ($c_i^* > c_{i+1}^*$ for all i) if $\frac{\partial^2 u(w,h)}{\partial w \partial h} > 0$, and constant ($c_i^* = c_{i+1}^*$ for all i) if $\frac{\partial^2 u(w,h)}{\partial w \partial h} = 0$.*

Proof. Assume $\frac{\partial^2 u(w,h)}{\partial w \partial h} < 0$. Since by definition $h_i > h_{i+1}$ for all i, we have $\frac{\partial u(w,h_i)}{\partial w} < \frac{\partial u(w,h_{i+1})}{\partial w}$ for any given wealth level w. However, by assumption $\frac{\partial u^2(w,h)}{\partial w^2} < 0$, that is the marginal utility of wealth for any given level of health h is decreasing in wealth, and so in order for $\frac{\partial u(w_i^*,h_i)}{\partial w} = \frac{\partial u(w_{i+1}^*,h_{i+1})}{\partial w}$ as (6) requires, we must have $w_i^* < w_{i+1}^*$, that is, $c_i^* < c_{i+1}^*$. The other two cases are proven in the same way. ■

This is an important result. Although at the outset, we argued that the cross derivative of utility with respect to wealth and health is hard to sign, other authors have hypothesised that it is non-negative. This assumption does imply quasi-concavity of the utility function, but as we have just pointed out, also implies that the optimal compensation function is not increasing. That is, an optimal compensation scheme will only offer greater compensation for more severe accidents when the cross derivative is negative.

Corollary 4 *A full compensation schedule is never optimal if $\frac{\partial^2 u(w,h)}{\partial w \partial h} \geq 0$.*

Proof. The proof is immediate from propositions 1 and 3. ■

A particular intuition of this result can be shown by considering a concrete utility function for which full compensation is optimal. Begin by noting the similarity between the set of equations defining full compensation (2), and the first order conditions that define the optimal compensation schedule, (6). If we write the wealth of the individual in each accident state under full compensation as w_i^f, then once the final equation (either a budget constraint or that compensation in the no-accident state is zero) has been decided, the set of equations defining full compensation are:

$$u(w_i^f, h_i) = u(w_0^f, h_0) \quad i = 1, ..., n$$

On the other hand, the set of equations defining optimal compensation is:

$$\frac{\partial u(w_i^*, h_i)}{\partial w} = \frac{\partial u(w_0^*, h_0)}{\partial w} \quad i = 1, ..., n$$

Clearly, if the full compensation scheme is set subject to a budget constraint, then the two systems will give the same final solution when, for each level of health, the derivative of utility is equal to utility, that is, when:

$$\frac{\partial u(w,h)}{\partial w} = u(w,h) \text{ for all } h$$

i.e. when $u(w,h) = v(h)e^{w}$ where $v(h)$ is a function of only health. Clearly, for this case utility is convex in wealth, and so is excluded from our assumption set.

4.1 State organized compensation systems

Given the above maximization problem, the optimal indemnity schedule clearly depends on initial wealth and the budget, and so can be written as:

$$c_i^*(w_0, B) \ \ i = 0, 1, ..., n$$

Now, if we are considering a state organized social security scheme, then we would also like to have $c_0^* = 0$, and the system would be financed by taxes on incomes. In Spain, financing of social security is done using a constant proportional tax on income. In the terms of this model, the insured must pay some fixed proportion, say α, of his initial wealth to the system, and the system is then financed by the proceeds of this tax, so that here $B = \alpha\widehat{w}$, where $\widehat{w}$ is the individual's monetary wealth after all taxes other than that corresponding to accident compensation have been paid. Hence, in this model, we have $w_0 = (1-\alpha)\widehat{w} = \widehat{w} - B$.

Note that such a system will be self-financing because the equations defining the optimum are not different from those of the previous section, due to the fact that even under the new definition of initial wealth, initial wealth is not dependent upon the choice variables, c. Hence, what we are really concerned with here is a re-definition of what we understand by a self-financing system, in such a way that the final indemnity schedule corresponds to a null indemnity in the no-accident state.

The optimal indemnity schedule can now be written as $c_i^*(\widehat{w} - B, B) = c_i^*(B) \ \ i = 0, 1, ..., n$. Now, from all the options defined by the different possible values of B, the organizers of the scheme can simply choose that one that sets $c_0^*(B) = 0$, in order to satisfy both optimality with respect to the budget and the socially expected aspect that no compensation is given in the no-accident state. We now show that such a situation indeed exists.

The principal problem to study in this section is the existence of a B such that the optimal indemnity schedule is characterized by $c_0^*(B) = 0$. Note that the existence of such a situation cannot be simply assumed, since an increase in B will have two effects, firstly it will enable all indemnity payments to be increased, but secondly it will require a greater financing, and so will imply a lower value of w_0. The problem will be studied under the assumptions that, for any given B, the indemnity schedule is optimal in the sense of the previous section, and B is non-negative.

We begin by noting how an increase in B affects expected utility in the optimum. In order to do this, we define the indirect utility to be $\sum_{i=0}^n p_i u(w_i^*(B), h_i) \equiv V(B)$, and we consider the derivative:

$$\frac{\partial V(B)}{\partial B} = \sum_{i=0}^{n} p_i \frac{\partial u(w_i^*(B), h_i)}{\partial w} \frac{dw_i^*}{dB}$$

Proposition 5 *An increase in B in a self financed scheme does not affect expected utility in the optimum.*

Proof. Since $w_i^*(B) = \widehat{w} - B + c_i^*(B)$, we get $\frac{dw_i^*}{dB} = -1 + \frac{dc_i^*}{dB}$. Using this, and the first order conditions from the optimization programe, we can write:

$$\frac{\partial V(B)}{\partial B} = \lambda \sum_{i=0}^{n} p_i \left(-1 + \frac{dc_i^*}{dB} \right) = \lambda \left(\sum_{i=0}^{n} p_i \frac{dc_i^*}{dB} - 1 \right)$$

But, from the fact that the budget constraint is always saturated in an optimum, we get:

$$\sum_{i=0}^{n} p_i \frac{dc_i^*}{dB} = 1$$

and so $\frac{\partial V(B)}{\partial B} = 0$, that is, an increase in the size of the budget will not affect optimal expected utility. ■

Since changes in the budget do not affect expected utility, we are justified in searching for a budget size that can accomodate the socially acceptable norm that compensation in the no-accident state should be 0.

Firstly, if $B = 0$, then it is quite obvious that $c_i = 0$ for all i must be a feasible solution to the original maximization problem. Furthermore, from proposition 3, if we assume $\frac{\partial u(w,h)}{\partial w \partial h} < 0$, then the optimal indemnity schedule is increasing, in the sense that more severe accidents lead to greater compensation, then it must be so that with $B = 0$, the optimal compensation in the no-accident state is strictly negative, $c_0^*(0) < 0$. Now consider what happens to $c_0^*(B)$ as B increases.

Proposition 6 $\frac{dc_i^*(B)}{dB} = 1$ *for all* i*.*

Proof. The first order conditions for an optimum indemnity are:

$$\frac{\partial u(w_i^*, h_i)}{\partial w} = \frac{\partial u(w_j^*, h_j)}{\partial w} \text{ for all } i, j = 0, 1, 2, ., n \text{ and for all } B$$

Now, derive this expression with respect to B to get:

$$\frac{\partial^2 u(w_i^*, h_i)}{\partial w^2}\left(\frac{dc_i^*}{dB} - 1\right) = \frac{\partial^2 u(w_j^*, h_j)}{\partial w^2}\left(\frac{dc_j^*}{dB} - 1\right) \text{ for all } i, j = 0, 1, 2, ., n \text{ and for all } B$$

Since $\frac{\partial^2 u(w,h)}{\partial w^2} < 0$ it holds that $sign\left(\frac{dc_i^*}{dB} - 1\right) = sign\left(\frac{dc_j^*}{dB} - 1\right)$ for all $i, j = 0, 1, 2, ., n$ and for all B. Hence, for all i we must have either $\frac{dc_i^*}{dB} > 1$, or $\frac{dc_i^*}{dB} < 1$, or $\frac{dc_i^*}{dB} = 1$. However, since $\sum_{i=0}^{n} p_i \frac{dc_i^*}{dB} = 1$, the first two options cannot hold. ■

Proposition 6 is interesting in its own right. It states that, not only will the expected value of the indemnity that the individual ends up recieving be equal to the premium payment B, but also any increase in the premium must end up back in the insured's pocket, since exactly one state of nature must occur.

Finally, we now have:

Proposition 7 $c_0^*(B) = 0$ *for* $B = -c_i^*(0)$*.*

Proof. From proposition 6, we know that $\frac{dc_0^*(B)}{dB} = 1$, that is, $c_0^*(B)$ is linear in B with slope equal to 1, i.e. $c_0^*(B) = c_0^*(0) + B$. ■

Proposition 7, does not depend on whether or not the optimal indemnity schedule is increasing, that is, it does not depend on the validity of the Viscusi-Evans proposition. However, if the optimal indemnity schedule is increasing, that is, if $\frac{\partial^2 u(w,h)}{\partial w \partial h} < 0$, then in a system in which accident compensation is financed by income taxes, the budget that will correspond to a case of the optimal indemnity in the no-accident state being 0 will be strictly positive, since we know that in this case $c_i^*(0) < 0$.

Finally, we note that, while the following formulation cannot be found as the result of a constrained optimization program, the optimal accident compensation scheme that is financed by taxes as above, is simply the solution to the set of n equations in n unknowns:

$$\begin{aligned} \frac{\partial u(w_i^*, h_i)}{\partial w} &= \frac{\partial u(w_{i+1}^*, h_{i+1})}{\partial w} \quad i = 0, 1, 2, ..., n-1 \\ c_i^* &= 0 \end{aligned}$$

The budget that is required to finance this system is given by $B = \sum_{i=0}^{n} c_i^* p_i$, so that the required tax rate is simply $\alpha = \frac{B}{\overline{w}}$.

5 Conclusions

In this paper we have considered how accident compensation schemes are organized. In particular, we showed that the Viscusi-Evans proposition, that marginal utility of wealth is not decreasing in health, leads to the unlikely situation of an optimal indemnity schedule that is not increasing in accident size, that is, more severe accidents will correspond to lower indemnities. Such a situation could correspond to a system in which the objective was to provide full compensation, but then we can state that, since full compensation schedules are always increasing, they can never be optimal under the Viscusi-Evans proposition.

As far as optimal schemes are concerned, we have shown that, if the system is financed by taxes on the insured's wealth, then changes in the budget do not affect the expected utility of the insured in the optimum. Furthermore, conditional on the optimal schedule being increasing (that is, on the Viscusi-Evans proposition being false), then the organizer can always find a positive budget size such that the optimal indemnity schedule corresponds to no indemnity in the no-accident state.

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