Asymmetric Information, Transaction Cost, and Externalities in Competitive Insurance Markets*

Jerry W. Liu

Department of Finance, University of Notre Dame, Notre Dame, IN 46556-5646

wliu2@nd.edu

Mark J. Browne

School of Business, University of Wisconsin, Madison, WI 53706

mbrowne@bus.wisc.edu

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Abstract

The Rothschild and Stiglitz adverse selection model is considered a classic among papers on the theory of markets with asymmetric information. However, this model relies on the unrealistic assumption that there are no profits or transaction costs. This paper modifies the model by incorporating various forms of transaction costs, which are either a fixed per policy fee or variable costs proportional to the premium and expected indemnity amount. Proportional costs bring distinct consequences to insurance buyers' behavior and partial coverage becomes the optimal choice for consumers. Departure from full coverage significantly changes the dynamics of Rothschild and Stiglitz's arguments. We can see a new separating equilibrium if customer groups have heterogeneous risk aversion or heterogeneous endowment wealth. In these two scenarios, neither high-risk individuals nor low-risk individuals cause any externality to the other group in the market. It is demonstrated that, under more realistic hypotheses, the standard result in the analysis of the insurance market under adverse selection can be reversed.

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I. Introduction

In their seminal paper, Rothschild and Stiglitz (1976) provide a framework for analyzing the problem of adverse selection in an insurance market. The model has become a standard in related literature. Joseph Stiglitz, together with George Akerlof and Michael Spence, was awarded the Nobel Prize in Economics in 2001 for their analyses of markets with asymmetric information. Assuming there is zero cost (or expected profit) and agents buy full coverage, Rothschild and Stiglitz make the conclusion that information asymmetry may lead to market failure: *"The high-risk (low ability, etc.) individuals exerted a dissipative externality on the low-risk (high ability) individuals…"*

Insurance policies, however, seldom provide complete protection against potential losses. On the other hand, empirical evidence to date has not always been consistent with the major implications of the Rothschild and Stiglitz model (1976). In a recent study, Chiappori and Salanié (2000) test whether contracts with more comprehensive coverage are chosen by agents with higher accident probabilities. They find no statistical evidence to support this prediction.

Several factors constitute explanations for partial insurance coverage. First of all, insurance companies bear significant proportion costs that could be as high as 30% of their income. Many papers, such as Arrow (1971) and Mossin (1968), propose that partial coverage is optimal when the premium contains a fixed-percentage loading. Secondly, as the required return for their capital investments, insurers do take profit out of their operations. Moreover, the risk-neutral hypothesis on the insurer part may not always hold. When risks are correlated with market risk, firms will demand a risk premium, since they are not diversifiable, for shareholders to take the risk. Finally, moral hazard, which refers to the impact of insurance on incentives to reduce risk, also leads to less than full insurance. To summarize, partial insurance coverage is the reality of the world and adverse selection should be looked at in such context.

In addition to the zero-profit hypothesis, other assumptions in the Rothschild and Stiglitz (1976) model are simplified. People may differ not only by their risk, but also by their preferences, risk aversion, wealth, loss severity, etc. de Meza and Webb (2001) assume that risk-averse people tend to buy more coverage while the reckless people often put less value on insurance protection. In their setting, pooling equilibrium becomes possible. Wambach (2000) adds exogenous, unobservable wealth differences in addition to the unobservable accident probabilities. When differences in assets are large, partial-pooling equilibria could exist and might involve positive profits. The case of severity risk is studied in Doherty and Schlesinger (1995). They conclude that adding this severity term will decrease the likelihood of the market achieving equilibrium as suggested in the Rothschild and Stiglitz model.

This study integrates transaction costs into Rothschild and Stiglitz's framework by systemically introducing linear cost functions. These costs include both constant costs and proportional costs as a percentage of premium or indemnity amount. Insurance profit can also be incorporated into this framework if we describe profit as a transaction cost. Our model shows a clear picture of the transaction cost effects on insurance buyers' behavior. To insurance consumers, the aftermath of paying transaction costs can be considered to be (a) Fixed cost that reduces the endowment wealth of the insurance buyers, (b) Proportional costs that change the slope of the fair-market line, (c) Customers still buy complete insurance for a reduced fee when they must pay a constant cost, (d) People tend to choose partial coverage when they must pay a proportional fee.

Rothschild and Stiglitz (1976) conclude that high-risk individuals always cause an externality on low-risk individuals in a market with asymmetric information. In their setting, both types of individuals buy complete insurance and high-risks always prefer the low-risk contract, which is located on the certainty line and provides more wealth in both states of nature. Incorporating transaction costs forces both types to move away from the the certainty line and brings significant change to the dynamics of Rothschild and Stiglitz's arguments.

With the presence of proportional costs, we introduce heterogeneous risk aversions into the model. When high-risks have a much higher level of risk aversion than low-risks, the two customer groups' optimal points become farther apart, which could lead to a new separating equilibrium. In what we call *NE equilibrium*, no group causes any negative externality to the other group in the market. Both types of customers will automatically purchase the first-best policy designed for them and the information problem disappears. In the scenario that high-risks are less risks averse than low-risks, the negative externality is aggravated as it is the high-risks who move closer to the endowment point. However, when difference in risk aversion is substantial and loss probabilities are close, the information problem disappears again.

Not only people can have different levels of risk aversion, they can have distinct amounts of endowment wealth. With the presence of proportional costs, we introduce heterogeneous wealth into the standard adverse selection model. There are two possible combinations – high-risks are wealthier than low-risks or high-risks are simply less affluent. In the first case, *NE equilibrium* exists when the difference in wealth is large enough while in the second case the information problem becomes worse under most circumstances. However, even in the second case, we can still find *NE equilibrium,* when the difference in wealth is large and the difference in loss probabilities is small.

Our study provides new insights into adverse selection models. We give concrete examples in which the Rothschild and Stiglitz externality result can be reversed. Our paper shows that transaction costs can be one explanation for empirical findings. With the presence of transaction cost, high-risk types do not have to be the ones that choose more comprehensive coverage.

The remainder of this paper is organized as follows: Section II gives an overview of Rothschild and Stiglitz model assumptions. Section III analyzes the effect of having both constant cost and proportional costs. The case of heterogeneous risk aversion and the new separating equilibrium are presented in Section IV. Section V studies wealth effect. Finally, conclusions are presented in Section VI.

II. The Model

The following model is based on that of Rothschild and Stiglitz (1976) and only the skeleton necessary to understand the question is presented here. The reader could consult the original work for a more detailed account.

- The market consists of two types of customers: low-risks and high-risks with different accident probabilities $p^H > p^L$.
- Customers know their accident probabilities, but the insurance companies do not. Insurance companies cannot discriminate among their potential customers on the basis of their characteristics.
- Both low-risks and high-risks have same endowment wealth W. This assumption is relaxed in section V in which we allow one risk group to have more endowment wealth than the other group in the market.
- If an accident occurs, the loss will be d. To avoid problems of bankruptcy, the value of d is assumed to be lower than endowment wealth W.
- An insurance company writes a contract $a = (a_1, a_2)$, where a_1 is the premium amount and a_2 is the net indemnity amount (premium deducted). If an accident occurs, the individual will receive a total indemnity of $a_1 + a_2$. Neither the insurance premium nor indemnity could become be a negative number and we call an insurance contract $(a_1,\ a_2)$ *admissible* if and only if $a_1 \geq 0$ and $a_2 \geq 0$.
- The insurance policy is an exclusive contract, which means that consumers cannot buy insurance from multiple insurers. The nature result of this

assumption is that an insurer can observe the total amount of coverage purchased by any customer.

- Insurers are risk neutral. One supporting argument is that company stocks are owned by those people who themselves are well diversified in stock holdings.
- Individual's wealth in two states of nature (accident or no accident) is indicated as a vector (W_1, W_2). If no accident happens, individual's wealth is the original endowment wealth minus the premium paid to insurance company. That is,

$$W_1 = W - a_1 \tag{2.1}$$

If an accident happens, people will suffer a loss of d and receive a reimbursement of amount $a_1 + a_2$ from the insurance company.

$$W_2 = W - d + a_2 \tag{2.2}$$

- The equilibrium concept to be applied is taken from Rothschild and Stiglitz (1976), which is defined as a set of contracts such that, when customers choose contracts to maximize expected utility, (i) no contract in the equilibrium set makes negative expected profits and (ii) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. This definition is equivalent to Nash equilibrium of a static game where insurers are the only players. Other equilibrium concepts, such as Wilson (1977) and Miyazaki (1977), have been developed in the literature. Our result can be extended under these equilibrium concepts. However, to focus on our main point, we restrict the equilibrium to be the original Rothschild and Stiglitz definition.
- Both types of customers have the same state-independent utility function U. The customers prefer more wealth than less ($U' > 0$), and they are risk-averse

($U'' < 0$). This assumption implies that all the customers in the market have the same risk preference and level of risk aversion, which is not necessary true in the real world. Later, in section IV, we modify this assumption and introduce heterogeneous risk aversion into the model.

- Absolute risk aversion is represented by

$$r(x) = -\frac{U''(x)}{U'(x)} \tag{2.3}$$

If the absolute risk aversion $r(x)$ is a decreasing function of wealth level x, then the utility is showing decreasing absolute risk aversion (DARA). Constant absolute risk aversion (CARA) corresponds to $r(x)$ being a constant. Similarly, we can define increasing absolute risk aversion (IARA). The realistic situation is that absolute risk aversion is decreasing with wealth. However, in section IV, we use CARA in order to eliminate the wealth effect.

- The expected utility of a type t individual ($t = H, L$) who purchases contract $\boldsymbol{a}$ is written as $V^t(\boldsymbol{a})$:

$$V^t = (1 - p^t)U(W - a_1) + p^t U(W - d + a_2) \tag{2.4}$$

By (2.4), the marginal rate of substitution for the two types is

$$\mathrm{MRS}^t = -\frac{dW_2}{dW_1} = \frac{(1 - p^t)U'(W_1)}{p^t U'(W_2)} \tag{2.5}$$

Obviously, $\frac{1 - p^L}{p^L} > \frac{1 - p^H}{p^H}$, the low-risk indifference curve is everywhere steeper than the high-risk indifference curve.

III. Transaction Costs

In the Rothschild and Stiglitz model, insurance firms make zero expected profits. Consequently, for each insurance policy sold, insurers charge a premium of the amount a_1, which is equal to the expected claim, $p(a_1 + a_2)$. The insurer's profit can be expressed as

$$p = (1-p)a_1 - pa_2 = 0 \qquad (3.1)$$

This hypothesis is clearly unrealistic. Most insurance policies have a price tag that is more than the cost of claims. According to the Insurance Information Institute: in 1998, net claims accounted for merely $59 of every $100 earned in private passenger auto insurance premiums in the United States. Commissions, however, accounted for $18, and on average, $4 were paid for other costs of settling claims. The insurer usually kept a profit of $11 and the remaining amount is attributable to taxes, policy issuance costs, and company operation expenses.

To consider the effort of expense loading, a term representing cost function should be added to the right side of the budget equation (3.1). In the rest of this section, we discuss possible forms of transaction costs and how they will change an individual's behavior.

A. Constant Cost

Insurers normally charge their customers for their daily expenses, which include salaries and expenses associated with secretarial help, computers, and other aspects of company operations. These costs should be averaged into each insurance contract. For each policy, this is a fixed amount.

$$\text{cost} = a \tag{3.2}$$

When there are no transaction costs, as indicated in Rothschild and Stiglitz (1976), people buy full insurance at actuarial odds. Do people pay for full insurance coverage under a fixed cost? With fixed transaction costs, the individual's optimization problem becomes (thereafter Problem I):

$$Max:\ E(U) = (1-p)U(W - a_1) + pU(W - d + a_2)$$

$$Subject\ to:\ a_1(1-p) - a_2 p = a$$

$$(1-p)U(W - a_1) + pU(W - d + a_2) \geq (1-p)U(W) + pU(W - d)$$

$$a_1 \geq 0,\ a_2 \geq 0$$

In Problem I, the first condition is the budget constraint and the second condition states that people need to see an improvement in their welfare before they pay for any amount of insurance. The final *admissible* condition guarantees that insurance premium will not become negative.

PROPOSITION 1. *Optimal solution for Problem I is* $W_1^* = W_2^* = W - pd - a$ *when* $a \leq \sqrt{d^2 p(1-p) + (pd + \frac{1}{r})^2} - (pd + \frac{1}{r})$; *Otherwise* $W_1^* = W$ *and* $W_2^* = W - d$.

Proof. See Appendix.

Proposition 1 states that, when the fixed charge is not too large, people still buy full insurance, which means that they will have the same amount of wealth in both states of nature. However, when the burden of fixed charge outweighs the benefit of having insurance protection, people choose not to pay for any insurance.

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Insert FIG. 1. about here

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As we can see in figure 1, the increase in the premium relocates the individual's starting point from E to F as if the individual's endowment wealth is reduced by the amount a. Without any cost, the optimal point is at a^* with the wealth level $W - pd$ in both states of nature, and it moves down to the point b^* ($W - pd - a$, $W - pd - a$) when a fixed charge is imposed. However, the slope of the "fair-odds line" (a_2/a_1) does not change. The reciprocal of this slope, $q(a) = a_1/a_2$, is defined in Rothschild and Stiglitz (1976) as *cost per unit coverage* or *price of insurance*. The new market odds line Fb^* is parallel to the original "fair-odds line" Ea^*. In figure 1, optimal point b^* is located at the intersection of the 45° line and the new market price line. Each customer will buy complete insurance and will have equal income (reduced by the amount a compared to the no cost case) in both states of nature. This result confirms many earlier works in insurance literature.

The effect of a constant cost can be viewed as a reduction in people's wealth. A rational customer will not buy a small amount of insurance since he would become worse off should he purchase insurance near point F. From another perspective, a fixed transaction cost lowers an individual's highest attainable expected utility level.

As we can see, the higher the fixed cost, the lower is the highest attainable expected utility level. The trend goes on until the fixed cost exceeds a_{Limit} in the following equation, in which case customers will buy no insurance.

$$a_{Limit} = \sqrt{d^2 p(1-p) + (pd + \frac{1}{r})^2} - (pd + \frac{1}{r})$$

Obviously, a_{Limit} decreases when $d^2 p(1-p)$ becomes smaller, which means that the incentive to purchase coverage, under a constant cost function, is the lowest when people have a small risk (in both loss amount and loss probability).

B. Proportional Cost

Expense loading in insurance rates also includes commissions and other marketing expenses, premium taxes and fees, loss adjustment and litigation expenses, and insurance profits. Sales commissions for agents and brokers depend on the total premium collected while premium taxes vary from two to four percent of the premium in different states. When an accident does happen and claims are filed, claim related expenses, such as loss adjustment and litigation expenses, could become substantial. Expenses associated with the claim settlement are closely correlated with the total claim amount, $a_1 + a_2$. Moreover, normal insurance profits are also part of the bill, and they are often expressed as a fraction of the premium or coverage amount. In this subsection we consider the case where

$$\text{Cost} = \text{b}a_1 + p\text{c}(a_1 + a_2) \tag{3.3}$$

When transaction costs are proportional to the insurance premium and claim amount, the slope of the "fair-odds line" (absolute value) becomes

$$\frac{a_2}{a_1} = \frac{1-b-p(1+c)}{p(1+c)}$$

Proportional costs turn the "fair-odds line" counter-clockwise downward. In figure 2, we can see that the budget line is turned downward around the endowment point from EA to EB. For consumers, this downward turn of the budget line means they are facing a hike in the price of insurance.

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Insert FIG. 2. about here

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Under proportional transaction costs, the individual seeks to maximize expected utility (Problem II):

$$Max: \quad E(U) = (1-p)U(W-a_1) + pU(W-d+a_2)$$

$$\textit{Subject to:} \quad a_1(1-p) - a_2 p = \mathrm{b}a_1 + p\mathrm{c}(a_1 + a_2)$$

$$a_1 \geq 0, \; a_2 \geq 0$$

PROPOSITION 2. *In any admissible optimal solution to Problem II,* $W_1^* > W_2^*$.

Proof.

The first order condition for Problem II is

$$\frac{U'(W-a_1)}{U'(W-d+a_2)} = \frac{1-b-p(1+c)}{(1+c)(1-p)} \tag{3.4}$$

Reorganize the budget constraint in Problem II,

$$[1-b-p(1+c)]a_1 = p(1+\mathrm{c})a_2$$

To make any non-zero optimal contract *admissible*, it is necessary to have $1-b > p(1+c)$. Under this condition, with b and c not equal to zero at the same time, it is trivial to show that $\frac{1-b-p(1+c)}{(1+c)(1-p)} < 1$. Then we have

$$U'(W-a_1) < U'(W-d+a_2)$$

Since $U'' < 0$, we can conclude that, for any admissible optimal contract

$$W-a_1^* > W-d+a_2^*$$

Proposition 2 shows that individuals will have more wealth in the non-loss state than in the loss state and they will not buy full insurance. With a proportional transaction cost, potential losses are not fully covered since market price for insurance is above the actuarial fair value. In figure 2, optimal coverage moves from a^*, which is on the certainty line, to the point b^*.

C. Other Cost Functions

Besides company operating expenses and marketing expenses, expense loading in insurance rates include insurance profits, loss adjustment and litigation expenses. The real expense loading might be more complicated than the two simple cases that we have discussed. For example, expense loading in most insurance policies are most likely not in a linear form. We feel that our points are best illustrated in the stylized linear situation and therefore we will not let readers be distracted by the more complicated nonlinear functions. However, it is indicated in insurance literature, such as Raviv (1979), that full coverage is not optimal under nonlinear costs.

As a matter of fact, insurance policies purchased in private markets seldom provide policyholders with complete coverage against losses. For instance, a common policy contract often contains upper limits on the coverage, with loss reimbursement only up to that limit, and a deductible, with which the insurer is only responsible for the excess of loss over a certain, predetermined level. The incomplete coverage phenomena have been extensively discussed in the literature, for example, Mossin (1968), Arrow (1971), Raviv (1979), Huberman, Mayers, and Smith (1983), and Young and Browne (1997).

In the real world we hardly observe an insurance policy that provides full coverage. Besides transaction costs, there are several other considerations that lead to less-than full coverage. First of all, if full protection is provided, the insured person has no incentive to avoid the misfortune and may act to bring it on under certain circumstances. Insurers incorporate deductible, co-payment, and upper-limit clauses into the policy to reduce the moral hazard problem. Secondly, insurers are not risk-neutral as stated in the model, as they often impose "safety loading" into the premium. Finally, as an investor, insurers do expect returns to their capital and, as we have seen in the automobile statistics, an insurer's profit share could become substantial.

As one of the major topics in insurance economics, the problem of adverse selection has long been studied under the zero-profit assumption of the Rothschild and Stiglitz (1976) model. Because of this unrealistic hypothesis, full insurance coverage becomes the choice of the universe. A second look at the Rothschild and Stiglitz (1976) model, under non-zero profit assumption, is necessary since we can hardly find an insurance policy that provides full coverage. Moreover, moving away from the full coverage can significantly change the dynamic of the Rothschild and Stiglitz arguments. As we can see in the rest of this paper, the well-regarded externality result could be reversed under many scenarios when heterogeneity is introduced into the model.

IV. Risk Aversion

Rothschild and Stiglitz (1976) assume people possess identical utility function, which implies that all individuals have the same degree of risk aversion. Nevertheless, insurance customers can differ in both loss probability and attitude toward risk.

Multidimensional adverse selection problem of such kind has drawn the attention of many economists since the birth of Rothschild and Stiglitz (1976) model. In the second part of their paper, Rothschild and Stiglitz conclude that pooling equilibrium cannot exist when high-risk people are more risk-averse than low-risk individuals. An opposite assumption is made by de Meza and Webb (2001). They argue that more risk-averse people tend to buy more coverage while the reckless people often put less value on insurance protection. In their setting, pooling equilibrium becomes possible. Many other articles, such as Smart (2000) and Villeneuve (2001), examine the same multidimensional problem under different assumptions. However, none of these papers questions the externality conclusion of the canonical model.

In this section, we extend the model under the following suppositions:

1) The market consists of two types of customers – high-risks and low-risks with different loss probabilities $p^H > p^L$.

2) Two types of customers have the same utility function but different levels of risk aversion. High-risk utility function is $U^H(x)$ and the low-risk utility function is $U^L(x)$, with $r^H(x)$ and $r^L(x)$ as the respective absolute risk aversion.

3) One group is, on average, more risk averse than the other group and there are two possibilities. In Scenario A, high-risk individuals are more risk averse than low-risks, $r(x)^H > r(x)^L$; while in Scenario B the relation is reversed.

4) Insurance firms face proportional costs in the form of

$$\text{Cost} = ba_1 + pc(a_1 + a_2) \tag{4.1}$$

5) Obviously, the absolute risk aversion depends on the wealth level. At this stage, the wealth effect, which is analyzed in the next section, brings only background noise. We delete the wealth factor by assuming people have constant absolute risk aversions (CARA)[1]. According to Pratt (1964), constant risk aversion leads to the negative exponential utility function,

$$U^t(x) = -e^{-r^t x}, \; t = H, L \tag{4.2}$$

This section proceeds as follows: As the first step of our study, we examine basic properties of high-risk and low-risk indifference curves. Then we inspect the multidimensional adverse selection problem under no transaction cost. Finally,

[1] Although our discussion is based on CARA, our result can be easily generalized to other utilities like DARA. All we get from the CARA assumption is the simplicity, which is necessary for our illustration.

proportional cost is added into the question and the concept of non-externality separating equilibrium is introduced. At the end, we give an illustrative example.

A. Indifference Curves

Our procedure is mainly graphical. We need a clear picture of the indifference curves of two risk types. The slope of the indifference curve is represented by the marginal rate of substitution, defined as,

$$\mathrm{MRS}^{\mathrm{t}} = -\frac{dW_2}{dW_1} = \frac{(1-p^t)U^{t'}(W_1)}{p^t U^{t'}(W_2)},\ \mathrm{t = H, L} \tag{4.3}$$

Under CARA, MRS is $[(1-p^t)/p^t]\times e^{-r^t(W_1-W_2)}$ and the ratio of high-risk MRS vs. low-risk MRS becomes,

$$\frac{\mathrm{MRS}^{\mathrm{H}}}{MRS^L} = (\frac{1-p^H}{p^H} / \frac{1-p^L}{p^L})\times e^{-(r^H-r^L)\times(W_1-W_2)} \tag{4.4}$$

Obviously, $\frac{1-p^H}{p^H} / \frac{1-p^L}{p^L}$ is less than 1 and the picture will become clear, once we know about the second part of (4.4). As we know, people are not allowed to have more wealth in the loss state than in the no-loss state, which means $\mathrm{W}_2 \le W_1$. Now we will discuss the relative slope of two types' indifference curves under the two scenarios proposed in assumption (3).

Scenario A - High-risks are more risk averse than low-risk people ($\mathrm{r}^{\mathrm{H}} > \mathrm{r}^{\mathrm{L}}$).

In this case the second factor, $e^{-(r^H-r^L)\times(W_1-W_2)}$, is less than one and $\mathrm{MRS}^{\mathrm{H}} < \mathrm{MRS}^{\mathrm{L}}$. The high-risk indifference curve is always flatter than low-risk indifference curves.

Scenario B - High-risks are less risk averse than low-risks ($\mathrm{r}^{\mathrm{H}} < \mathrm{r}^{\mathrm{L}}$).

Our major interest is whether high-risk indifference curve is still flatter than low-risk indifference curves. The answer, nevertheless, can be either a yes or a no.

In Scenario B, the second factor of the equation (4.4), $e^{(r^L - r^H)\times(W_1 - W_2)}$, is greater than one. But the first factor, $\frac{1-p^H}{p^H} / \frac{1-p^L}{p^L}$, is less than 1. It becomes difficult to decide which indifference curves are flatter. If we hold $\mathrm{p}^{\mathrm{H}}, \mathrm{p}^{\mathrm{L}}$ constant, the second factor increases as the differences between r^{H} and r^{L}, W_1 and W_2 become larger. At some point, the second factor overpowers the first factor, $\frac{\mathrm{MRS}^{\mathrm{H}}}{MRS^L} \geq 1$, and the high-risk indifference curve becomes steeper. Otherwise, when the differences are small, the high-risk curve is flatter.

Our second interest lies in whether the two indifference curves cross only once. If the indifference curve of one class is always flatter than the other class's indifference curve, the two indifference curves will cross only once. Otherwise, the indifference curves cross twice. In Scenario B, double-cross becomes a possibility.

B. Heterogeneous risk aversion with no cost

In this subsection, we introduce different risk aversions into the Rothschild and Stiglitz model, assuming transaction costs.

A simple argument, similar to that of Rothschild and Stiglitz (1976), establishes that *there cannot be a pooling equilibrium*. Under most circumstances, at any possible pooling point, two indifference curves have distinct slopes. Even in the least likely case that two indifference curves have the same slope at the potential pooling point, the two indifference curves will diverge in the area next to the pooling point. Another contract, which lies in between these two indifference curves, will attract one type away from the pooling point. It could be either the low-risks or the high-risks who are attracted away.

The argument for separating equilibrium remains almost the same. Although two types have different degrees of risk aversion, both of them regard full insurance, a^{H} and a^{L} in figure 3, as the first-best choice. The high-risks prefer the low-risk contract a^{L}, because it provides more consumption on both states of nature. Since insurance firms cannot tell who is the bad type, they can only offer the low-risks another partial coverage contract g^{L}, at the intersection of high-risk indifference curve and low-risk budget line. In this way, high-risk individuals inflict negative externality to low-risks.

The argument for separating pooling equilibrium has a slight change in another unique circumstance. In Scenario B, low-risk indifference curve could become flatter than that of high-risks at g^{L}. In this case, low-risks will prefer another point h^{L}, the tangent point of two difference curves. The contract h^{L} is profitable to the insurer since it is located below the low-risk budget line.

C. Optimal choice with transaction costs

Now we introduce the proportional cost into the model. We assume the cost function is $\mathrm{b}\alpha_1 + pc(\alpha_1 + \alpha_2)$. In Rothschild and Stiglitz (1976), people choose full coverage under zero cost and the degree of risk averse cannot affect individual behavior. Will risk aversion affect the optimal choice under proportional cost?

PROPOSITION 3. *With cost function* $\mathrm{b}a_1 + p\mathrm{c}(a_1 + a_2)$, *increases in the risk aversion level will lead to increases in optimal insurance coverage amount* a_1^* *under CARA.*

Proof.

When $U(x) = -e^{-rx}$, the optimal solution for Problem II is

$$a_1^* = \frac{p(1+c)}{1-b}[d - \frac{1}{r}\log\frac{(1-p)(1+c)}{1-b-p-pc}] \tag{4.5}$$

$$a_2^* = \frac{1-b-p-pc}{p(1+c)}[d - \frac{1}{r}\log\frac{(1-p)(1+c)}{1-b-p-pc}] \tag{4.6}$$

Obviously, $\frac{da_1^*}{dr} > 0$ and $\frac{da_2^*}{dr} > 0$. Both a_1^* and a_2^* are increasing with the risk aversion r.

Proposition 3 states that, under proportional costs, people who are more risk averse tend to purchase more insurance. Less risk-averse individuals will buy a small amount of insurance that provides incomplete protection. Although we prove this property under CARA, this result applies to other utility functions. This conclusion has been comfirmed in the insurance literature in papers such as Schlesinger (2000).

D. No-Externality Separating Equilibrium

We now move on to the multidimensional adverse selection problem with both proportional costs and heterogeneous risk aversion. The argument for pooling equilibrium is the same as the no cost case and we will concentrate on the mechanism of separating equilibrium.

First, we consider separating equilibrium under Scenario A. In Scenario A, high-risks are more risk averse than low-risks and high-risk indifference curve is flatter everywhere than the low-risk indifference curve.

The separating equilibrium, however, is different from the standard Rothschild and Stiglitz separating equilibrium in the following aspects: First of all, both types of individuals, under proportional costs, will not view full insurance as the first-best choice. Both types move away from the 45-degree certainty line. Secondly, the less risk averse, low-risk people tend to move closer to the endowment point. The high-risk stay close to the certainty line since they are more risk averse.

Will high-risks still prefer low-risks optimal contract to their own contract? To answer this question, we need to set up a formal framework.

For a type t individual ($t = H, L$), $a^{t*}(a_1, a_2)$ is defined as the solution to the following problem (Problem III):

$$Max: \quad E(U) = (1 - p^t)U^t(W - a_1) + p^t U^t(W - d + a_2)$$

$$Subject\text{ to}: \quad a_1(1 - p^t) - a_2 p^t = b a_1 + pc(a_1 + a_2)$$

$$a_1 \geq 0\text{, and } \quad a_2 \geq 0$$

A type s individual's ($s = H, L$) expected utility at type t optimal point is written as $V^s(a^{t*})$. For example, a high-risk individual's expected utility at a low-risk optimal point is:

$$\mathrm{V}^H[a^{L*}] = (1 - p^H)U[W - a_1^*(p^L)] + p^H U[W - d + a_2^*(p^L)]$$

DEFINITION 1. *A separating equilibrium in which neither group causes any negative externality to the other group in the market, thereafter* ***NE equilibrium****, satisfies the following conditions:*

$$V^H[a^{H*}] \geq V^H[a^{L*}] \tag{4.7}$$

$$V^L[a^{L*}] \geq V^L[a^{H*}] \tag{4.8}$$

If $V^H[a^{H*}]$ is greater than $V^H[a^{L*}]$, high-risks will not prefer low-risks' optimal consumption bundle. Meanwhile, we need low-risks to stay with a^{L*}. If both conditions (4.7) and (4.8) are satisfied, the insurance market will reach a state of equilibrium by an automatic separation of two types of consumers. In that case, neither high-risk individuals nor low-risk individuals will cause any externality to the other group.

LEMMA 1. *$V^L[a^{L*}] \geq V^L[a^{H*}]$ always holds under linear cost function.*

Proof. See Appendix.

Lemma 1 states that low-risks will always prefer their own optimal consumption bundle to that of the high-risks. This result should hold as long as the price of insurance $q(a)$ is lower for the low-risk type. There is no need to worry about the possibility that low-risks could bring negative effects on the high-risks. It only remains to see whether high-risks affect low-risks in the market.

Existence of NE Equilibrium

PROPOSITION 4. Under cost function $Cost = ba_1 + pc(a_1 + a_2)$, when high-risks are more risk averse than low-risk and the difference in risk aversion is large enough, NE equilibrium exists.

Instead of offering a proof, we show how to construct the NE Equilibrium:

Step 1 - Find out the high-risks optimal coverage contract a^H, shown in figure 4. The high-risk indifference curve that passes a^H must intersect the low-risk market odds line at some point, and call it g^L.

Step 2 - According to proposition 3, low-risk optimal choice a^L decreases as the low-risks reduce their risk aversion. Let the low-risk risk aversion to be small enough and make a^L fall between g^L and the endowment point E.

At this time, high-risks will prefer its own optimal consumption bundle to a^L, the low-risk's optimal consumption bundle. By lemma 1, we have $V^L[a^{L*}] \geq V^L[a^{H*}]$, which means low-risks will not prefer high-risk contract. Therefore, this is a *NE equilibrium*.

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Insert FIG. 4. about here

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Proposition 4 states that, in Scenario A, when the difference in risk aversions is large enough, the negative externality disappears and the low-risks can enjoy their first best choice as if there were no high-risks in the market.

In other cases, where the high-risk indifference curve passes below the low-risk optimal point a^L, high-risks nonetheless pose negative externality to the low-risk. However, the degree of externality is alleviated since a^L is already very close to the endowment point.

Having examined Scenario A, we move on to Scenario B. In Scenario B, high-risk individuals are less risk averse than low-risks. High-risk indifference curve could be either steeper or flatter than the low-risk indifference curve depending the parameters. Since high-risks are more risk averse, they tend to purchase less insurance and their optimal point is closer to the endowment point, as shown in figure 5. Under most circumstances, the negative externality is aggravated since low-risks have to take a much lower insurance coverage contract. In this case, the mechanism for separating equilibrium remains same as that of the standard model.

A special case that needs attention is when the low-risk indifference curve is flatter than high-risk at g^L. Under such circumstance, low-risks will prefer another point, h^L, on the high-risk indifference curve. Contract h^L is profitable to the insurer since it is located below the low-risk budget line.

However, interesting enough, it is still possible to find a *NE equilibrium* when high-risk individuals are more risk averse.

PROPOSITION 5. *In Scenario B, when high-risks are less risk averse than low-risks, the following conditions lead to NE equilibrium:*

1) *Difference in risk aversion is large.*
2) *Difference in loss probabilities is small.*

The NE Equilibrium can be constructed by the following steps:

Step 1 - Find the high-risks optimal coverage contract a^H, shown in figure 5.

Step 2 - The high-risk indifference curve that passes a^H must intersect 45 degrees line at some point. Find this point and call it A.

Step 3 – Turn the low-risk budget line counter clock wise. When the low-risk loss probability is close enough to the high-risk loss probability, the low-risk budget line intersect the 45 degree line below A. Name the intersection of high-risk indifference curve and low-risk budget line as b^H.

Step 4 – Move low-risks' optimal point toward the 45-degree certainty line. We can always do this because, according to Proposition 3, people buy more insurance when they become more risk averse. When risk aversion is high enough, the low-risk optimal consumption point a^L can fall above b_2 on the low-risk budget line.

At this time, high-risks will prefer its own optimal consumption bundle, a^H, to the low-risk's optimal consumption bundle, a^L. By lemma 1, we have $V^L[a^{L*}] \geq V^L[a^{H*}]$, which means low-risks will not prefer high-risk contract. Therefore, this is a NE equilibrium for this pair of individuals.

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Insert FIG. 5. about here

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Illustrative Example for NE Equilibrium

Here, we provide an illustrative example with which we show NE eqilibrium exists under both Scenario A and Scenario B. Let every customer in the insurance market have an initial wealth of W =1. When an accident occurs, the individual suffers a loss of d = 0.9. Both the low- and high-risks have utility function of the form $U(x) = -e^{-rx}$.

First consider Scenario A - high-risks are more risk averse. Suppose that low-risks have loss probability of p^L = 0.1 and that high-risks have p^H = 0.2. High-risks are less tolerant toward risk and their absolute risk aversion is set as r^H = 4. On the other side the low-risks have a lower absolute risk averse, which is assumed to be r^L = 0.5.

When insurance companies provide actuarially fair policies, i.e. $Cost = 0$, low- and high-risks want full coverage and they are willing to pay 0.09 and 0.18 for the premium, respectively. In case a loss of 0.9 happens, they will get full reimbursement and receive a payment of 0.9 from the insurance company. However, the high-risk individuals will have a gain of 0.011 in the expect utility if they buy the low-risk policy and pay a lower premium. Insurance companies are forced to offer partial coverage to low-risk individuals due to the existence of high-risks in the market. High-risk individuals exert a negative externality on the low-risk individuals making them worse off.

Now, let us consider the more realistic case that insurance companies include a proportional transaction cost, of the form $ba_1 + pc(a_1 + a_2)$, in the premium. Suppose both b and c are 10%. Thus, both the high- and low-risks will not buy full insurance since the premum is not actuarially fair. While high-risks have to pay a premium a_1^H of 0.2 and in case a loss of 0.9 happens, they will receive a reimbursement of 0.84 from the insurance company. Low-risks will scale down their premium payment (a_1^L) to 0.04 and can only receive a check of 0.45 if they suffer a loss of 0.9. Compared to high-risks, the best coverage that low-risks can receive offers much lower protection. Thus, high-risk people do not like the low-risk policy any more. They will have a higher expected utility at their own policy ($V^H[a^{H*}] = -0.044$) than switching to the low-risk policy ($V^H[a^{L*}] = -0.046$). The low-risks, however, always prefer their own policy ($V^L[a^{L*}] = -0.63$) to the high-risk policy ($V^L[a^{H*}] = -0.66$). In this case, no group interferes with the other group in the market and the informational problem disappears. The market achieves a NE equilibrium.

Similary, under Scenario B when low-risks are more risk averse, the market can reach a state of NE equilibrium. Suppose that high-risks have risk aversion of $r^H = 0.4$ and the low-risk absolute risk averse is $r^L = 4$. Low-risk loss probability is assumed to be $p^L = 0.1$ and high-risk loss probabilty is close to the high-risk one with $p^H = 0.105$[2].

Again, NE equilibrium exists. In this case, high-risks will spend 0.042 on premium, while low-risks are willing to paying a premium of 0.103. In case a loss of $d = 0.9$ happens, high-risks will only receive a payment of 0.33 and the low-risks will get a

[2] During the searching process for the *NE equilibrium*, we have to set the high-risk loss probabilty very close to that of low-risks. Otherwise such *NE equilibrium* is not available.

payment of 0.84. Again, low-risk customers prefer their own policy ($V^L[a^{L*}]$ - $V^L[a^{H*}]$ = 0.012), and high-risk customers will automatically choose a high-risk policy ($V^H[a^{H*}]$ - $V^H[a^{L*}]$ = 0.0001). Again we find NE equilibrium exists when difference in risk aversion is large.

V. Wealth

In Rothschild and Stiglitz (1976), all agents have same amount of endowment wealth. But, in reality, people may differ in their financial status. In this section, we extend the model under the following set of assumptions:

1) The market consists of two types of customers – high-risks and low-risks with different loss probabilities $p^H > p^L$.
2) One type of customers is, on average, wealthier than the other group and there are two possibilities. In Scenario I, low-risk individuals are richer, $W^H > W^L$; while in Scenario II this relation is reversed.
3) Insurance firms face proportional costs in the form of

$$\text{Cost} = b a_1 + pc(a_1 + a_2) \tag{5.1}$$

4) Two types of customers have same utility function and the same level of risk aversion.
5) People have decreasing absolute risk aversion (DARA). This is a common hypothesis used in economic studies.

In the rest of this section, we will first exam the slope of indifference curves for the two types. Then, under the zero-profit assumption, we analyze the multidimensional adverse selection problem with heterogeneous wealth. In the end, we show that NE equilibrium exists.

A. Indifference Curves

Our graphical procedure requires that we have a clear picture of the indifference curves of the two types. Instead of looking at the indifference curves in the (W_1, W_2), we move into the ($-a_1$, a_2) space, in which our graph is greatly simplified. In the ($-a_1$, a_2) space, the marginal rate of substitution for the two types is represented by,

$$\mathrm{MRS}^{\mathrm{t}} = -\frac{da_2}{d(-a_1)} = -\frac{(1-p)U'(W^t - a_1)}{pU'(W^t - d + a_2)}, \ \mathrm{t} = \mathrm{H, L} \tag{5.2}$$

The ratio of high-risk MRS vs. low-risk MRS turns out to be,

$$\frac{\mathrm{MRS}^{\mathrm{H}}}{MRS^L} = \left(\frac{1-p^H}{p^H} / \frac{1-p^L}{p^L}\right) \times \left[\frac{U'(W^H - a_1)}{U'(W^H - d + a_2)} / \frac{U'(W^L - a_1)}{U'(W^L - d + a_2)}\right] \tag{5.3}$$

Obviously, the first part of (5.3) is less than 1. Let $Z(W) = \frac{U'(W - a_1)}{U'(W - d + a_2)}$, and we will find that

$$\frac{dZ(W)}{dW} = \frac{U'(W_2)}{U'(W_1)}[r(W_2) - r(W_1)] \tag{5.4}$$

Under proportional costs, $\mathrm{W}_1 > W_2$, and, with DARA, $\frac{dZ(W)}{dW} > 0$. The relative slopes of two indifference curves can be analyzed under two scenarios.

Scenario I - High-risks are poorer than low-risks. $W^H < W^L$

In this case, the second factor in equation (5.3) is less than one. Therefore, $MRS^H < MRS^L$. The indifference curve of the high-risks is flatter everywhere than that of low-risks. Their indifference curves can across once.

Scenario II - High-risks are richer than low-risks. $W^H > W^L$

In this case, the second factor in equation (5.3) is greater than one. Therefore, it is difficult to judge which indifference curve is flatter. However, we can hold p^H and p^L constant, the second factor increases when the difference between W^H and W^L become larger. As a result, the second factor could overpower the first factor and the high-risk indifference curve becomes steeper. On the other side, when the difference between W^H and W^L is small, the high-risk indifference curve is flatter. Based on what we have seen so far, wealth affect people's behavior through risk aversion. Under DARA, more wealth means less risk averse, and less wealth more higher degree of risk aversion.

B. Heterogeneous wealth with no cost

The situation here is almost exactly same as the heterogeneous risk aversion case in Section IV. Pooling equilibrium is not possible. For separating equilibrium, both types regard full insurance as the first-best choice and the low-risks have to take a second-best choice due to the presence of high-risks. A special case is that, in *Scenario II*, low-risk indifference curve is flatter than that of high-risks. The low-risk contract is profitable to the insurer.

C. Optimal choice with transaction costs

We now examine, under proportional costs, how wealth difference will affect people's choice.

PROPOSITION 6 *Under DARA, rich people buy less insurance, when there is a proportional cost of the form* $\mathrm{b}a_1 + p\mathrm{c}(a_1 + a_2)$.

Proof.

Rewrite the first order condition of Problem II as

$$\frac{U'(W - a_1(W))}{U'(W - d + a_2(W))} = \frac{1 - b - p(1 + c)}{(1 + c)(1 - p)} \tag{5.5}$$

Notice that

$$a_2(W) = \frac{[1 - b - p(1 + c)]}{p(1 + \mathrm{c})} a_1(W)$$

Take derivative with respect to W on both side of (5.5), we can get

$$\frac{da_1}{dW} = \frac{r(W_1) - r(W_2)}{r(W_1) + \dfrac{1 - b - p - pc}{p(1 + c)} r(W_2)} \tag{5.6}$$

Since $W_1 > W_2$, we have $r(W_1) < r(W_2)$ under DARA. Therefore, $\frac{da_1}{dW} < 0$.

Proposition 6 states that, under proportional costs, the richer people are, the less they purchase insurance. On the other side, poor individuals, everything else equal, will buy more insurance.

D. No-Externality Separating Equilibrium

We now proceed to analyze the Rothschild and Stiglitz (1976) arguments for equilibrium under proportional costs with heterogeneous wealth levels. Again, the argument for pooling equilibrium is the same as the no cost case and we will concentrate on the mechanism of separating equilibrium.

First, we discuss the separating equilibrium under Scenario I, in which high-risk individuals are poorer and low-risks are richer. In this case, high-risk indifference curve is flatter everywhere than the low-risk indifference curve.

The separating equilibrium, however, is totally different from RS arguments. The most dramatic change is that both types, under proportional costs, will not regard full insurance as their first-best choice. Both types move away from the 45-degree certainty line. The low-risks, who are wealthier and less risk averse, tend to move close to the endowment point. The high-risks, on the other hand, move down by a smaller distance. Now the question is: *Will high-risks prefer low-risks optimal contract to their own contract?*

PROPOSITION 7. *Under cost function* $Cost = b\alpha_1 + pc(\alpha_1 + \alpha_2)$*, when high-risks are poorer than low-risks and the difference in wealth level is large enough, NE equilibrium exists.*

Proposition 7 is, in nature, the same as Proposition 4. All we did is substituting risk aversion with wealth level. Being poorer is equivalent to being more risk averse while being wealthier means that the person is more tolerant toward risk. When difference in wealth levels is large enough, the externality problem disappears and we will reach *NE equilibrium*.

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Insert FIG. 6. about here

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This case is illustrated in figure 6. The high-risks, who are also less wealthy people, prefer to get more protection and buy the contract a^H. The low-risks, who are wealthier, prefer to buy a small amount of coverage, represented by a^L in figure 6. Since a^L is too close to the endowment point, high-risk will not purchase a^L although they can do so under the asymmetric information condition. Therefore, low-risks can enjoy their first best choice as if there were no high-risks in the market.

In other cases, where the high-risk indifference curve still passes below the low-risk optimal point a^L, high-risks pose negative externality to the low-risk. However, the degree of externality is weakened since a^L is already very close to the endowment point.

In Scenario II, where high-risks are affluent, they are less risk averse than low-risks. Generally speaking, they purchase less insurance and their optimal consumption point is very close to the endowment point. Under most circumstances, the negative externality is aggravated.

However, it is still possible to find a *NE equilibrium* in Scenario II, but the following two conditions have to hold,

(i) The difference in loss probability is small.

(ii) The difference in endowment wealth is large.

Illustrative Example for NE Equilibrium

Here, we provide an illustrative example with which we show *NE eqilibrium* exists. Suppose both the low- and high-risks have *log* utility, $U(x) = \log(x)$. Assume that low-risks have loss probability of $p^L = 0.1$ and they are relatively wealthy, with a endowment wealth of $W^L = 2$. High-risks, on the other hand, have a higher loss probability, $p^H = 0.2$, and they are less wealthy, $W^H = 0.75$. In case of an accident, both types will suffer a loss of $d = 0.5$.

When insurance companies provide *actuarially fair policies, i.e.* $Cost = 0$, low- and high-risks are willing to pay up to 0.05 and 0.1 for the premium, respectively, for full coverage. In case a loss of 0.5 happens, they will get full reimbursement and receive a payment of 0.5 from the insurance company. However, the high-risk individuals will have a gain of 1.1 in the expect utility if they buy the low-risk policy and pay a lower premium. Insurance companies are forced to offer partial coverage to low-risk individuals due to the existence of high-risks in the market. High-risk individuals exert a negative externality on the low-risk individuals making them worse off.

Now, let us consider the more realistic case that insurance companies include a proportional transaction cost, of the form $b\alpha_1 + pc(\alpha_1 + \alpha_2)$, in the premium. Suppose both b and c are 10%. Thus, both the high- and low-risks will not buy full insurance since the premum is not actuarially fair. While high-risks have to pay a premium α_1^H of 0.085 and in case a loss of 0.5 happens, they will receive a reimbursement of 0.349 from the insurance company. Low-risks will scale down their premium payment (a_1^L) to 0.012 and can only receive a check of 0.098 if they suffer a loss of 0.5. Compared to high-risks, the best coverage that low-risks can receive offers much lower protection. Thus, high-risk people do not like the low-risk policy any more. They will have a higher expected utility at their own policy ($V^H[a^{H*}]$= –0.46) than switching to the low-risk policy ($V^H[a^{L*}]$= –0.461). The low-risks, however, always prefer their own policy ($V^L[a^{L*}]$= 0.66) to the high-risk policy ($V^L[a^{H*}]$= 0.64). In this case, no group interferes with the other group in the market and the informational problem disappears. The market achieves a *NE equilibrium.*

Similary, under Scenario II, when low-risks are less wealthy, the market can reach a state of *NE equilibrium.* We assume that high-risks have an endowment wealth of W^H = 2.4 and the low-risks' financial status is represented by W^L = 1. Low-risks have loss probability is assumed to be p^L = 0.1. High-risks loss probabilty has to be very close to p^L, p^H = 0.105. We find the *NE equilibrium does exist*: high-risks will spend a_1^H = 0.0016 on premium payment, while low-risks is paying a premium a_1^L equal to 0.037. In case a loss of d = 0.5 happens, high-risks will only receive a net payment of 0.0127 and the low-risks will get a payment of 0.306. Still, low-risk customers will prefer their own policy ($V^L[a^{L*}] - V^L[a^{H*}] = 7.9\times10^{-3}$), and high-risk customers will automatically choose a high-risk policy ($V^H[a^{H*}]$ - $V^H[a^{L*}] = 1.7\times10^{-4}$).

VI. Conclusion

This article demonstrates that the externality conclusion of Rothschild and Stiglitz (1976) can be reversed when transaction costs are present. The long-held zero-profit hypothesis is in direct contradiction to reality. As we know that insurance policies seldom provide complete coverage. Market fictions are simply too important to be ignored. In our view, analysis of markets with informational problems should be conducted in a more realistic setting.

Incorporating transaction costs brings significant change to consumers' behavior. They move away from their full demand amount and purchase less than what they wanted to purchase in fictionless world. Diversity of customers means that care has to be taken on whether one group will cause negative effect to other groups in the market. It is illustrated that the special *non-externality separating equilibrium,* in which no group causes any negative externality to the other group in the market, does exist. The world is more complicated than the two simple cases that we have analyzed. For example, high risk group can be both less wealthy and more risk averse.

Rothschild and Stiglitz (1976) model is a construct of simplicity -- the market is competitive, firms are risk neutral and make zero profit. Furthermore, the model is a construct of homogeneity -- people have same endowment wealth, utility function (therefore risk aversion), and loss amount. These hypostheses lead to the externality result. Introducing complexity and heterogeneity, as we have showed with heterogenous risk aversion and endowment wealth, leads to deviations from their standard conclusion.

Our study provides new insights to econometrics of insurance under asymmetric information. Empirical evidence to date is not always consistent with the major implications of Rothschild and Stiglitz (1976). In a recent study, Chiappori and Salanié (2000) test whether agents with higher accident probabilities choose contracts with more comprehensive coverage. They find no statistical evidence to support this prediction. Our paper does show that transaction costs and heterogeneous risk aversion or wealth could be one explanation for Chiappori and Salanié's findings. The concept of non-externality separating equilibrium suggests future empirical work on asymmetric information problems when agents do not have identical characteristics.

Mathematical Appendix

Proof of Proposition 1

When there is a constant cost, individuals choose the optimal insurance coverage to maximize expected utility:

$$Max: \quad E(U) = (1-p)U(W - a_1) + pU(W - d + a_2)$$

$$Subject \;\; to: \quad a_1(1-p) - a_2 p = a \;\text{ or }\; a_1 = a_2 = 0$$

$$a_1 \geq 0,\; a_2 \geq 0$$

The last condition guarantees that there will be no reverse insurance and it is checked at the end of this proof. As to the other two restraints, optimal solution is first calculated based on the first condition, then compared with expected utility level at $a_1 = a_2 = 0$. The Lagrangian function for problem I is:

$$L = (1-p)U(W-a_1) + pU(W-d+a_2) - l[a_1(1-p) - a_2 p - a]$$

First order conditions give us the following equations:

$$U'(W-a_1) = -l$$

$$U'(W-d+a_2) = -l$$

Combining these two conditions,

$$U'(W-a_1) = U'(W-d+a_2)$$

Since $U'' < 0$, we can conclude

$$W - a_1 = W - d + a_2$$

Then, we will get

$$a_1 = pd + a$$

$$a_2 = (1-p)d - a$$

Purchasing the insurance policy moves the individual to the point:

$$a^* = (W - pd - a, W - pd - a)$$

Under the first constraint each individual will buy complete insurance under constant cost, but his or her final wealth in both states will be reduced by the same amount, a. To make sure this is also optimal over the set including $a_1 = a_2 = 0$, we use the following approximation to analyze of the welfare change from E to a^*,

$$V(a^*) - V(E) = U(W - pd - a) - [(1-p)U(W) + pU(W-d)]$$

Using Taylor expansion, we have

$$\Delta V \cong \frac{1}{2} p(1-p)d^2[-U''(W)] - a\{U'(W) + (pd + \frac{a}{2})[-U''(W)]\}$$

Setting the last equation equal to zero, we can solve for the explicit condition of purchasing insurance under constant cost (r as *risk aversion function)*,

$$a \le \sqrt{d^2 p(1-p) + (pd + \frac{1}{r})^2} - (pd + \frac{1}{r})$$

The final solution is

$$a^* = (W - pd - a, W - pd - a)\,, \quad \text{when } a \le \sqrt{d^2 p(1-p) + (pd + \frac{1}{r})^2} - (pd + \frac{1}{r})$$

$$a^* = (W, W - d)\,, \text{ when } a > \sqrt{d^2 p(1-p) + (pd + \frac{1}{r})^2} - (pd + \frac{1}{r})$$

Proof of Lemma 1

We are comparing the value of function $\mathrm{V}^L[a]$ over two points $a^{L*}[a_1(p^L),\ a_2(p^L)]$ and $a^{\mathrm{H}*}[a_1(p^H),\ a_2(p^H)]$. a^{L*} is the solution to the following problem:

$$Max:\ \ E(U) = (1-p^L)U(W - a_1) + p^L U(W - d + a_2)$$

$$\textit{Subject to}:\quad a_1(1-p^L) - a_2 p^L = a + b a_1^L + c a_2^L$$

$$a_1^L \ge 0\,,\ a_2^L \ge 0$$

Here, we can always rewrite the first constraint as:

$$a_2^L = \frac{(1 - p^L - b)a_1^L - a}{p^L + c}$$

Since the value of $E(U)$ increases when a_2 increases, we can change the question into:

$$Max:\ \ E(U) = (1-p^L)U(W - a_1) + p^L U(W - d + a_2)$$

$$Subject \text{ to}: \ a_2^L \leq \frac{(1 - p^L - b)a_1^L - a}{p^L + c}$$

Let S^L represent the set defined the contraint, it is represented by the area $AEGF$.

Similarly, Let S^H represent the high-risk set defined by

$$a_2^H \leq \frac{(1 - p^H - b)a_1^H - a}{p^H + c}$$

In figure 7, S^H is represented by the area $BEGH$ and we would like divide S^H into S_1^H which is $CEGF$ in Figure 10 and S_2^H, represented by $BCFH$.

As the first step, we show that $S_1^H \subset S^L$.

Since for every a_1 in $AEGF$, we have

$$\frac{(1 - p^H - b)a_1 - a}{p^H + c} < \frac{(1 - p^L - b)a_1 - a}{p^L + c}$$

Then we have $a_2^H < a_2^L$ for every a_1, therefore we can conclude that

$$S_1^H \subset S^L$$

Because $\text{V}^L[a^{L*}]$ is the maximum value for the function $\text{V}^L[a]$ over the set S^L, Low-risks prefer any a^{L*} to any contract in the set S_1^H.

Now we show low-risks prefer any a^{L*} to any contract in the set S_2^H. Take a look at the point A, which is the intersection of low-risk budget line and the $45°$ line. Compared to contract A, any contract within S_2^H must have less wealth in both states of nature. Therefore, low-risks will prefer A to any contract in the set S_2^H. Moreover, because $\text{V}^L[a^{L*}]$ is the maximum value for the function $\text{V}^L[a]$ over the set S^L which includes the contract A. If we use $X \succ Y$ to represent that low-risks prefer contract X to contract Y, then, for low-risks,

$$\boldsymbol{a}^{L*} \succ A \succ \textit{Any Contract in Set } S_2^L$$

Since high-risks' optimal contract must be in the set $S^H \equiv S_1^H \cup S_2^H$, low-risks will prefer $\boldsymbol{a}^{L*}$ to $\boldsymbol{a}^{H*}$

$$V^L[\boldsymbol{a}^{L*}] \geq V^L[\boldsymbol{a}^{H*}]$$

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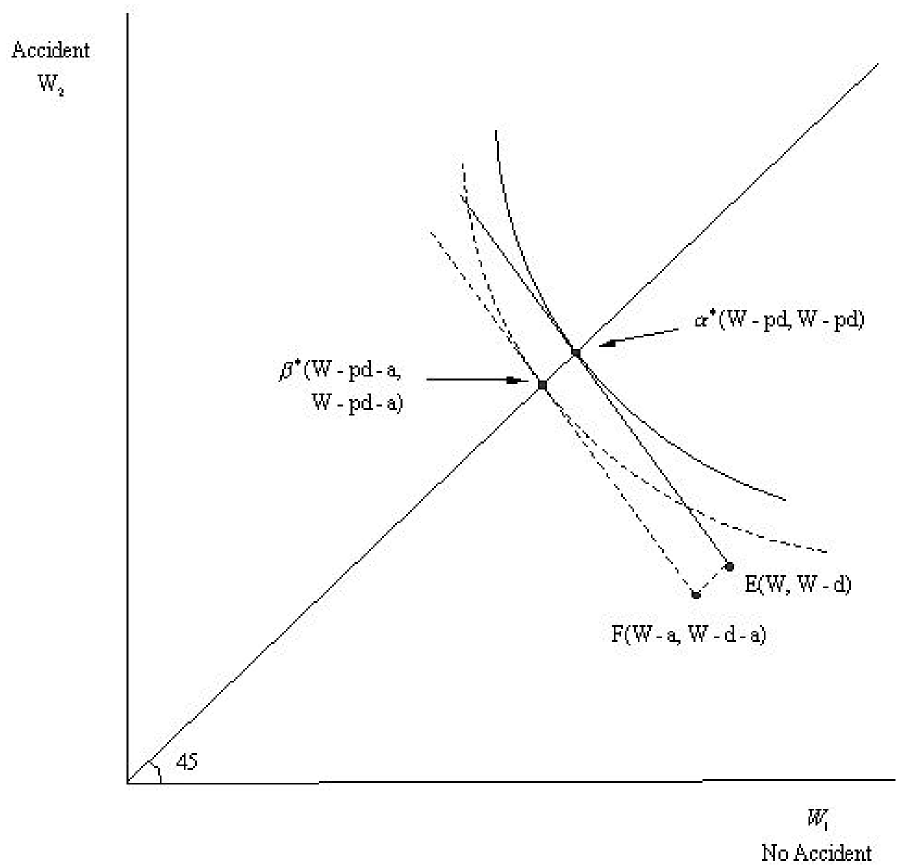

FIG. 1.- Optimal choice under constant cost

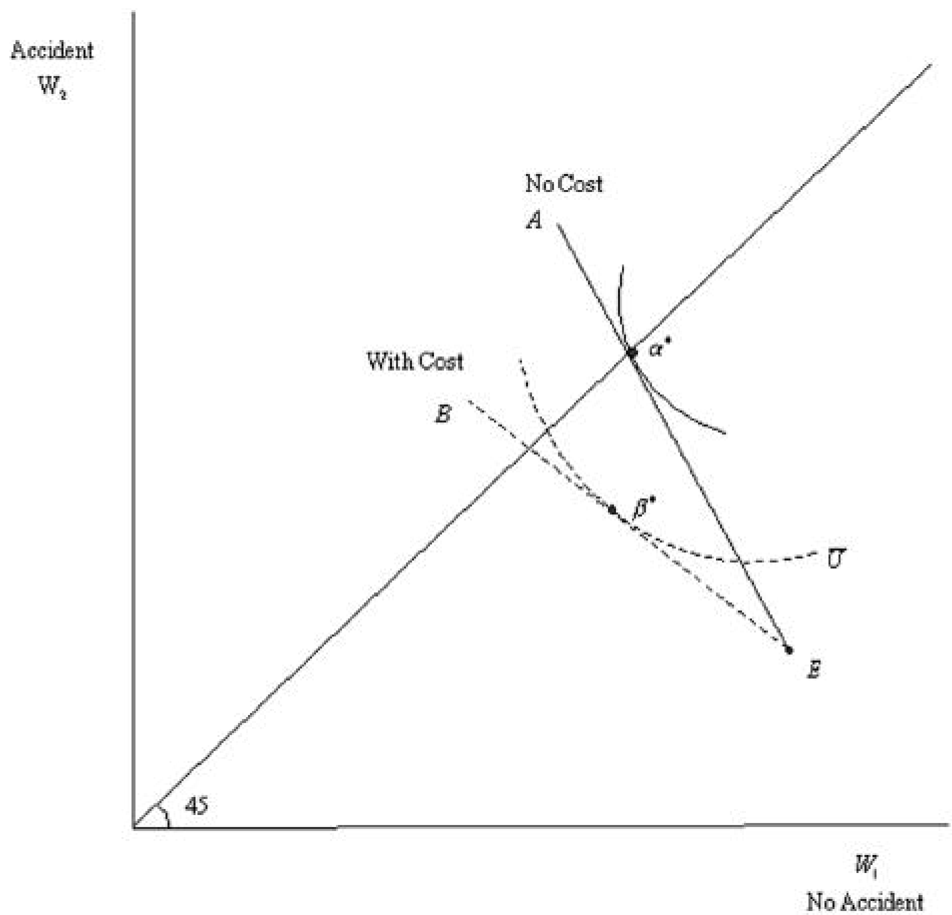

FIG. 2. – Optimal choice under proportional cost

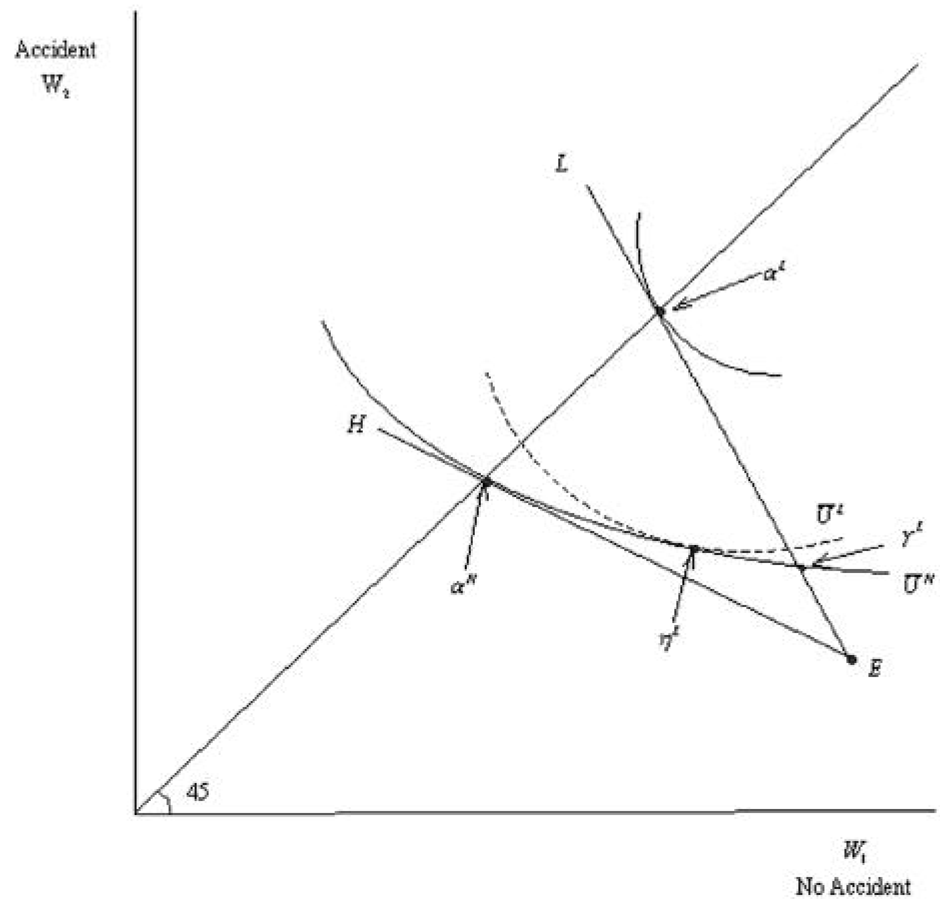

FIG. 3. – Separating equilibrium with heterogeneous risk aversion and zero cost

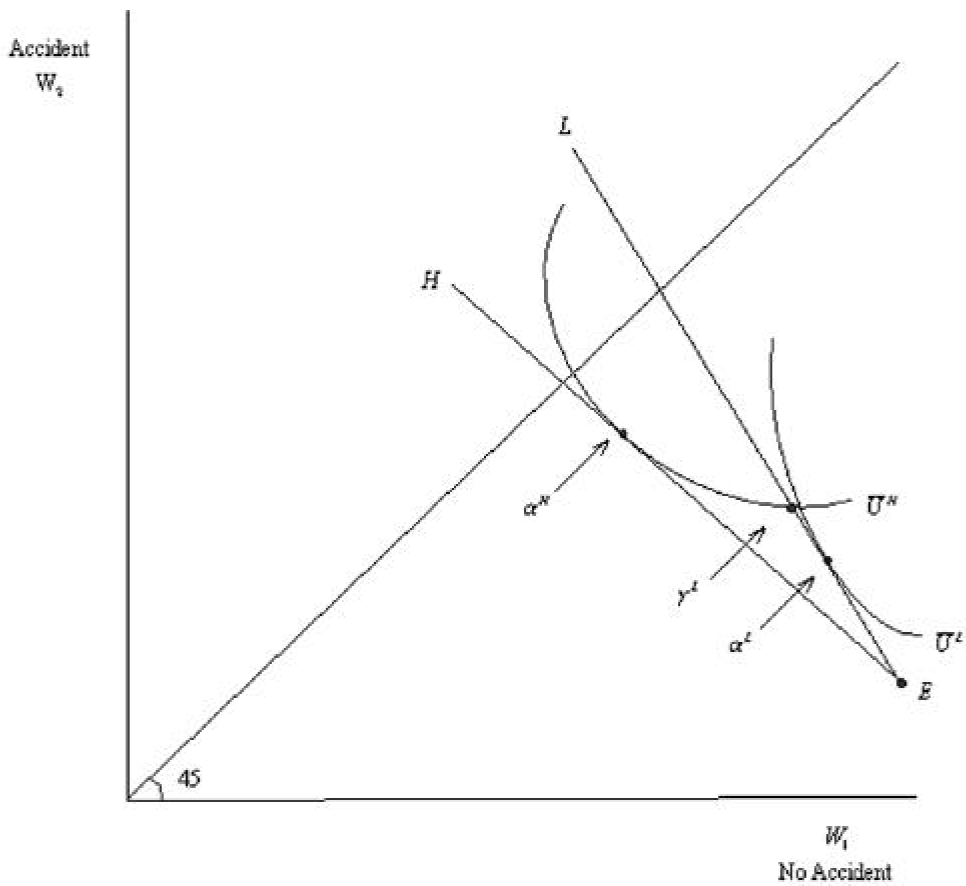

FIG. 4. - *NE Equilibrium* when high-risks are more risk averse

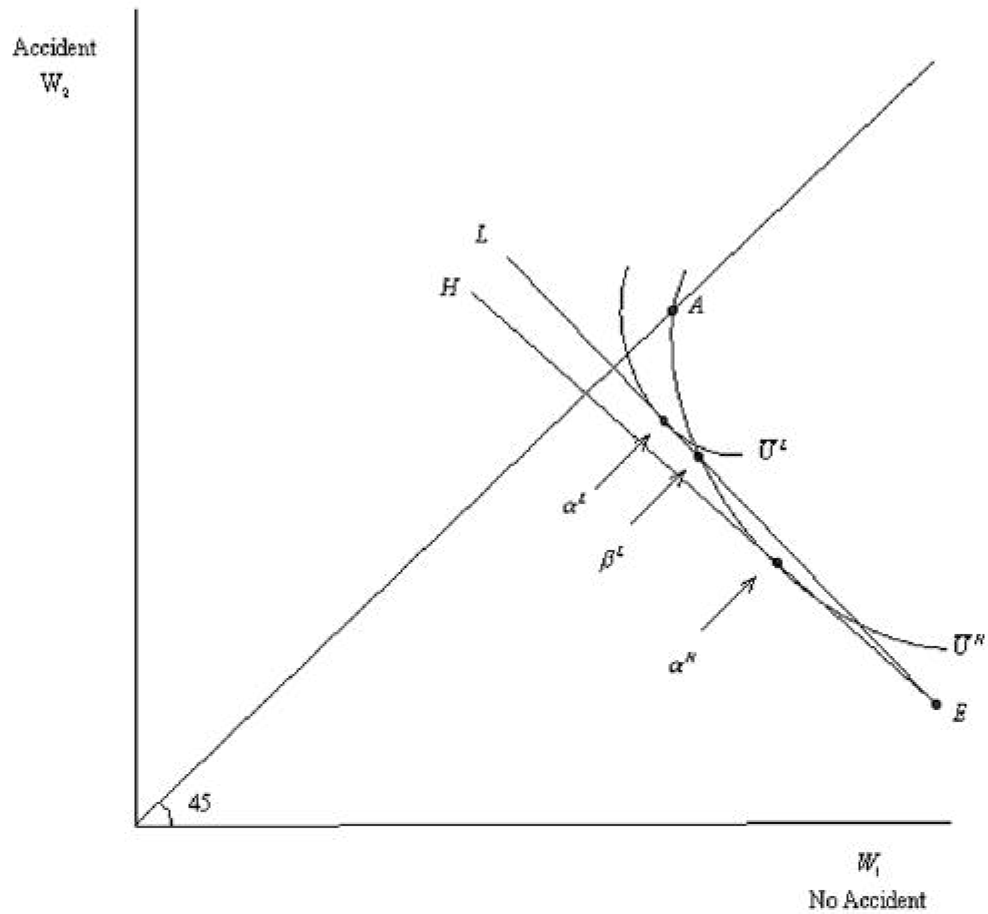

FIG. 5. – *NE Equilibrium* when high-risks are less risk averse

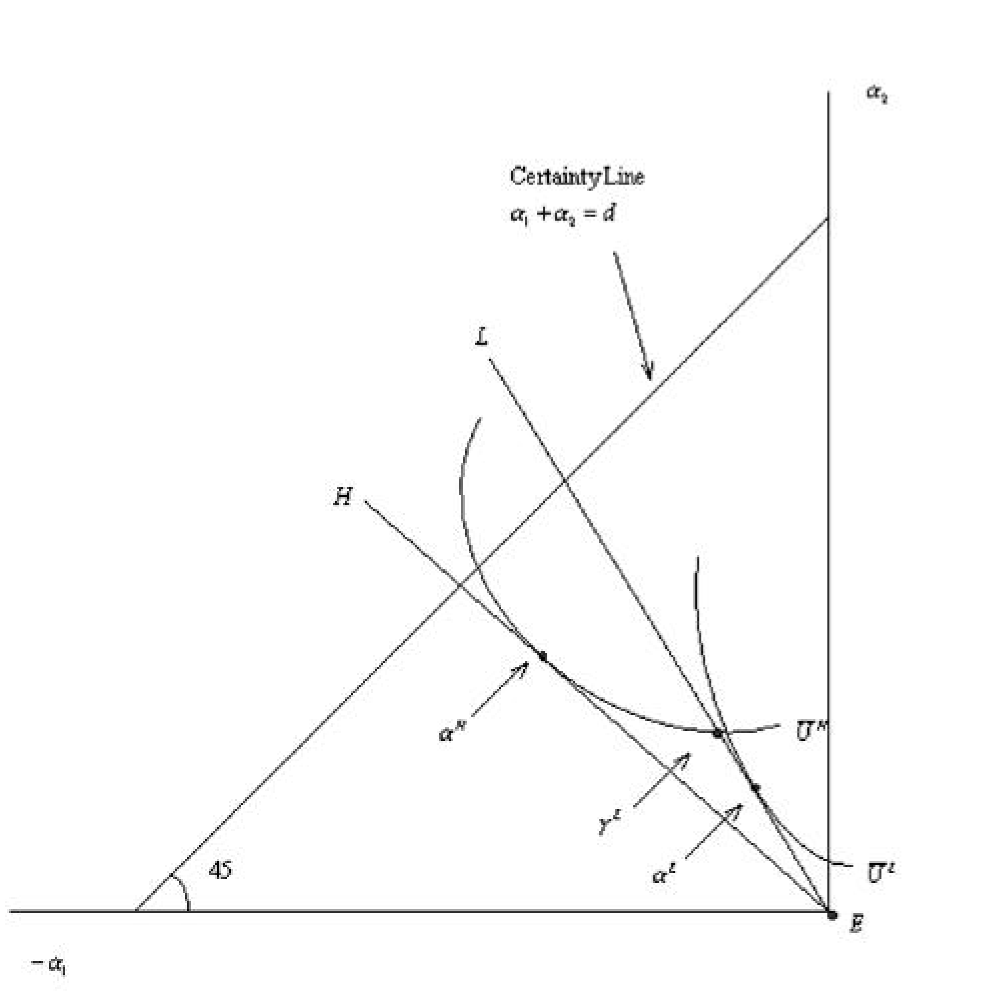

FIG. 6. – *NE Equilibrium* when high-risks are less wealthy

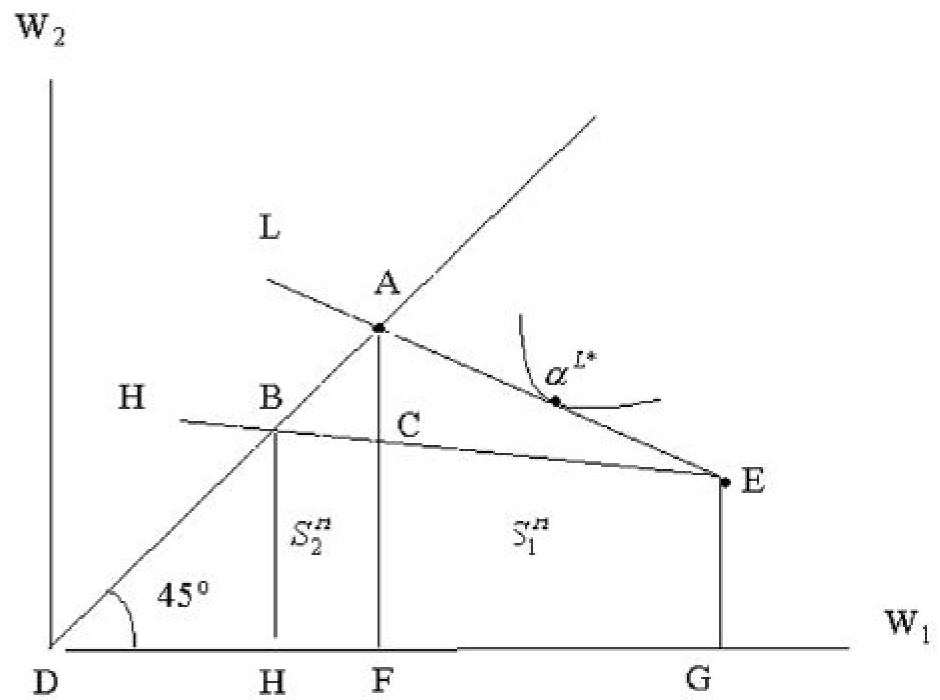

FIG. 7. – Low-risk types will not prefer high-risk optimal contract