Equilibrium portfolios with heterogeneous consumption externalities

Christian Gollier[1]
University of Toulouse (LEERNA, IDEI)

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Abstract

The present paper extends the Arrow-Debreu portfolio model to consumption externalities. It is assumed that each investor has a von Neumann-Morgenstern utility that is a function of her own consumption and of the average consumption in the group to which she belongs. Individual degrees of risk aversion and conformism are heterogeneous within each group and between the different groups in the economy. We show that, under some conditions on the degree of conformism in the economy, the optimal portfolio and consumption choices observed at equilibrium in each group with consumption externalities are equivalent to those that are optimal without any externality, but with an adjusted degree of risk aversion. If these conditions are not fulfilled, groups have no representative agent and the demand for pure zero-mean lotteries may be positive, thereby showing that not all diversifiable risks are washed away at equilibrium. We characterize the relationship between the distribution of conformism in the economy to the competitive allocation of risk and to the equity premium. We provide conditions for the two-fund separation property to hold.

Keywords: Consumption externalities, conformism, envy, portfolio choice, mutuality principle.

1 Introduction

A classical hypothesis is that agents are selfish in the sense that they care only about their own consumption. However, this hypothesis is challenged by introspection. For example, getting a rejection letter from an editor is harder to swallow in a good department with a high publication rate than in a department with a lower one. My young twin boys, Vincent and Simon, are much affected by the gifts received by their brother. It seems that Vincent being unlucky when Simon is unlucky has a smaller adverse effect on Vincent's own welfare than when Simon is lucky. This interdependence of preferences may be either genetic or generated by the environment. For example, it is often the case that mutual fund managers get bonuses that are made dependent to the performance of other fund managers (yardstick competition). In this paper, we examine how these interdependencies of preferences affect consumption and portfolio decisions together with asset prices.

The idea of comparing consumption is not new in economics. Carroll, Overland and Weil (1997) cited Adam Smith on this topic. But it is generally accepted that Veblen (1899) is the founder of this approach. Veblen stressed in particular that "the accepted standard of expenditure in the community or in the class to which a person belongs largely determines what his standard of living will be." Duesenberry (1949) examined consumption and saving decisions under this behavioral assumption. Van de Stadt, Kapteyn and van de Geer (1985) estimated a lifecycle consumption model in which both one's own consumption and the consumption of a reference group affect one's utility. They could not reject the hypothesis that utility is entirely relative. Kapteyn (2000) used consumption survey data to show that savings are lower if incomes in the reference group are higher.

We consider an economy which is partitioned in different groups of consumers. Groups are representative of households, village economies, social groups or nationalities. This partition is taken as given in this paper. The von Neumann Morgenstern utility function of an agent depends upon both his own consumption, and the average consumption in the group to which he belongs. We formally measure the degree of conformism of this agent by the increase in his consumption that leaves his marginal utility unchanged for a unit increase in the average consumption in his group.[1] My young twin boys

[1] Conformism should not be confused with envy. Envy, or jealousy, measures the effect

have a positive degree of conformism, i.e., they are conformist. When one of the two decides to spend his savings for something, it raises the willingness to consume of the other. Contrary to the existing literature, we do not restrict the functional form characterizing consumption externalities. Anticonformism is allowed. Moreover, we allow for heterogeneous degrees of risk aversion and conformism within each group and between different groups.

Our benchmark individual decision problem is a static Arrow-Debreu portfolio selection in an exchange economy. We assume that there is a single market for contingent claims where all members of all groups can either insure their individual risk and take a position on the undiversifiable risk in the economy. Each investor selects the portfolio that maximizes his expected utility. In this process, we assume that people form rational expectations about the optimal portfolio selected by others in their group. Because agents have heterogeneous preferences, equilibrium portfolios will usually not be limited to a proportional claim on the aggregate wealth, as in Gali (1994) or Campbell and Cochrane (1999). By enriching the set of assets to any contingent claim, we can explore richer portfolio strategies. In particular, this allows for examining portfolio diversification, the two-fund separation property, and portfolio insurance.

One of the contributions of this paper is to characterize the conditions under which individual and group demands for assets in this economy with consumption externalities can be duplicated by those coming from a classical economy with no externality, but with a cautiously selected distribution of individual degrees of risk aversion. From Constantinides (1982) and Wang (1996), these conditions imply the existence of a representative agent, either at the level of the group, and at the level of the entire economy. We link the degree of risk aversion of the representative agent to the distribution of the degrees of risk aversion and conformism in the corresponding group. Up to our knowledge, Chan and Kogan (2000) are the only authors to examine the problem of preferences aggregation in the presence of externalities, but the only source of heterogeneity in their model comes from differences in the (constant) relative risk aversion.

In section 3, we examine portfolio strategies in a specific reference group. We first show that if all consumers in the group have a degree of conformism less than unity, they never want to accept zero-mean gambles. In other words,

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of other's consumption on one's utility, not on one's marginal utility.

agents hedge all diversifiable risks. We show that, under this condition, there is a representative agent for this group whose attitude toward risk depends upon the distribution of conformism. More specifically, the degree of risk tolerance of the representative agent is an increasing function of the mean degree of conformism in the group. More conformist groups are less risk-averse. Because conformist consumers suffer less from a loss when others suffer the same loss, conformism induce more risk-taking.

When some agents in the reference group have a degree of conformism larger than unity, it may be an equilibrium strategy not to hedge diversifiable risk. People may intentionally accept second-degree stochastically dominated portfolios. This may explain for example why people do not diversify their portfolio internationally. Consumption externalities can thus explain the so-called international diversification puzzle (Baxter and Jermann (1997), Shore and White (2002)). We also show that investors with a large degree of conformism may find it optimal to be short in stock, in spite of the positive equity premium.

In section 4, we endogenize state prices by considering the economy as a whole. We derive the equilibrium allocation of the aggregate risk in the economy together with the equilibrium asset prices. We show how to derive the representative agent of the economy as a whole, together with the equity premium. We finally provide conditions for the two-fund separation property to hold in the economy. In section 5, we examine a consumption-saving model by using the fact that the static Arrow-Debreu portfolio problem is equivalent to a dynamic consumption-saving problem under certainty. This allows us to reinterpret our result to either live-cycle consumption behaviors or to the determinants of the risk free rate in the economy.

2 Description of the model

The economy consists of N groups of consumers. To keep the model simple, consumption externalities are assumed to prevail within each group, but not between groups. Agents have heterogeneous preferences that are indexed by $\theta \in \Theta$. Agents of type θ have a von Neumann-Morgenstern utility function $U(c, C, \theta)$, where c is the agent's consumption and C is the average consumption in her group.[2] We assume that U is twice differentiable with respect to (c, C) and that U_{11} is strictly negative: agents are risk-averse towards their own consumption when C is certain. Different groups may have different structures of their population. Let $\widetilde{\theta}_i$ denote the random variable characterizing the distribution of characteristics θ in group i, $i = 1, ..., N$. Finally, let η_i denote the proportion of agents of group i in the economy.

Except for consumption externalities, the model is a standard static Arrow-Debreu portfolio choice. There are two dates. Consumption takes place only at date 1. There is some uncertainty about the state s that will prevail at that date. Agents have common beliefs about $\widetilde{s}$, the random variable characterizing the state. Agents of type θ are endowed with a positive quantity $\omega(s, \theta)$ of the single consumption good in the economy. At date 0, there is a complete market of Arrow-Debreu securities to which all agents have access. The problem of agent θ in group i is to trade contingent assets in order to maximize her expected utility at date 1, given her expectation about the average consumption in her group:

$$c_i(., \theta) \in \arg\max_{c(.,\theta)} \quad EU(c(\tilde{s}, \theta), C_i(\tilde{s}), \theta) \tag{1}$$

$$s.t. \qquad E\pi(\tilde{s})\left[c(\tilde{s}, \theta) - \omega(\widetilde{s}, \theta)\right] = 0, \tag{2}$$

where $\pi(s)$ is the state price per unit of probability of state s. The first-order condition for problem (1) is written as

$$U_1(c_i(s, \theta), C_i(s), \theta) = \lambda_i(\theta)\pi(s) \qquad \forall s. \tag{3}$$

Because we assumed that U is concave in c, this condition combined with the budget constraint (2) is necessary and sufficient to characterize the optimal portfolio strategy, for a given distribution of $C_i(\tilde{s})$. We assume that a solution to this problem exists for all $\theta \in \Theta$ and all i when $C_i(.)$ is the equilibrium consumption plan in group i.

We assume rational expectation in each group. Thus, the equilibrium for group i requires that

$$C_i(s) = Ec_i(s, \widetilde{\theta}_i) \qquad \forall s, \tag{4}$$

[2] The fact that agent $\theta' s$ utility depends upon the distribution of other agents' consumption only through its mean is a standard simplification in the literature on internal and external habit formation. It is clear that this simplification is very helpful in our model.

where the expectation is taken with respect to $\widetilde{\theta}_i$, the random variable characterizing the distribution of types in group i. Solving the system of equations (2), (3) and (4) yields the equilibrium allocation of risk in group i, given the distribution of state prices $\pi(\widetilde{s})$.

This partial equilibrium model is solved in the next section. In the second part of the paper, we solve for the competitive equilibrium prices by adding the market clearing conditions

$$\sum_{i=1}^{N} \eta_i E\left[c_i(s,\widetilde{\theta}_i) - \omega(s,\widetilde{\theta}_i)\right] = 0 \qquad \forall s. \tag{5}$$

This can be rewritten as

$$\sum_{i=1}^{N} \eta_i C_i(s) = \sum_{i=1}^{N} \eta_i E\omega(s,\widetilde{\theta}_i) =_{def} \omega(s) \qquad \forall s, \tag{6}$$

where $\omega(s)$ is the average wealth in the economy in state s.

Let us go back to the characteristics of the consumption externalities embedded in the utility function U. In addition to the assumption that the utility is increasing and concave with respect to one's own consumption level, the model allows for either envy ($U_2 < 0$) or altruism ($U_2 > 0$). The agent whose own consumption is risk free is averse to the group's average risk if U is concave with respect to C. Finally, the supermodularity of U with respect to (c, C), i.e. $U_{12} > 0$, means that an increase in average consumption in the group increases the marginal utility of one's own consumption. Agents want to "keep up with the Joneses" in that case. Conformism is another word often used in this context. We can define a local index of conformism as follows:

$$\Gamma(c, C, \theta) = \left.\frac{dc}{dC}\right|_{U_1} = -\frac{U_{12}(c, C, \theta)}{U_{11}(c, C, \theta)}. \tag{7}$$

Γ is the increase in one's own consumption that leaves one's marginal utility unchanged after a unit increase of the mean consumption in the economy. We say that an agent is anti-conformist if Γ is uniformly negative. It is said to be conformist if Γ is strictly in between 0 and 1. Finally, he is "over-conformist" in the case where Γ is uniformly larger than 1.

The simplest case of consumption externalities is obtained with an additive specification: $U(c, C, \theta) = u(c - \theta C, \theta)$, as considered for example by

Campbell and Cochrane (1999) and Shore and White (2002). In this case, the degree of conformism $\Gamma(c, C, \theta)$ is equal a constant θ.[3] Following Gali (1994) and Carroll, Overland and Weil (1997) for example, we will often illustrate our results by using the special case of multiplicative externalities:

$$U(c, C, \theta) = u(cC^{-\theta}) \tag{8}$$

for the increasing and concave outer utility function $u(z) = z^{(1-\gamma)}/(1-\gamma)$. The parameter $\theta \geq 0$ is an index of the degree of consumption externalities. When $\theta = 0$, there is no externality, while $\theta = 1$ means that the agent focuses only on his consumption relative to the average consumption in the group. For $\theta \in [0, 1]$, both the absolute and the relative consumption matter as the argument of the outer utility is a geometric mean of the two. This specification yields

$$\Gamma(c, C, \theta) = \theta \frac{\gamma - 1}{\gamma} \frac{c}{C}. \tag{9}$$

Thus, agents are conformist ($\Gamma > 0$) if relative risk aversion is larger than unity. Under this condition, the degree of conformism is less than unity only if C/c is larger than $\theta(\gamma - 1)\gamma^{-1}$.

3 Portfolio management within a group

In this section, we consider a specific group i, and we take state prices as given. We characterize the demand for risky assets for each member of this group, and from the group as a whole. We address two different questions. First, we examine whether risk-averse agents want to fully insure against all risks when insurance prices are actuarially fair. Second, we determine the degree of risk aversion of the representative agent of the group, thereby characterizing the demand for risky assets by the group.

Before examining these questions, it is noteworthy that the existence of a solution to program (1) is not guaranteed for all $\theta \in \Theta$. For example, consider the additive case $U(c, C, \theta) = u(c - \theta C, \theta)$ where $\theta C(s)$ can be considered as

[3]Parameter θ can also be interpreted as a degree of envy, as $\theta = dc/dC \mid_U$ measures the compensation for a unit increase of average consumption in the group to preserve agent θ's utility. With additive externalities, there is no difference between the notions of envy and conformism.

the "minimum level of subsistence" in state s when $u_1(z,\theta)$ tends to infinity when z tends to zero. A solution to program (1) exists only when

$$E\pi(\widetilde{s})\omega(\widetilde{s},\theta) \geq \theta E\pi(\widetilde{s})C(\widetilde{s}),$$

i.e., when the market value of the endowment exceeds the market value of the endogenous minimum level of subsistence θC. When there is only one group ($N = 1$), condition (6) implies that $C(s) = \omega(s)$ for all s. The above condition puts an upper bound $E\pi(\widetilde{s})\omega(\widetilde{s},\theta)/\omega(\widetilde{s})$ to the degree of conformism θ. This upper bound equals unity in the egalitarian economy ($E\pi(\widetilde{s})\omega(\widetilde{s},\theta)$ independent of θ). In the other classical case with multiplicative externalities $U(c,C,\theta) = u(cC^{-\theta},\theta)$ and $\lim_{z\to 0} u_1(z,\theta) = +\infty$, it is easy to check that a solution to program (1) exists with $c(.,\theta) > 0$ whenever $C(.) > 0$.

3.1 Hedging and insurance demand under actuarially fair prices

Without consumption externalities, it is well-known that risk-averse agents fully insure risks when insurance is actuarially priced. This means that $c_i(s,\theta) = c_i(t,\theta)$ whenever $\pi(s) = \pi(t)$. Those agents that would not follow this rule would face a state-contingent consumption plan that would be second-order-stochastically dominated by those who fully insure the risk. Under risk aversion, this would be disliked. In other words, there is no demand for zero-mean lottery games in groups with no consumption externalities.

When consumption externalities are introduced, the full insurance strategy is still an equilibrium under actuarial pricing. This is straightforward to check: if everyone in group i believes that the others will purchase full insurance, this implies that $C_i(s) = C_i(t)$. In such a situation, the second variable in the U function can be treated as a non-stochastic parameter, and we are back to the classical case without any consumption externality. This implies that full insurance is optimal if the others purchase full insurance. Thus, full insurance is an equilibrium when asset prices are actuarially fair.

However, the existence of consumption externalities may yield multiple equilibria, potentially without full hedging by some members of the group. In the following Proposition, we show that this is not possible if all agents have a degree of conformism less than unity.

Proposition 1 *Suppose that the degree of conformism is uniformly less than unity within group i:* $\Gamma(c, C, \theta) < 1$ *, or* $U_{11}(c, C, \theta) + U_{12}(c, C, \theta) < 0$*, for all* (c, C) *and for all* θ *in the support of* $\widetilde{\theta}_i$*. Then, full insurance is the only equilibrium strategy in group i under fair prices:* $\pi(s) = \pi(t) \implies c_i(s, \theta) = c_i(t, \theta)$*, for all* θ*.*

Proof: If $\pi(s) = \pi(t) = \pi$, then , from condition (3), we must have that

$$U_1(c_i(s, \theta), C_i(s), \theta) = U_1(c_i(t, \theta), C_i(t), \theta) = \lambda_i(\theta)\pi \tag{10}$$

for all θ. Suppose that $C_i(s) \neq C_i(t)$. Define function $\psi(C, \theta)$ in such a way that $U_1(\psi(C, \theta), C, \theta) = \lambda_i(\theta)\pi$. Because, by assumption,

$$\left.\frac{d\psi}{dC}\right|_{U_1} = -\frac{U_{12}(\psi, C, \theta)}{U_{11}(\psi, C, \theta)} < 1 \tag{11}$$

for all relevant θ, we obtain that

$$\begin{aligned} c_i(s, \theta) - c_i(t, \theta) &= \psi(C(s), \theta) - \psi(C(t), \theta) \\ &= \int_{C_i(t)}^{C_i(s)} -\frac{U_{12}(\psi(C, \theta), C, \theta)}{U_{11}(\psi(C, \theta), C, \theta)} dC \\ &< C_i(s) - C_i(t). \end{aligned} \tag{12}$$

Taking the expectation with respect to $\widetilde{\theta}_i$ yields

$$C_i(s) - C_i(t) = E\left[c_i(s, \widetilde{\theta}) - c_i(t, \widetilde{\theta})\right] < C_i(s) - C_i(t), \tag{13}$$

a contradiction. We must thus have that $C_i(s) = C_i(t)$. Because U_1 is strictly decreasing in c, condition (10) yields the result. ■

When no agent is over-conformist, full insurance is the only equilibrium strategy under actuarial prices. Notice that it can be shown that the same result holds if all agents in the group are over-conformists. But this result does not need to hold when some agents have a low degree of conformism ($\Gamma < 1$), and others have a large degree of conformism ($\Gamma > 1$). In such a situation, there is still an equilibrium with full insurance. But other equilibria may exist. The intuition for this can be obtained as follows: suppose that there are some over-conformist agents in the group. Suppose also that a higher average consumption is expected in state s than in state t. It implies that over-conformist agents are confronted to the following dilemma. Because they are risk-averse, they dislike the risk they would take by not purchasing the same amount of contingent claims in the two states. But, because of their beliefs of a larger average consumption in the group in state s, they are more eager to consume in that state than in the other state t. It may be possible that this second effect dominates the risk aversion effect, thereby inducing them to take the risk. This would fulfill their initial beliefs of a larger average consumption in state s.

To illustrate, consider the following numerical example. Agents have constant relative risk aversion and multiplicative externalities: $U(c, C, \theta) = u(cC^{-\theta})$ and $u(z) = z^{(1-\gamma)}/(1-\gamma)$. We consider a group with two types of equal size. The first type is neutral, with $\theta_1 = 0$. The second type faces consumption externalities with $\theta_2 = 5$. There are two equally likely states, s and t, with $\pi(s) = \pi(t) = 1$. Finally, we assume that $\gamma = 2$ and $\omega(x, \theta) = 1$ for $x = s, t$ and $\theta = \theta_1, \theta_2$. Because of risk aversion, the neutral type does not take any risk at equilibrium, with $c(s, \theta_1) = c(t, \theta_1) = 1$. The optimality condition for type θ_2 is written as

$$c^{-2}\left[\frac{1+c}{2}\right]^5 = (2-c)^{-2}\left[\frac{3-c}{2}\right]^5, \tag{14}$$

where $c(s, \theta_2) = c$ and $c(t, \theta_2) = 2 - c$. We can check that equation (14) has three solutions. The first one is $c = 1$, which corresponds to the classical solution with full insurance. The other two symmetric solutions correspond to an equilibrium with risk-taking: $c(s, \theta_2) = 0.2357$ and $c(t, \theta_2) = 1.7643$. In spite of the aversion to risk on their own consumption, agents of type 2 are willing to take zero-mean risks at that alternative equilibrium. This is because they anticipate that others will do the same, with an average consumption in states s and t equaling respectively $C(s) = 0.6179$ and $C(t) = 1.3821$. Their risk aversion is dominated by their over-conformism, which induces them to consume much more in state t than in state s. They do not insure their risk in spite of the fair insurance pricing.

This example could explain the so-called "equity home bias", the tendency of investors to hold few international assets. This implies that their portfolio is second-order stochastically dominated by other more diversified portfolios. The fact that they do not hedge this zero-mean noises could be due to the existence of over-conformist agents in the country. Our explanation differs from the one given by Shore and White (2002), in spite of the fact that their story also relies on (additive) consumption externalities. In Shore and White (2002), there is no over-conformist agents but a subgroup of investors are forced to invest in a specific home biased portfolio. The other conformist investors then prefer to bias their own portfolio towards domestic assets.

3.2 The individual assets demand

From Proposition 1, we know that if the degree of conformism is uniformly less than unity, individual consumptions depend upon the state only through the corresponding state price. A standard result of the expected utility model without any consumption externality is that individual state consumptions are decreasing with the state price, under risk aversion. Moreover, as shown by Dybvig (1988), to any feasible consumption plan $c(.)$ that is nonincreasing in π, there exists an increasing and concave utility function for which this plan is optimal.[4] The derivative of c with respect to π, taken in absolute value, is a local measure of the demand for risky assets. The larger $|c'|$, the larger the sensitivity of individual consumption to changes in the state of nature. In a two-state model, $|c'|$ is proportional to the demand for equity. At the lower limit, when c is independent of π, the investor does not take any risk, i.e., he invests in a risk-free portfolio.

In this section, we examine the impact of conformism on the implicit degree of risk tolerance of the members of the group. Suppose that Γ is uniformly less than unity in group i. From Proposition 1, this implies that individual consumptions depend upon the state only through the state price π. Without loss of generality, let us number each state by the price of the Arrow-Debreu security associated to it: $\pi(s) = s$ for all s. Fully differentiating condition $U_1(c_i(\pi, \theta), C_i(\pi), \theta) = \lambda\pi$ and eliminating the Lagrangian multiplier λ yields

$$\frac{dc_i}{d\pi}(\pi, \theta) = -\frac{1}{\pi}T(c_i, C_i, \theta) + \Gamma(c_i, C_i, \theta)\frac{dC_i}{d\pi}, \tag{15}$$

[4] In an Arrow-Debreu framework, risk-loving EU-maximizers would select a corner solution for their portfolio, investing all their wealth in the Arrow-Debreu security associated with the smallest π. This implies that no EU consumer would ever select a portfolio c that is not decreasing in π.

where

$$T(c,C,\theta) = -\frac{U_1(c,C,\theta)}{U_{11}(c,C,\theta)}$$

is the Arrow-Pratt degree of absolute risk tolerance of agent of type θ (see Arrow (1965) and Pratt (1964)). Equation (15) is interesting on its own. Its left-hand side measures the risk borne by agent θ in group i. The first term in the right-hand side is the classical term that one would obtain without any consumption externality: $c'(\pi) = -T(c(\pi))/\pi$. In a classical group, the optimal risk borne by any of its members is proportional to her degree of absolute risk tolerance. The second term of the right-hand side of equation (15) takes into account of conformism. To measure it, we need to solve for $dC_i/d\pi$.

Taking the expectation of equality (15) with respect to the group distribution of characteristics $\widetilde{\theta}_i$ yields

$$E\left[\frac{dc_i}{d\pi}(\pi,\widetilde{\theta}_i)\right] = -\frac{1}{\pi}E\left[T(c_i,C_i,\widetilde{\theta}_i)\right] + \frac{dC_i}{d\pi}E\left[\Gamma(c_i,C_i,\widetilde{\theta}_i)\right], \qquad (16)$$

or, equivalently, since the left-hand side equals $dC_i/d\pi$ by (4),

$$\frac{dC_i}{d\pi} = -\frac{1}{\pi}\frac{E\left[T(c_i(\pi,\widetilde{\theta}_i),C_i(\pi),\widetilde{\theta}_i)\right]}{1-E\left[\Gamma(c_i(\pi,\widetilde{\theta}_i),C_i(\pi),\widetilde{\theta}_i)\right]}, \qquad (17)$$

If we assume that $U_{11}+U_{12}$ is negative, or $\Gamma < 1$, we obtain that C_i is decreasing in the state price. Combining this with equation (15) yields the following result.

Proposition 2 *Suppose that $0 \leq \Gamma(c,C,\theta) < 1$ for all (c,C) and for all θ in the support of $\widetilde{\theta}_i$. Then all individual consumptions in group i are decreasing in the state price per unit of probability.*

When Γ is uniformly between 0 and 1, i.e., when all consumers in group i are conformists, all agents in this group have a positive demand for risky assets ($dc_i/d\pi \leq 0$). In other words, they behave as if they would be classic expected-utility maximizers (Dybvig (1988)). We can evaluate their implicit degree of risk tolerance by combining equations (15) and (17) to obtain that

$$\frac{dc_i}{d\pi}(\pi,\theta) = -\frac{1}{\pi}\left[T(c_i,C_i,\theta) + \Gamma(c_i,C_i,\theta)\frac{E\left[T(c_i,C_i,\widetilde{\theta}_i)\right]}{1-E\left[\Gamma(c_i,C_i,\widetilde{\theta}_i)\right]}\right]. \tag{18}$$

By analogy, we see that the bracketed term in equation (18) can be interpreted as the implicit degree of risk tolerance of agent θ in group i, evaluated at $c_i(\pi,\theta)$. Notice that this equation does not provide a closed-form solution for optimal strategies, as it is in fact a system of differential equations that may be difficult to solve.

When Γ is not uniformly in $[0,1]$, some investors may prefer to purchase more consumption in more expensive states. Equivalently, they want to be short on equities, in spite of the positive equity premium. Let us illustrate this by considering again the group that we examined in the previous section. There are two subgroups of equal size where agents have a utility function of type (8), with $\gamma = 2$. The first type is neutral to externalities ($\theta = 0$) whereas the other group has a θ equaling 5. This implies that their degree of conformism is larger than unity in a wide range of the consumption ratio c/C. Let us consider the case where there are two equally likely states, $s = 1$ or 2, with $\pi(1) = 1$ and $\pi(2) = 2$. Finally, we assume that $\omega(s,\theta_1) = \omega(t,\theta_1) = 1$ and $\omega(s,\theta_2) = \omega(t,\theta_2) = 4$. The following table describes an equilibrium portfolio strategy.

	cheap state $\pi_1 = 1$	expensive state $\pi_2 = 2$
$\theta_1 = 0$ (neutral)	$c = 1.243$	$c = 0.879$
$\theta_2 = 5$ (over-conformist)	$c = 0.163$	$c = 5.919$

As expected, neutral agents consume more in the cheap state than in the expensive one. But over-conformists have a low consumption level in that state. At equilibrium, there exist unexploited feasible risk-sharing contracts that would induce a mean-preserving reduction in risk on consumption for the two types. Transferring consumption from neutral agents to over-conformists in state 1 with a symmetric transfer in state 2 would reduce all consumption risks in the sense of Rothschild and Stiglitz (1970).

This economy can be reinterpreted as one with a risk free asset and one risky asset whose rate of return in excess of the risk free rate is either -33%

or +67% with equal probabilities. At equilibrium, neutral agents optimally invest 36.4% of their wealth in the risky asset. But over-conformist agents prefer to be short on the risky asset, with a negative investment in the risky asset representing 143.9% of their wealth. In spite of the positive expected excess return of the risky asset, the risk-averse over-conformist agents have a negative demand for it. Again, this allocation cannot be an equilibrium in a classical economy with no externality, since in that economy, all investors would have a positive demand for the risky asset, independent of their degree of risk aversion.

The existence of multiple equilibria is a standard feature of rational expectation models. It is noteworthy that this numerical example provides an illustration of this phenomenon. Indeed, there is another equilibrium where the all over-conformist agents select a portfolio with $c = 2.582$ in the cheap state and $c = 4.709$ in the expensive state. These agents are short on equities, but less than in the equilibrium exhibited above. In fact, there is an infinite number of equilibria that are indexed by the proportion of over-conformist agents that take the shortest position.

3.3 Implicit agents' risk tolerance

In Proposition 2, we have seen that agents behave as if they would be neutral to externalities when Γ is uniformly in $[0, 1]$. It can be useful to estimate the implicit degree of risk tolerance of these agents. To keep things simple, let us assume that the support of $\widetilde{\pi}$ is in a small neighborhood of unity. Therefore, all $c_i(\pi, \theta)$ are in a small neighborhood of $z(\theta) = E\widetilde{\pi}\omega(\widetilde{\pi}, \theta)$. In other words, we are assuming here that optimal portfolio risks are small, and that the risk-free rate is zero. Using (18), it implies that we can measure the risk borne by agent θ as

$$\left| \frac{dc_i}{d\pi}(1, \theta) \right| = T_i(z(\theta), \theta), \tag{19}$$

where the implicit degree of risk tolerance $T_i(z, \theta)$ is defined by

$$T_i(z, \theta) = T(z(\theta), Ez(\widetilde{\theta}_i), \theta) + \Gamma(z(\theta), Ez(\widetilde{\theta}_i), \theta) \frac{E\left[T(z(\widetilde{\theta}_i), Ez(\widetilde{\theta}_i), \widetilde{\theta}_i)\right]}{1 - E\left[\Gamma(z(\widetilde{\theta}_i), Ez(\widetilde{\theta}_i), \widetilde{\theta}_i)\right]}. \tag{20}$$

This leads to the following Proposition.

Proposition 3 *Assume that Γ is uniformly in $[0,1]$ in group i and that the support of $\widetilde{\pi}$ is in a small neighborhood of unity. Then, everything else unchanged, the implicit risk tolerance of agent θ in this group is increasing in her degree of conformism, in the average degree of conformism in the group and in the average degree of absolute risk tolerance in the group.*

In other words, the equilibrium risk exposure is increasing in these three parameters. For conformist agents, part of the risk taken on one's own portfolio is offset by the portfolio risk taken by others. In bad states (large π), the marginal utility of wealth is reduced by the low average wealth that is expected in those states (U_{12} is positive). This induces conformist agents to reduce their already low demand for the contingent claims associated to these states. Seen from ex-ante, this means taking more risk. This is as if people would be more risk-tolerant. Agents with a larger degree of conformism react more in this way, thereby explaining why conformism and risk tolerance go together.

The provision "everything else unchanged" is important in the above proposition. For example, an increase in conformism raises implicit risk aversion T_i only if the absolute risk tolerance T is maintained unchanged. This comparative statics exercise may be difficult for the classical specification of consumption externalities that are used in the literature. To illustrate, let us consider the additive/CRRA case $U(c,C,\theta) = u(c - \theta C)$ and $u(w) = w^{1-\gamma}/(1-\gamma)$. It yields $T(c,C,\theta) = (c-\theta C)/\gamma$ and $\Gamma(c,C,\theta) = \theta$. Suppose that all agents have the same γ and the same $z(\theta) = z$, but different θ. Equation (20) yields

$$T_i(z,\theta) = z\frac{1-\theta}{\gamma} + \theta\frac{z\frac{1-\overline{\theta}}{\gamma}}{1-\overline{\theta}} = \frac{z}{\gamma}.$$

An increase in the degree of conformism $\theta = \Gamma$ has no effect on the optimal risk taking in this case. This is because an increase in θ also reduces absolute risk tolerance T. This reduction exactly counterbalances the effect discussed in the above proposition.

We can alternatively examine the multiplicative/CRRA case with $U(c,C,\theta) = (cC^{-\theta})^{1-\gamma}/(1-\gamma)$ for which $T(c,C,\theta) = c/\gamma$ and $\Gamma(c,C,\theta) = \theta(\gamma-1)c/\gamma C$. In this case, we obtain that

$$T_i(c,C,\theta) = \frac{z}{\gamma}\frac{\gamma + (\theta - \overline{\theta})(\gamma-1)}{\gamma - \overline{\theta}(\gamma-1)}.$$

Assuming that $\overline{\theta}$ is in $[0, 1]$, the denominator of the second ratio is positive and the absolute risk tolerance T_i is increasing in Γ which is proportional to $\theta(\gamma - 1)$.

An interesting point that can be observed by looking at equations (17), (18) or (20) is that the degree of conformism, as the degree of absolute risk tolerance (Wilson (1968)), can be averaged up in a complete market framework. From (17), both an increase in the average degree of conformism and an increase in the average absolute risk tolerance in the group raise the riskiness of the average portfolio in the group. The offsetting effect of the externality is thus larger in those groups with a larger $E\Gamma$ or ET. Thus the individual demands for risk are larger in these groups.

3.4 The representative agent of the group

From equation (17), we directly get the following result.

Proposition 4 *If $\Gamma(c, C, \theta) < 1$ for all (c, C) and for all θ in the support of $\widetilde{\theta}_i$, then the average consumption in group i is decreasing in the state price.*

When Γ is uniformly less than unity, but not necessarily positive for all members of the group, it may be possible for some anti-conformist consumers to have a negative demand for risky assets ($dc_i/d\pi > 0$). However, the above Proposition states that the average consumption in the group must be decreasing with respect to the state price. In other words, there must exist an externality-free representative agent for the group. The average demand $C_i(.)$ for contingent assets in group i must be the same than the one from a classical agent with a concave utility function v_i endowed with the average wealth in that group $\omega_i(\pi) = E\omega(\pi, \widetilde{\theta}_i)$ for all π. Using equation (17), the degree of absolute risk tolerance of the group i's representative agent would equal

$$T_{v_i}(C_i(\pi)) = \frac{E\left[T(c_i(\pi, \widetilde{\theta}_i), C_i(\pi), \widetilde{\theta}_i)\right]}{1 - E\left[\Gamma(c_i(\pi, \widetilde{\theta}_i), C_i(\pi), \widetilde{\theta}_i)\right]} \tag{21}$$

in state π. This formula extends Wilson's well-known property that, in a classical economy with complete markets, the absolute risk tolerance of the representative agent is just the mean of the absolute risk tolerances of the investors in the corresponding state at the competitive equilibrium. Again, condition (21) is not a closed-form solution since it requires solving the set of differential equations (18). But the same trick can be used as above. If we assume that the support of $\widetilde{\pi}$ is in a small neighborhood of unity, and if all agents in the group have the same initial wealth z, the degree of absolute risk tolerance of the representative agent evaluated at z equals

$$T_{v_i}(z) = \frac{E\left[T(z, z, \widetilde{\theta}_i)\right]}{1 - E\left[\Gamma(z, z, \widetilde{\theta}_i)\right]}. \tag{22}$$

We see that the risk tolerance of the group is increasing in the average degree of conformism in that group. The intuition is immediate from the previous section.

4 General equilibrium

In the previous section, we were mainly interested in individual portfolio decisions within a specific group. We now aggregate these individual demands for risky assets in order to determine equilibrium prices and the equilibrium allocation of risk. If the degrees of conformism is uniformly less than unity in the population, we have seen that to each group i, there is an equivalent increasing and concave utility function v_i that "represents" it. Thus, the problem is to find the competitive price kernel $\pi(\tilde{s})$ such that $\sum_{i=1}^{N} \eta_i C_i(s) = \omega(s)$ for all s, where $C_i(.)$ is the solution of the following program:

$$\max_{C(.)} Ev_i(C(\tilde{s})) \qquad s.t. \quad E\pi(\tilde{s})C(\tilde{s}) = E\pi(\tilde{s})\omega_i(\tilde{s}). \tag{23}$$

This problem is nothing else than the standard Arrow-Debreu model where groups would be taken as the elementary agents.

4.1 The mutuality principle

Wilson (1968) examines the properties of the equilibrium allocation of risk in an economy without any consumption externality. An important property of this equilibrium is the so-called mutuality principle. It states that if there are two states with the same stock of the good in the economy, each agent consumes the same amount of it in these two states: $\omega(s) = \omega(t)$ implies

$c_i(s, \theta) = c_i(t, \theta)$ for all θ and i. Under the mutuality principle, all resources are mutualized in a common pool, and the allocation of consumption in a given state depends only upon what is available in the pool in that state, not upon the distribution of the contributions to the pool in that state.

Proposition 5 *The mutuality principle holds in an economy when $\Gamma(c, C, \theta)$ is uniformly less than unity: the competitive allocation of consumption in the economy is the same in any pair of states (s, t) with the same average wealth. Moreover, the state prices will be the same: $\omega(s) = \omega(t)$ implies $\pi(s) = \pi(t)$.*

Proof: When Γ is less than unity, v_i is concave, and the competition between groups can be analyzed as a standard Arrow-Debreu equilibrium problem for which we know that the mutuality principle holds.[5] This implies that if $\omega(s) = \omega(t)$, than $C_i(s) = C_i(t)$ for all i and $\pi(s) = \pi(t)$. Using Proposition 1, it implies that $c_i(s, \theta) = c_i(t, \theta)$ for all θ and i. ■

All states with the same average wealth leads to the same allocation of consumption, and to the same equilibrium state price. In particular, if the average wealth is state independent, the competitive allocation is such that all agents purchase full insurance against their own risk. More generally, the above Proposition means that all diversifiable risks are washed away at equilibrium. We conclude that the price kernel and individual consumptions are single-valued functions of the state-contingent average wealth in the economy. Without loss of generality, we hereafter rank states by the average wealth available in these states: $s = \omega(s)$ for all s. In other words, state prices and individual consumptions depend upon the states only through the average wealth available in the corresponding state: $\pi = \pi(\omega)$ and $c_i = c_i(\omega, \theta)$.

4.2 The equity premium

The sensitivity of the state price to changes in the average wealth measures the equilibrium price of risk. Those who accept to reduce their consumption in poorer states get a premium in terms of expected consumption level that is increasing in $\mid d\pi/d\omega \mid$. How large is this risk premium at equilibrium? The standard method to answer this question relies on the notion of the representative agent. Imagine an economy where all agents are identical for their preferences and for their state-contingent endowment $\widetilde{\omega}$. Moreover, assume

[5] See for example Gollier (2001).

that they are indifferent to the consumption of others. The utility function on their consumption is denoted $V(.)$. The equilibrium price kernel $\pi(.)$ in this homogeneous economy without consumption externality is fully described by condition $V'(c(\omega)) = \lambda\pi(\omega)$. Given the market-clearing conditions $c(\omega) = \omega$ for all ω, we get the equilibrium condition $\pi(\omega) = \lambda^{-1}V'(\omega)$. The sensitivity of the price kernel to changes in C is thus measured by

$$-\frac{\pi'(\omega)}{\pi(\omega)} = -\frac{V''(\omega)}{V'(\omega)} = [T_V(\omega)]^{-1}. \tag{24}$$

Consider an economy with a small macroeconomic risk $\widetilde{\omega} = z + \widetilde{\varepsilon}$, with $E\widetilde{\varepsilon} = 0$. The price of equity, i.e. the price of the promise to get the per capita wealth $\widetilde{\omega}$ in each state, equals $P_s = E\widetilde{\omega}\pi(\widetilde{\omega})$, which is approximately equal to $z[\pi(z) + \sigma^2_{\widetilde{\varepsilon}}\pi'(z)]$. The price of bonds, i.e. the value of a sure endowment z, equals $P_b = Ez\pi(\widetilde{\omega}) \simeq z\pi(z) > P_s$. Using these approximations together with condition (24) implies that the risk premium equals

$$\frac{P_b - P_s}{P_b} \simeq \frac{\sigma^2_{\widetilde{\varepsilon}}}{T_V(z)} \tag{25}$$

The smaller the degree of risk tolerance, the larger the sensitivity of prices to changes in ω and the larger the risk premium. Suppose now that the price kernel obtained in this homogeneous economy be the same than the one of an heterogeneous economy with consumption externalities. Then, we say that the neutral agent V is representative of this heterogeneous economy. In particular, the concavity of V characterizes the degree of risk aversion of this Society with externalities.

Corollary 1 *Suppose that $\Gamma(c, C, \theta)$ is uniformly less than unity in the economy. Then, there exists a risk-averse representative agent of this economy whose local degree of risk tolerance is measured by*

$$T_V(\omega) = \frac{1}{N}\sum_{i=1}^{N} \frac{E\left[T(c_i(\omega, \widetilde{\theta}_i), C_i(\omega), \widetilde{\theta}_i)\right]}{1 - E\left[\Gamma(c_i(\omega, \widetilde{\theta}_i), C_i(\omega), \widetilde{\theta}_i)\right]} \tag{26}$$

Proof: Property (26) is obtained by applying equation (21) twice. We first use it for the economy with the N agents representative of their corresponding group. Because there is no consumption externality among them, applying condition (21) to this economy yields

$$T_V(\omega) = \frac{1}{N}\sum_{i=1}^{N} T_{v_i}(C_i(\omega)), \tag{27}$$

where $T_{v_i}(.)$ is the absolute risk tolerance of the representative agent of group i. Replacing T_{v_i} by its expression in (21) concludes the proof.■

Observe that we get back the original formula of Wilson (1968) when there is no externality in the economy. Indeed, when $\Gamma \equiv 0$, the market's absolute risk tolerance equals the average degree of absolute risk tolerance of the investors.

Equation (26) tells us how to aggregate preferences of the different groups in the economy. Conformism raises the degree of absolute risk tolerance in Society. It reduces the equity premium and it raises the Society's willingness to take risk. The intuition is that for conformist agents, the risk exposure of the others plays the role of insurance of one's own risk exposure. In the bad states where the prices of contingent claims are high, the average wealth is small and the marginal utility of consumption is smaller due to the consumption externality. This helps investors accepting to sacrifice consumption in these states. It is noteworthy that the degree of risk tolerance in the economy depends upon the individual degrees of conformism in a non-linear way. This is only when there is a single group in Society that it depends upon the mean degree of conformism.

4.3 The competitive allocation of risks

We can now turn to the characterization of competitive allocations of risk in the economy with consumption externalities. Combining equations (18), (24) and (26) yields

$$\frac{dc_i}{d\omega}(\omega,\theta) = \frac{dc_i}{d\pi}\frac{d\pi}{d\omega} = \frac{T(c_i,C_i,\theta) + \Gamma(c_i,C_i,\theta)\dfrac{E\left[T(c_i,C_i,\widetilde{\theta}_i)\right]}{1-E\left[\Gamma(c_i,C_i,\widetilde{\theta}_i)\right]}}{\dfrac{1}{N}\sum_{j=1}^{N}\dfrac{E\left[T(c_j,C_j,\widetilde{\theta}_j)\right]}{1-E\left[\Gamma(c_j,C_j,\widetilde{\theta}_j)\right]}}. \tag{28}$$

The derivative of c_i with respect to ω measures the size of risk borne by each individual at equilibrium. It must equal unity on average. In the absence of any consumption externality in the economy, the right-hand side of this complex equality simplifies to just $T(c_i,\theta)/ET(c,\widetilde{\theta})$. Agents with a degree of risk tolerance larger (smaller) than average bear a share of the macroeconomic risk larger (smaller) than average. Equation (28) extends this property to the case of consumption externalities. It is easy to interpret when there is only one group in the economy, in which case it simplifies to

$$\frac{dc}{d\omega}(\omega,\theta)=\frac{T(c,\omega,\theta)}{ET(c,\omega,\widetilde{\theta})}+\Gamma(c,\omega,\theta)-E\Gamma(c,\omega,\widetilde{\theta})\frac{T(c,\omega,\theta)}{ET(c,\omega,\widetilde{\theta})}. \tag{29}$$

Solving this system of differential equations yields the set of Pareto efficient allocations of risk in the economy. Any such solution that satisfies the set of individual budget constraints is a competitive equilibrium.

The first term in the right-hand side of this equality is the classical one (Wilson (1968)). The effect of consumption externalities appears in the second and third terms. The second term measures how agent θ's degree of conformism affects his own equilibrium risk exposure. We see that an increase in the conformism of an agent increases his equilibrium risk exposure. The reason is now well understood: an increase in conformism makes the other's risk exposure a better insurance against agent θ's own risk. He is therefore more risk prone. The third term determines the effect of the other's degree of conformism on agent θ's equilibrium risk exposure: this effect is negative. This is because the increase on conformism in the economy reduces the equilibrium price of risk. It affects negatively the individual demand for risk.

Going back to the more general condition (28), we can also compare the share of the undiversifiable risk taken by two agents with the same positive degree of conformism living in two different groups. If the two groups have the same mean degree of risk tolerance, the agent living in the group with a larger mean degree of conformism bears a larger share of the social risk. The intuition is that this agent faces peers that take more risk on average, inducing him to be more risk-prone.

4.4 Two-fund separation and portfolio insurance

Cass and Stiglitz (1970) characterized the conditions under which individual equilibrium consumptions are linear in the average wealth in the economy, when there is no externality. Linearity is very useful, since it means that this equilibrium can be attained with just one risk free asset (in zero net supply) and another fund gathering all state contingent assets ("the market"). Cass and Stiglitz proved that this two-fund separation property holds only if all agents have an absolute risk tolerance that varies linearly with wealth, and with the same slope: $T(c, \theta) = t(\theta) + c/\gamma$. Preferences are HARA with identical shape parameter (ISHARA).[6] When this property does not hold, investors select at equilibrium a portfolio which is nonlinear in average wealth ω. Such a portfolio can be obtained by purchasing the two funds, plus call and put options on the risky fund. Purchasing a put option is like buying insurance against the underlying asset, i.e., this is portfolio insurance. Leland (1980) examines the problem of who should buy portfolio insurance at equilibrium. This is equivalent to determining the conditions under which an individual equilibrium consumption is a concave function of ω. In this section, we reexamine these questions when consumption externalities are present.

The starting point is equation (28) which measures the slope of function $c(., \theta)$. Determining whether c is concave, linear, or convex in ω would require the full differentiation of this equation. This exercise does not yield any useful and interpretable result. This is why we hereafter limit our analysis to the special case of a single group ($N = 1$) with additive consumption externalities:[7]

$$U(c, C, \theta) = u(c - \theta C, \theta). \tag{31}$$

It yields a constant degree of conformism $\Gamma(c, C, \theta) = \theta$.[8] This allows us to

[6]For these ISHARA preferences, the relevant domain of consumption is $[-\gamma t(\theta), +\infty[$ when $\gamma > 0$, and $]-\infty, -\gamma t(\theta)]$ when γ is negative (as is the case for the quadratic utility function for example).

[7]The case of multiplicative externalities is more complex. In the special case where $U(c, C, \theta) = (cC^{-\theta})^{1-\gamma}/(1-\gamma)$ as in Gali (1994), condition (29) yields

$$\frac{dc}{d\omega} = \frac{c}{\omega}\left[1 + \frac{\gamma - 1}{\gamma}\left(\theta - \frac{E\widetilde{\theta c}}{\omega}\right)\right]. \tag{30}$$

When there is some heterogeneity in the degree of conformism, the bracketted term is not constant with respect to ω, and the two-fund separation property does not hold.

[8]Parameter θ can also be interpreted as a degree of envy, as $\theta = dc/dC\mid_U$ measures the compensation for a unit increase of average consumption in the group to preserve agent θ's utility. With additive externalities, there is no difference between the notions of envy

simplify condition (29) which can be rewritten as

$$\frac{dc}{d\omega}(\omega,\theta) = \left[1-\overline{\theta}\right]\frac{T(c(\omega,\theta)-\theta\omega,\theta)}{ET(c(\omega,\widetilde{\theta})-\widetilde{\theta}\omega,\widetilde{\theta})}+\theta, \tag{32}$$

where $\overline{\theta} = E\widetilde{\theta}$ is the average degree of conformism in the economy. Fully differentiating with respect to ω yields

$$\frac{\left[ET(c-\widetilde{\theta}\omega,\widetilde{\theta})\right]^2}{1-\overline{\theta}}\frac{d^2c}{d\omega^2} = T'(c-\theta\omega,\theta)\left[\frac{dc}{d\omega}-\theta\right]ET(c-\widetilde{\theta}\omega,\widetilde{\theta}) \tag{33}$$

$$-T(c-\theta\omega,\theta)E\left[T'(c-\widetilde{\theta}\omega,\widetilde{\theta})\left[\frac{dc}{d\omega}-\widetilde{\theta}\right]\right]. \tag{34}$$

It can be simplified to

$$\frac{d^2c}{d\omega^2}(\omega,\theta) = \frac{\left[1-\overline{\theta}\right]^2 T(c-\theta\omega,\theta)}{\left[ET(c-\widetilde{\theta}\omega,\widetilde{\theta})\right]^2}\left[T'(c-\theta\omega,\theta)-\frac{ET'(c-\widetilde{\theta}\omega,\widetilde{\theta})T(c-\widetilde{\theta}\omega,\widetilde{\theta})}{ET(c-\widetilde{\theta}\omega,\widetilde{\theta})}\right]. \tag{35}$$

We conclude that the equilibrium allocation of risk is linear in ω if and only if

$$T'(c(\omega,\theta)-\theta\omega,\theta) = \frac{E\left[T'(c(\omega,\widetilde{\theta})-\widetilde{\theta}\omega,\widetilde{\theta})T(c(\omega,\widetilde{\theta})-\widetilde{\theta}\omega,\widetilde{\theta})\right]}{ET(c(\omega,\widetilde{\theta})-\widetilde{\theta}\omega,\widetilde{\theta})} \tag{36}$$

for all ω and θ. This means that the derivative of absolute risk tolerance with respect to the consumption net of the externality must be independent of the agent's type θ and of average wealth ω. Therefore, the utility function u must be ISHARA, as in the classical case. In the ISHARA case, the two-fund separation property holds, independent of the distribution of $\widetilde{\theta}$. In such a situation, the complete market framework is unnecessary to obtain the first-best allocation of risk. A risk free asset and a fund representing the market are enough. There is no demand for portfolio insurance. The following proposition summarizes our finding in the additive case.

and conformism.

Proposition 6 *Consider the additive externality case (31). The two-fund separation property holds in this economy if preferences are ISHARA, i.e., if* $\exists t(.) : \Theta \to R$, $\exists \gamma \in R : T(c, \theta) = t(\theta) + c/\gamma$ *for all* $\theta \in \Theta$ *and all* c *in the domain of* u. *When this condition is not satisfied, there exists at least one vector* $(\widetilde{\theta}, \omega(\widetilde{s}, \widetilde{\theta}))$ *such that the equilibrium sharing of the macroeconomic risk* $\widetilde{\omega}$ *is nonlinear.*

Two special cases are useful to consider. In the first case, all individual utility functions are exponential, where $T(c, \theta) = t(\theta)$ is independent of c. The sensitivity of agent $\theta' s$ consumption to a change in the average wealth is then given by

$$\frac{dc}{d\omega}(\omega, \theta) = \left[1 - \overline{\theta}\right] \frac{t(\theta)}{Et(\widetilde{\theta})} + \theta, \tag{37}$$

which is indeed independent of ω. We see that the individual equilibrium exposure to the macroeconomic risk is increasing both in the degree of conformism of the agent and to the degree of absolute risk tolerance (if $\overline{\theta}$ is less than unity). This is combined with a lump-sum transfer $c_0(\theta)$ that is determined in a way to satisfy the individual budget constraint.

The second interesting case is when all agents have the same power utility function $U(c, C, \theta) = (c - \theta C)^{1-\gamma}/(1 - \gamma)$. In this second case, the only accepted source of heterogeneity is for the degree of conformism θ. In that case, it is easy to solve the set of differential equations (29) to prove that the equilibrium allocation of risk is just proportional: $c(\omega, \theta) = k(\theta)\omega$, where $k(\theta)$ is selected in order to satisfy the budget constraint of agent θ. Each agent gets a fixed proportion of the aggregate wealth in the economy at equilibrium. Interestingly enough, this proportion is independent of the degree of conformism in the economy.

When the set of utility function is not ISHARA, some agents will sell portfolio insurance to others. As in Leland (1980), those who will buy portfolio insurance are those whose sensitivity of the absolute risk tolerance to changes in net consumption is smaller than average ($ET'T/ET$). Indeed, equation (35) tells us that c is concave in ω in that case.

4.5 Numerical illustration with multiplicative over-conformism

In the following numerical example, we examine the case of multiplicative externalities and ISHARA preferences. We show that the equilibrium shar-

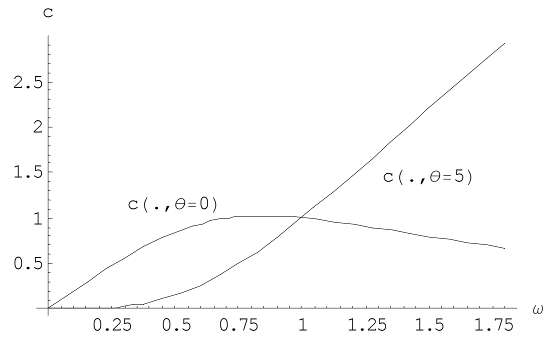

Figure 1: The equilibrium allocation of risk.

ing rules is nonlinear, and that some consumption plans can even be locally decreasing in the aggregate wealth for over-conformist agents. As in the previous numerical illustrations, there is a single group with two types of agents. The two types have the same size. Consumption externalities are multiplicative as stipulated in equation (8), and relative risk aversion is constant and equal to two. The first type is neutral to externalities ($\theta = 0$) whereas the other type exhibits $\theta = 5$, which yields over-conformism over a wide range of relative consumption. We consider a competitive allocation of risk where the Lagrangian multipliers associated to the budget constraint is equal to unity for both agents. There is a distribution of initial wealth that sustains such an equilibrium. This allocation is described in Figure 1. Observe that individual consumptions are increasing with the state wealth in poor states. But for larger wealth levels, over-conformist agents want to outperform the market so much that each dollar increase of average wealth in the economy yields an increase of their consumption by more than two dollars.[9] That is possible only because neutral agents accept to reduce their consumption in these states. They accept this because, under the pressure of the demand by over-conformists, the equilibrium state prices are increasing in the average wealth in this region. This is seen in Figure 2 where we have drawn the state prices as a function of average wealth.

We see in Figure 2 that the state price as a function of the average wealth

[9]The convexity of $c(., \theta = 5)$ shows that overconformist agents purchase call options on the aggregate wealth of the economy.

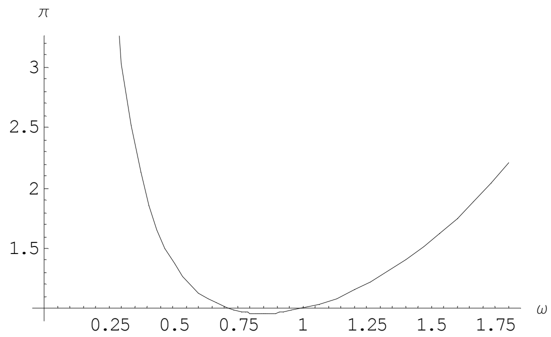

Figure 2: The equilibrium price kernel.

in the economy is single valued. However, because it is not a monotone function of it, its inverse is not single valued. This observation provides new insights on portfolio decisions that we presented earlier. In Figure 3, we draw the optimal portfolios of the two agents. Consider an economy with two possible average wealth in the economy. As seen in Figure 2, it may be possible that the same state price be associated to these two different states. This is due to the fact that over-conformist agents do not insure this risk in spite of the fair pricing. Over-conformist agents may opt for one branch or the other of their C-shaped optimal portfolio structure in Figure 3, or to any mixture of them as long as their budget constraint is satisfied. This is only if they follow the lower branch of the curve that the equilibrium exhibits the same characteristics of a classical economy without any externality.

5 Another interpretation of the model: the consumption-saving problem

These results can be reinterpreted in the framework of the consumption-saving problem. Consider an economy where people live from date $t = 0$ to $t = T$. The felicity of agent θ in group i is a function $U(c, C_i, \theta)$ of his own consumption c, and of the average consumption C_i in the group. In this framework, the concavity of U with respect to c or C is representative of preferences for the smoothing of the corresponding variable through time.

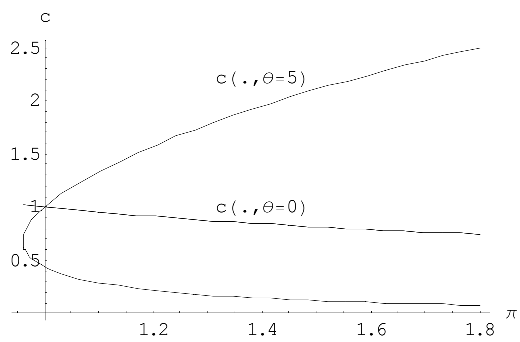

Figure 3: Equilibrium demands for contingent claims.

Agent θ has an endowment $\omega(t,\theta)$ of the consumption good at date t. Let $\rho(t)$ denote the gross rate of interest at date 0 for a zero-coupon bond maturing at t. Agent θ in group i selects the consumption plan $\{c(t,\theta)\}_{t=1,\ldots,T}$ which maximizes the discounted value of his felicity, given his expectation about the average consumption plan in the group:

$$\max_{c_1,\ldots,c_T} \quad \sum_{t=0}^{T} \beta^t U(c_t, C_i(t), \theta) \tag{38}$$

$$s.t. \quad \sum_{t=0}^{T} \frac{c_t - \omega(t,\theta)}{(\rho(t))^t} = 0, \tag{39}$$

where β is the discount factor. An allocation $(\rho(.), c_1(.,.), \ldots, c_N(.,.))$ is an equilibrium if

1. $c_i(.,.)$ is a solution of the above program given prices $\rho(.)$ and beliefs $C_i(.)$ about what the others in the group will consume over time, for all i and θ;

2. there is rational expectation: $C_i(t) = Ec_i(t,\widetilde{\theta}_i)$ for all t and i;

3. credit markets clear: $\sum_i \eta_i E\omega(t,\widetilde{\theta}_i) = \sum_i \eta_i c_i(t,\widetilde{\theta}_i)$ for all t.

Kapteyn (2000) provides an analytical solution to this problem with exogenous interest rates when U is quadratic in (c, C), in the case of two consumers and two dates. Observe now that problem (38) (39) is formally equivalent to the one that we examined earlier in this paper. Consider a discrete random variable $\tilde{s}$ with support $\{0, 1, ..., T\}$. The probability of $\tilde{s} = t$ is $p_t = \beta^t / b$ with $b = \sum_{t=0}^{T} \beta^t$. Let also $\pi(t)$ be equal to $b(\beta\rho_t)^{-t}$. For this specification of $\tilde{s}$ and $\pi(.)$, program (1) (2) is equivalent to (38) (39). We can thus use the results presented in this paper to characterize the equilibrium consumption and saving decision in each group.

Let us limit our discussion to the problem of a specific group that take prices $\rho(.) = \rho$ as given. Without consumption externalities, the characteristics of the optimal consumption path are well-known: If $\beta\rho = 1$, i.e., if the risk free rate of the economy equals the rate of pure preference for the present, the optimal consumption is constant through time under the assumption that $U_{11} < 0$. More generally, consumption is increasing (decreasing) with time if $\beta\rho$ is larger (less) than unity. Finally, under the condition that $\beta\rho > 1$, an increase in the concavity of U increases initial consumption $c(0, \theta)$. From the above-mentioned analogy, we immediately obtain the following results:

1. Under the assumption that the degree of conformism is less than unity, all equilibrium consumptions are constant through time if $\beta\rho = 1$.

2. Suppose that $\beta\rho$ is larger than unity. Under the assumption that the degree of conformism is less than unity, the average consumption in the economy is increasing with time. If in addition Γ is positive, then all individual consumptions are increasing with time. An increase in the average degree of conformism in the group raises its initial saving.

If we go to the price formation, we derive from our earlier results that, in a growing economy, the equilibrium risk free rate is negatively affected by the average degree of conformism in the economy. Finally, it is straightforward to combine the saving problem and the portfolio problem into a single dynamic saving-portfolio problem à la Arrow-Debreu, at least in the finite horizon case. This dynamic version of the model presented in this paper is useful to discuss the combined puzzles of the equity premium and of the riskfree rate, as first stated by Weil (1989). The riskfree rate puzzle comes from the fact that a very low aversion to consumption fluctuations is required to explain the low historical riskfree rate in the U.S. economy, under a reasonable rate of pure preference for the present. Weil shows that the equity premium and the riskfree rate observed during the past century can be explained only if consumers have strong preferences for the *future*. In an economy with no uncertainty on growth, conformism raises the riskfree rate and it can thus contribute to explaining the low interest rate. But it also reduces the equity premium. Our results are thus not sufficient to determine the effect of conformism on the rate of pure preferences for the present that is compatible with the data.

It should be noticed here that our extension to a dynamic framework with external habit formation does not include a recent trend of the literature in which external and internal habit formation are mixed.[10] Habit formation is internal when C is a (discounted) average of the individual's own consumption in the past. The loss of time separability of individual objective functions raises specific questions that are outside the scope of this paper.

6 Concluding remark

When individual consumption must be increased by less than the increase in per capita consumption to maintain marginal utility unchanged, equilibrium saving and portfolio behaviors in an economy with consumption externalities are observationally equivalent to the behaviors of externality-free agents with an adjusted degree of risk aversion. An important consequence of this result is that the measurement of the risk aversion is context-dependent. With consumption externalities, it is not irrational for an expected-utility maximizer to be very risk-averse towards an individual-specific risk, and very risk-tolerant towards market risks. Conformist agents want to be lucky when others are lucky. If offered the same risk opportunities, they are more willing to take these risks. More conformist individuals and groups are less risk-averse.

The dependence of the attitude towards risk to the context creates a measurement problem. The standard experiment to measure risk aversion of subjects is based on the elicitation of the certainty-equivalent of a *subject-specific* lottery $\widetilde{x}$, and that is fine to measure the subject's degree of risk aversion towards his own consumption risk. If, on the contrary, all closely

[10]See for example Campbell and Cochrane (1999).

related subjects are offered to participate to the *same* lottery $\widetilde{x}$, the certainty equivalent will not be a good measurement of this. Similarly, the observation of the demand for insurance and of the structure of households portfolios will be affected by such externalities, and they will not allow to measure the degree of risk aversion to households' consumption risks. This could explain why the large literature on measuring risk aversion has been so disappointing in the past, by offering point estimates that are so distant from each others.

Consumption externalities could also explain why people purchase lottery tickets and why they do not diversify their portfolio internationally. If some members of the reference group have a degree of conformism that is larger than unity, accepting zero-mean risks may be an equilibrium strategy in this group. We also showed that some members in this group may decide optimally to be short on stocks, in spite of their risk aversion and of the positive equity premium. These behaviors cannot be explained by the classical expected utility theory.

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Comments on referee 1's report

I thank referee 1 for the time spent in reading and commenting my paper. Here are some comments on his/her helpful report:

I agree that the extension to a dynamic version of the model, in particular when the time horizon is infinite and if we follow the recent trend of the literature in which internal and external habit formation are mixed. I am now more cautious in section 5 about this extension.

1. In the new version of the paper, I address the problem of the existence of a solution to program (1) in the beginning of section 3, at least in the classical special cases of multiplicative and additive externalities.

2. I tried to clarify the results on the two-fund separation property. In the additive case, I summarize the results in Proposition 6. I make clear in particular that if the ISHARA condition is not satisfied, I can always exhibit an economy with a nonlinear equilibrium sharing of the macroeconomic risk. To answer to a specific question of the referee, this includes the case of heterogeneity in the curvature parameter γ which does not satisfy the ISHARA condition. Equation (29) provides the general tool that should be used to check whether a specific preferences set yields the 2-fund separation property. In a footnote, I explore for example the alternative multiplicative case which yields sharing rule (30).

3. My comment on the riskfree rate puzzle was indeed misleading. I had in mind an economy with no uncertainty. In such an economy, conformism raises the interest rate. I agree that Weil's puzzle is more sophisticate than that. At the end of section 5, I now write in a less ambitious way:

   "Finally, it is straightforward to combine the saving problem and the portfolio problem into a single dynamic saving-portfolio problem à la Arrow-Debreu, at least in the finite horizon case. This dynamic version of the model presented in this paper is useful to discuss the combined puzzles of the equity premium and of the riskfree rate, as first stated by Weil (1989). The riskfree rate puzzle comes from the fact that a very low aversion to consumption fluctuations is required to explain the low historical riskfree rate in the U.S. economy, under a reasonable rate of pure preference for the present. Weil shows that the equity premium and the riskfree rate observed during the past century can be explained only if consumers have strong preferences for the *future.* In an economy with no uncertainty on growth, conformism raises the riskfree rate and it can thus contribute to explaining the low interest rate. But it also reduces the equity premium. Our results are thus not sufficient to determine the effect of conformism on the rate of pure preferences for the present that is compatible with the data."

4. (minor comment 1) When there is no heterogeneity in the economy, as in Gali (1994) or Campbell and Cochrane (1999), it is not restrictive to limit the portfolio choice to one riskfree asset and a claim on the aggregate wealth. Thus, it is true that my contribution is not to examine a complete market framework, but rather to introduce heterogeneous preferences. I changed the introduction accordingly.

5. I corrected the typos, and I added two useful references with a comment on the link to my work for each of them. Schroder and White (2002) has not been added because that paper focus only on internal habit formation, not on external ones.

Comments on referee 2's report

I acknowledge the nice and constructive comments of referee 2. I tried to improve the paper following his/her advices, in particular in the direction of providing 3 numerical examples when the degree of conformism is not in $[0, 1]$. They are scattered at different places in the paper to illustrate different points:

- the non-hedging of diversifiable risks (page 9),
- the preference for a short position on equities, in spite of risk aversion and a positive equity premium (page 12),
- and the violation of the two-fund separation property (section 4.5).

I use heterogeneous multiplicative externalities and a CRRA utility function to do this. I hope that showing through these examples that economies with heterogeneous consumption externalities are not isomorphic to standard EU economies provides more strength to the paper.

Here are my reaction to the different advices given by referee 2 (the numbers refer to numbering in the report):

3i) Two examples of multiple equilibria are provided, first in page 12, and second in section 4.5. See in particular Figure 3.

3ii) This has been a very helpful comments. I added a paragraph in section 3.3 in which I solve equation (20) (previously (17)) for the additive and multiplicative cases (under CRRA). The results are quite stricking!

4) In the new Proposition 6, I show that the ISHARA condition is necessary to get the two-fund separation property, at least for the additive case. The numerical example in section 4.5 shows that there is no hope to get a simple answer in the multiplicative case.

5) The fact that agent $\theta' s$ utility depends upon the distribution of other agents' consumption only through its mean is a standard simplification in the literature on internal and external habit formation. It is clear that this simplification is very helpful in our model. There is certainly much to do to relax this assumption, but I prefer to leave this for future research.

6) The example on page 9 illustrates the fact that agents may prefer not to diversify diversifiable risks, as is the story behind international diversification puzzle. For the riskfree rate puzzle, I am now much more cautious about whether external effects can solve the problem, since we cannot really disentangle the riskfree rate puzzle and the equity premium puzzle. See the end of section 5.