Insurance, Self-Protection, and the Economics of Terrorism*

Darius Lakdawalla[†]
RAND and NBER

George Zanjani[‡]
Federal Reserve Bank of New York

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Abstract

This paper investigates the rationale for public intervention in the terrorism insurance market. It argues that government subsidies for terror insurance are aimed, in part, at discouraging self-protection and limiting the negative externalities associated with self-protection. Cautious self-protective behavior by a target can hurt public goods like national prestige if it is seen as "giving in" to the terrorists, and may increase the loss probabilities faced by others if it encourages terrorists to substitute toward more vulnerable targets. We argue that these externalities distinguish the terrorism insurance market and help to explain why availability problems in this market have engendered much stronger government responses than similar problems in other catastrophe insurance markets.

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[†]RAND, 1700 Main Street, Santa Monica, CA 90407. darius@rand.org

[‡]Capital Markets Function, Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045. george.zanjani@ny.frb.org

1 Introduction

Soaring prices for terrorism coverage after September 11, 2001, led to calls for government intervention in the insurance market. Such appeals are not uncommon after significant shocks to the insurance industry. Examples include the crisis in the liability insurance market of the late 1980's (sparked by asbestos-related lawsuits) and the crisis in the catastrophe reinsurance market of the early 1990's (sparked by Hurricane Andrew), both of which were accompanied by failed campaigns for federal intervention. The push for federal involvement in terrorism insurance, however, succeeded.

The success had precedent: Governments intervene much more aggressively in markets for terror risk than in markets for natural disaster risk. For example, both the Israeli and British governments participate in terrorism coverage (McCool, 2001), although they shun direct involvement in other apparently similar catastrophe markets. France and Germany enacted government backing for terrorism insurance in the wake of the September 11 attacks, and Australia followed suit with a similar proposal after the Bali bombing. Likewise, the U.S. government's explicit commitment of $100 billion (over three years) to back the terror insurance market[1] dwarfs the aggregate of similar federal and state commitments for insurance against natural disasters.[2]

[1]The Federal terror insurance bill will remain in effect for three years. Federal liability is capped at $90 billion in the first year, $87.5 billion in the second year, and $85 billion in the third year. However, total Federal payouts over the life of the bill are capped at $100 billion (Lindlaw, 2002).

[2]Comparisons are difficult, but the following suggests that the existing natural disaster commitments are nowhere close to the terrorism commitment. The main flood program is the National Flood Insurance Program, which allows the National Flood Insurance Fund to borrow up to $1.5 billion from the U.S. Treasury (see *NFIP Program Description*, FEMA, 8/1/2002). The main earthquake program, the California Earthquake Authority, is backed by existing and contingent industry assessments of several billion dollars (McCool, 2001). Florida's Citizens Property Insurance Corporation, the main bulwark for hurricane risk, lists claims-paying resources of about $9 billion, with the ability to make additional annual premium-based assessments on insurers and policyholders (http://www1.citizensfla.com on 11/20/02).

This characterization appears to be true of implicit commitments as well: Ex post payments to victims were far more generous in the case of the September 11 attacks than in previous disasters.[3]

What accounts for this aggressive public intervention? The terror insurance market shares the availability and affordability problems common to other catastrophe insurance markets,[4] but evidently these problems are only part of the story. This paper argues that a key motivation for public involvement in the terror insurance market lies not in the insurance market itself, but in a closely associated market—the market for self-protection. Self-protection behavior can have negative social consequences not considered by the individual, and that behavior can be encouraged by problems in the insurance market when insurance and self-protection are substitutes. Hence, although economists usually compare terrorism insurance to natural catastrophe insurance when discussing public intervention, we argue that better analogies are drawn with deposit insurance and war-risk life insurance. In these markets, the *immediate* concern is not the efficiency of the markets themselves, but the harmful self-protective behaviors engendered by a lack of insurance.[5]

The effects of self-protection (Ehrlich and Becker, 1972) extend beyond the cal-

[3]The Federal Emergency Management Agency (FEMA) provides natural disaster relief in any or all of the following forms: low-interest loans; cash grants of up to $15,000; unemployment compensation; food; temporary housing assistance; unemployment benefits; tax-deductibility of certain casualty losses for individuals and/or excise tax relief; counseling and legal advice (data are from http://www.fema.gov/library/dproc.shtm, on 11/15/02). These provisions pale in comparison to compensation paid to the 9/11 victims, who have received an average award of $1.5 million from the September 11 Victims' Compensation Fund (this information is from http://www.usdoj.gov/victimcompensation/payments.html, on November 13, 2002.)

[4]For a review of explanations for the problems in catastrophe insurance markets, see Froot (1997). For contrasting views on the inefficiency of the insurance/reinsurance markets and the case for government intervention, see Gron and Sykes (2002), or Brown et al. (2002).

[5]Hirshleifer (1953) provides one of the first analyses of insurance and self-protection decisions in the context of war risk.

culus of individual optimization problems in two respects. First, an agent's decision to self-protect may affect the loss probabilities and insurance prices faced by other agents. Protection by one agent may increase the risks faced by others if terrorists substitute toward other targets. This is similar to the external effects spawned by anti-theft devices, which may influence risk and insurance prices by affecting the behavior of thieves (see, e.g., Shavell, 1991; Eck, 1993; Ayres and Levitt, 1998), and automobile safety features, which may change prices by affecting the behavior of protected drivers or the damages associated with accidents (Peltzman, 1975). Second, self-protection decisions may lead to externalities that extend beyond the insurance market. For example, individuals moving away from a high-profile downtown area might hurt national prestige or morale by appearing to capitulate to terrorism. A related example is self-protection by bank depositors, which may lead to bank failures (through bank runs) and, ultimately, the liquidation of unrelated businesses (Diamond and Dybvig, 1983). These external effects of self-protection are largely absent with other catastrophe risks such as windstorm and earthquake.

Government policy can limit externalities in the market for self-protection through the use of taxes or subsidies. The obvious and best approach is to apply taxes, subsidies, or regulations directly to the self-protection behaviors that generate externalities, in the tradition of Pigovian welfare economics (Pigou, 1932). However, if self-protection is difficult to define, monitor, or measure, this may not be feasible. Instead, the government can influence self-protection choices by taxing or subsidizing the purchase of insurance. When insurance and self-protection are substitutes from the perspective of the individual, a policy aimed at encouraging the purchase of insurance simultaneously discourages self-protection, and vice-versa. Unfortunately, however, while insurance subsidies can improve welfare, they cannot restore a first-best equilibrium, because they themselves create distortions in the insurance market. Therefore, it can often be in the public interest to use other policy tools in conjunction with insurance market intervention, such as publicly provided protection against terror risk.

The rest of this paper is organized as follows. Section 2 shows that externalities in the market for protection can call for intervention in the insurance market when it is difficult to tax or subsidize protection directly. We also analyze the circumstances under which insurance and self-protection are substitutes or complements. Section 3 shows how rational behavior by a terrorist group creates externalities in protection, and characterizes the relationship between the direction of externalities and the nature of terror risk. Finally, Section 4 examines public protection as a complement to intervention in insurance markets.

2 Public Policy and the Market for Protection

We start by building a framework to study how individuals use self-protection and insurance in the presence of terror risk. We then show how externalities in protection can be limited through intervention in the insurance market. Even if the market for terror insurance functions perfectly, taxing or subsidizing insurance can be Pareto-improving in the presence of externalities in protection. When there are negative externalities in protection, insurance subsidies may be used.

2.1 The Inefficiency of Private Decisions

Consider an economy with N agents facing a risk of terrorist attack. The j-th agent faces risk of experiencing a loss of fixed size L and allocates wealth W between expenditures on insurance coverage I_j—available at a price π_j for every dollar of coverage—and on self-protection s_j. Self-protection is purchased at a per unit price of k, and the individually-chosen levels of self-protection jointly determine the probabilities of loss faced by each of the agents. Self-protection refers to actions or investments that lower the probability of a successful terrorist attack, such as fences, guards, metal detectors, or background checks. The probability of loss for j is written as: $p_j(s_1, ..., s_N)$.

Agent j's probability of loss is decreasing in her own chosen level of self protection. As we show in Section 3, the cross-effects $\frac{\partial p_j}{\partial s_k}$ are negative, so long as terrorists can observe protection levels. For the sake of completeness, we also allow for the case of positive cross-effects, which (as we show later) can obtain if protection is sufficiently hard to observe and terrorist groups are "moderate."

The probability of loss affects the price of insurance, written as:

$$\pi_j(s_1, ..., s_N; \alpha) \equiv p_j(s_1, ..., s_N) + \alpha,$$

where $0 \leq \alpha \leq 1$ reflects other influences on the price of insurance such as administrative expenses, capital costs, and information costs. We assume that the pricing reflects "real" costs in the sense that they cannot be avoided by government intervention.

Finally, we include a public good damaged by self-protection. We write this as $V(s_1, ..., s_N)$. The public good represents another channel of external effects. For example, the public good may represent "national prestige," which may suffer when individuals protect themselves by failing to build landmarks in downtown areas or by not patronizing cultural events.

In this environment, agent j solves:

$$\max_{s_j, I_j} \; \{p_j U_{0j}(W - L + (1 - \pi_j) I_j - k s_j) + \\ (1 - p_j) U_{1j}(W - \pi_j I_j - k s_j) + V(s_1, ..., s_N) \; \}, \tag{2.1}$$

where U_{0j} and U_{1j} are increasing and concave functions representing agent $j's$ utility of wealth in the loss and no-loss states, respectively. If all functions are differentiable, the individual optimality condition for the choice of insurance is the familiar

$$\pi_j = \frac{p_j U'_{0j}}{(1 - p_j) U'_{1j} + p_j U'_{0j}}. \tag{2.2}$$

When pricing is actuarially fair ($\pi_j = p_j$), this condition equates marginal utilities across states. The condition for the choice of self-protection is:

$$\frac{\partial p_j}{\partial s_j} (U_{0j} - U_{1j}) - \left[p_j U'_{0j} + (1 - p_j) U'_{1j} \right] (k + \frac{\partial \pi_j}{\partial s_j} I_j) + \frac{\partial V}{\partial s_j} = 0. \tag{2.3}$$

The first term is positive and reflects self-protection's mitigating impact on the probability of loss. The second term reflects the net marginal pecuniary cost of self-protection, including any premium reductions realized. The last term reflects the marginal impact of the individual's choice on his private valuation of the public good. The agent balances the marginal benefit associated with loss prevention (the first term), with the marginal out-of-pocket cost (the second term) and the marginal impairment of the public good (the third term).

We now study how individual choices may be socially inefficient. Consider the social planner's problem, where, for simplicity, the Pareto weights have been equalized across agents:

$$\max_{s_1,\ldots,s_N;I_1,\ldots I_N} \ \{\sum_{j=1}^{N} \ [p_j U_{0j}(W - L + (1-\pi_j)I_j - ks_j) + \\ (1-p_j)U_{1j}(W - \pi_j I_j - ks_j) \ ] + NV(s_1,...,s_N) \ \} \tag{2.4}$$

This is simply the sum of the utilities of the N agents. The last term is the individual valuation of the public good multiplied by the total number of individuals, representing the total social value of the public good.

The social optimality condition for the choice of insurance is identical to the private condition in Equation 2.2. However, when there are external effects on the risk of terrorism ($\frac{\partial p_j}{\partial s_k} \neq 0$) or public goods V, private choices for *protection* will depart from efficient choices. The socially optimal level of s_j is given by:

$$\frac{\partial p_j}{\partial s_j}\left(U_{0j} - U_{1j}\right) - \left[p_j U_{0j}^{'} + (1-p_j)U_{1j}^{'}\right](k + \frac{\partial \pi_j}{\partial s_j} I_j) + \frac{\partial V_j}{\partial s_j} + ... \\ \sum_{i \neq j}\left[\frac{\partial p_i}{\partial s_j}\left(U_{0i} - U_{1i}\right) - \left(p_i U_{0i}^{'} + (1-p_i)U_{1i}^{'}\right)\frac{\partial \pi_i}{\partial s_j} I_i + \frac{\partial V_i}{\partial s_j}\right] = 0, \tag{2.5}$$

The second line, which is not present in individual j's problem, measures the externalities on the risks faced by others, and on the public good accruing to other agents.

These represent the two sources of external effects.

First, the individual does not consider her effects on the probabilities of loss faced by other agents and the associated prices of insurance. The total externality is measured by two components: the direct equilibrium impact of self-protection on the utility of others, and the impact on other agents' insurance premiums. If individuals are fully insured, the externality consists solely of the increase in remedial expenditures on insurance. Conversely, when there is no terror insurance, the externality consists solely of the direct effect on others' utility.[6]

Second, the individual does not consider her impact on public goods that accrue to other agents. The public good in this case represents national prestige, morale, or war objectives. For example, it might be individually rational for a firm to self-protect by moving away from a high-profile downtown area, but this might be viewed by the public (or policymakers) as contrary to national objectives. From the national point of view, there may be too much caution and too few social incentives to encourage risk-taking.[7] This represents at least part of the reason why excessive self-protection might lead to publicly provided insurance.

[6]While the change in the insurance premium is, strictly speaking, a pecuniary externality, it reflects an underlying technical (or real) externality, namely the effect of other agents' self-protection. The underlying technical externality has a pecuniary impact and a non-pecuniary impact, and thus its total cost includes both these components.

[7]These kinds of considerations could also be present in the case of natural disasters, but at a more regional scale. The risk of earthquake in California might encourage the state of California to subsidize earthquake insurance (which it does to some small extent) in order to attract investment or to prevent emigration.

2.2 Corrective Public Policy in the Insurance Market

The privately optimal level of protection may not be socially optimal, but taxes, subsidies, or restrictions aimed at self-protection may not be cost-effective or even feasible. Self-protection against terrorism entails a wide range of activities—such as beefing up security, avoiding public places, relocation, and emigration—all of which may be difficult to describe and regulate in a policy aimed at promoting the national interest. For example, it might be hard to distinguish between the builder that avoids a downtown area because of terrorism and one who is not good at building downtown high-rises. An employer who performs background checks on employees might be concerned about terrorists in one case, but thieves in another. Moreover, even if it were possible to isolate the relevant behaviors, the result would be a legal regime akin to the tax code, which provides myriad incentives for circumvention and litigation.

When the government cannot implement Pigovian taxes to correct failures in the market for protection, it can improve welfare by manipulating the price of insurance, which affects the private demand for protection. In the discussion that follows, we focus on the case where insurance and self-protection are substitutes and the government is subsidizing insurance to correct negative externalities in protection. Our results hold exactly in reverse for the case of positive externalities. The results can also be adapted to handle other cases, such as the "no moral hazard" case where insurance and self-protection are complements.

Consider a government that has access to lump-sum taxation τ in order to subsidize insurance prices. The subsidy rate, $f_j \equiv f(\phi, \pi_j)$, is controlled by the government through the choice of ϕ, with $\frac{\partial f}{\partial \phi} > 0$, $f(0, \pi_j) = 0$, and $0 \leq \frac{\partial f}{\partial \pi_j} \leq \phi$. Agent j pays the price $\pi_j - f_j$. This general specification can accommodate a variety of subsidy schemes. In particular, it allows an "additive" subsidy as in

$$f(\phi, \pi_j) \equiv \phi,$$

where the government offers a fixed contribution per unit of coverage purchased. It also supports a "multiplicative" subsidy such as

$$f(\phi, \pi_j) \equiv \phi\pi_j,$$

where the government pays for a fixed fraction of total insurance costs.

In the presence of a subsidy, agent j solves:

$$\max_{s_j, I_j} \quad \{p_j U_0((W - \tau_j - L + (1 - \pi_j + f(\phi, \pi_j))I_j - ks_j) + ... \tag{2.6}$$
$$(1 - p_j)U_1((W - \tau_j - (\pi_j - f(\phi, \pi_j))I_j - ks_j) + V(s_1, ..., s_N)\ \}.$$

The private first order conditions now become:

$$p_j U_0' - [\pi_j - f(\phi, \pi_j)] \left[(1 - p_j)U_1' + p_j U_0'\right] = 0, \tag{2.7}$$

$$\frac{\partial p_j}{\partial s_j}(U_0 - U_1) - \left[p_j U_0' + (1 - p_j)U_1'\right](k + \left(\frac{\partial \pi_j}{\partial s_j} - f_\pi \frac{\partial \pi_j}{\partial s_j}\right) I_j) + \frac{\partial V}{\partial s_j} = 0. \tag{2.8}$$

A budget-balancing subsidy (i.e., one for which $\tau_j = f(\phi, \pi_j)I_j$) raises insurance consumption, because its only direct effect is to lower the marginal cost of insurance. As can be seen in Equation 2.7, the effective price of insurance (the second term) is decreasing in the amount of the subsidy. Simply put, the own-price elasticity of insurance is negative, and a budget-balancing subsidy affects only the price of insurance and not individual income.

Insurance subsidies will also discourage protection. The appendix proves this in detail, but a less detailed exposition is illustrative. The relationship between subsidies and protection has two components: a direct effect on the returns to protection, and a cross-effect between the demands for insurance and protection (i.e., the presence or absence of moral hazard). Equation 2.8 demonstrates that the returns to protection depend not on the price of insurance (π_j), but on the premium reduction from self-protection ($\frac{\partial \pi_j}{\partial s_j}$). An additive subsidy will not affect this, but a multiplicative subsidy will lower it, as the government assumes a share of the total premium reduction. Therefore, subsidies weakly lower the returns to protection, where the strict relationship holds only for multiplicative subsidies.

The cross-effect concerns the impact of the increase in insurance consumption on the demand for self-protection. If insurance and self-protection are substitutes, moral hazard exists: the increase in insurance will discourage investments in protection. Empirical evidence from other insurance markets suggests that this situation of moral hazard prevails in a variety of contexts.[8] However, it is worth discussing

[8] While the literature is voluminous, a few examples suggest the robustness of this result. Dionne and Gagne (2002) provide evidence of moral hazard in automobile theft insurance. Bolduc and et al. (2002) document it in Workers' Compensation policies. Zweifel and Manning (2000) provide a review of the literature on moral hazard in health insurance markets. Pearson (2002) investigates the emergence of moral hazard in the newly formalized British insurance markets of the eighteenth and nineteenth centuries.

the theoretical conditions that yield moral hazard. In the Appendix, we show that moral hazard will generally tend to exist at the point of actuarially fair pricing.[9] We also analyze the more complex case of unfair pricing, showing that subsidies will be associated with moral hazard at least under certain conditions.[10]

When insurance subsidies discourage self-protection ($\frac{\partial s_j^*}{\partial \phi} < 0$), the government can improve welfare by using subsidies to correct negative externalities in protection. Denote the optimal choices of agent j by I_j^* and s_j^*. The lump sum taxes are fixed in advance of the agent's maximization (i.e., the agent takes these as given), but must be consistent with government budget balance, so that $\tau_j = f(\phi, \pi_j)I_j$. Define $p_j^* \equiv p_j(s_1^*, ..., s_N^*)$, $\pi_j^* \equiv \pi_j(s_1^*, ..., s_N^*; \alpha; \phi)$ and $V_j^* \equiv V(s_1^*, ..., s_N^*)$. With equal Pareto weights, the government chooses subsidies to solve:

$$\max_{\phi} \quad \sum_{j=1}^{N} \{p_j^* U_0(W - \tau_j - L + (1 - \pi_j^* + f(\phi, \pi_j^*))I_j^* - ks_j^*) + ... \\ (1 - p_j^*)U_1(W - \tau_j - (\pi_j^* - f(\phi, \pi_j^*))I_j^* - ks_j^*) + V(s_1^*, ..., s_N^*)\}. \tag{2.9}$$

When the budget remains balanced, subsidies have no direct effects on consumption. They operate through effects on insurance and self-protection choices. When the indi-

[9]More precisely, additive subsidies are neutral and multiplicative subsidies produce moral hazard. This contrasts with the findings of Ehrlich and Becker (1972). In the appendix, we reconcile our results to those of Ehrlich and Becker, showing that their finding of complementarity between insurance and self-protection at the point of fair pricing follows from assumptions that embed unusual properties into the expense loading. The two most common models of expense loading—additive to expected loss and proportional to expected loss—correspond to the additive and multiplicative subsidy cases analyzed in this paper.

[10]Unfair pricing introduces certain wealth effects in risk aversion that are absent from the fair pricing case. The conditions are used throughout the paper and are summarized in the appendix, with the major ones being 1) CARA utility, and 2) a (subsidized) price of less than 50 cents per dollar of coverage.

vidual chooses insurance and self-protection optimally (given the taxes and subsidies imposed by the government), the government's first order condition simplifies to

$$
\begin{aligned}
&-\sum_{j=1}^{N} f_j \frac{dI_j^*}{d\phi}\left[p_j^* U'_{0j} + (1-p_j^*)U'_{1j})\right] \\
&-\sum_{j=1}^{N} \frac{\partial f_j}{\partial \pi_j}\frac{\partial \pi_j^*}{\partial s_j}\frac{ds_j^*}{d\phi} I_j \left[p_j^* U'_{0j} + (1-p_j^*)U'_{1j})\right] + \\
&\sum_{j=1}^{N}\sum_{m\neq j}\frac{ds_m^*}{d\phi}\left[\frac{\partial p_j^*}{\partial s_m}(U_{0j}-U_{1j}) + \frac{\partial V_j^*}{\partial s_m} - \frac{\partial \pi_j^*}{\partial s_m} I_j^*(p_j^* U'_{0j} + (1-p_j^*)U'_{1j})\right] = 0
\end{aligned}
\tag{2.10}
$$

Subsidies have the benefit of limiting self-protection (as reflected in the third line of the equation). Its costs are distortions of the insurance market (first line) and the protection market (second line).

As the top line illustrates, without subsidies, $f_j = 0$, and there are no distortions in the insurance market. As subsidies rise, the private price of insurance falls below the efficient price, as individual agents do not consider the effect of their own insurance consumption decisions on their tax bill. The second line reflects the costs associated with distorting incentives for individual self-protection. These costs arise only with a multiplicative subsidy ($\frac{\partial f_j}{\partial \pi_j} > 0$), which causes agents to realize less than the full premium reduction from their self-protection efforts.

The third line reflects the benefit of lowering self-protection when it has external effects on the risks faced by others and on the public good. This term is positive if $\frac{\partial p_j^*}{\partial s_m} > 0$, $\frac{\partial V_j}{\partial s_m} < 0$, and $\frac{ds_m^*}{d\phi} < 0$. That is, when subsidies discourage self-protection and there are negative externalities associated with self-protection, this term is a benefit reflecting the extent to which subsidies limit the negative externalities. Furthermore, under these conditions, it is *always* optimal to subsidize insurance. Without a subsidy scheme, the top two lines are zero and the third line is positive. Intuitively, it is costless to distort an (unsubsidized and) efficient market, so that subsidies are always beneficial initially. This is true even if prices are actuarially fair. If the insurance market is inefficient, however, subsidies involve distortion costs that will depend on the nature of the inefficiency. If the price of insurance is inefficiently high, the distortion costs may be negative; if the price is inefficiently low, the distortion costs will be positive.

In principle, an alternative to Pigovian subsidies would be for the government to award a monopoly in terror insurance to a single firm and mandate participation. However, the important thing to note about this possibility is that it too requires government intervention, because it would not be privately stable. A monopolistic insurer could internalize the externalities associated with protection, but several problems arise. First, the government has to ban entrance by new firms. A monopolistic insurer would penalize customers for self-protection more than a competitive firm. Therefore, a private monopolist would always be vulnerable to entrance by a new competitor that offered insurance without this penalty: it would be privately optimal for customers to switch to the new insurer. Second, the government must mandate universal participation, because large customers would prefer to self-insure without paying a penalty for self-protection. Finally, the government would have to guarantee that the monopolist properly values all public goods. This would amount to public price-setting for the monopolist, which is essentially a more restrictive form of Pigovian subsidies.

3 Terrorist Behavior and Externalities

In this section, we present a simple model of terrorist behavior and use it to derive some important properties for the probability $p(s_1, ..., s_N)$ and, consequently, implications for the optimal public policy in the insurance market.[11] If terrorists can observe protective measures, private protection has negative effects on other targets. This calls for insurance subsidies by the logic of the previous section.

Consider a syndicate of terrorists, criminals, or soldiers with resources R. They pursue their objectives by allocating their resources between violent terror T and nonviolent activities A. Suppose that terrorists face a set of N targets, and that they generate damage $B(L)$ when they inflict the loss L on a particular target.[12] The probability of inflicting this loss is a function $\rho(s_j, r_j)$, where s_j is the self-protection expenditure of target j, and r_j is the level of resources spent by terrorists on attacking target j. The function thus satisfies $\rho_s < 0$ and $\rho_r > 0$, with the concavity conditions $\rho_{ss} > 0$ and $\rho_{rr} < 0$. Expenditures on self-protection thwart the effectiveness of terror investments, so that $\rho_{sr} < 0$. Given T and a set of self-protection levels, terrorists maximize expected damage according to:

[11] Analyses of terrorist behavior in the context of insurance risk are also presented in Woo (2002) and Major (2002).

[12] Introducing heterogeneity in the returns to attacking different targets does not greatly affect our results. In the language presented below, the only effect might be that it strengthens the "deterrence" effect; terrorists may substitute away from higher value targets to lower value ones, and the returns to terrorism would fall as a result.

$$
\begin{aligned}
D(T; s_1, ..., s_N) &= \max_{\{r_i\}} \sum_{i=1}^{N} \rho(s_i, r_i) B(L) \\
s.t. \sum_{i=1}^{N} r_i &\leq T
\end{aligned} \tag{3.1}
$$

The first order conditions of this problem equate the expected marginal productivity of terrorist resources across all targets:

$$
\rho_r(s_i, r_i) = \rho_r(s_j, r_j) \tag{3.2}
$$

Comparative statics in a symmetric equilibrium reveal the terrorists' "conditional input demands" for terror investments:

$$
\begin{aligned}
\frac{\partial r_i}{\partial s_i}|_T &= \frac{(N-1)(-\rho_{rs})}{N\rho_{rr}} < 0 \\
\frac{\partial r_j}{\partial s_i}|_T &= \frac{\rho_{rs}}{N\rho_{rr}} > 0
\end{aligned} \tag{3.3}
$$

With the total level of terror investments fixed at T, increases in self-protection by one target cause terrorists to substitute toward other targets. This is the "substitution" or "displacement" effect of protection. Each target's probability of attack falls with its own self-protection, but rises with the self-protection investments of others. However, the first-stage problem also makes it plain that protection reduces the payoff to terrorism. This leads to the "deterrence" effect crystallized in the second-stage of the terrorists' problem.

Suppose terrorists' total utility is equal to the expected terrorism damage they produce plus the (concave) returns to nonviolent activities, $\nu(A)$, which might also accord with their objectives. Examples include political pressure or propaganda. Expected damage is simply the $D(T; s_1, ..., s_N)$ function derived above. This leads to the terrorists' resource allocation problem:

$$\begin{aligned} &\max_{A,T} \nu(A) + D(T; s_1, ..., s_N) \\ &s.t.\, A + T \leq R \end{aligned} \tag{3.4}$$

Denoting the marginal utility of wealth as ω, this problem has the two first order conditions:

$$\begin{aligned} \nu_A &\leq \omega \\ D_T &= \omega \end{aligned} \tag{3.5}$$

An decrease in the marginal benefit of terrorism—because of additional protection—will lower the amount of terrorism chosen and encourage substitution toward nonviolent activities. This is the deterrence effect of protection.

The two effects, of deterrence and displacement, work together to govern the effect of protection on terror risk. We can write terror investments against a specific target as $r_i(s_i, T(s_i))$. Totally differentiating this expression in a symmetric equilibrium yields:[13]

[13] In a symmetric equilibrium, $\frac{\partial r_i}{\partial T} = \frac{1}{N}$, and $\frac{\partial T}{\partial s_i} = -\frac{\partial A}{\partial s_i}$.

$$\frac{\partial r_i}{\partial s_i} = \frac{\partial r_i}{\partial s_i}|_T - \frac{1}{N}\frac{\partial A}{\partial s_i} < 0$$
$$\frac{\partial r_j}{\partial s_i} = \frac{\partial r_j}{\partial s_i}|_T - \frac{1}{N}\frac{\partial A}{\partial s_i} > 0 \quad (3.6)$$

Protection by one target draws terror investments away from it and funnels them toward other targets and other nonviolent activities. Deterrence takes place insofar as private self-protection raises A and lowers the total amount of violent terror investments.[14] However, displacement ($\frac{\partial r_j}{\partial s_i}|_T$) dominates the deterrence effect ($\frac{1}{N}\frac{\partial A}{\partial s_i}$) in the sense that protection by one target increases the terror investments directed at other targets. This follows directly from the concavity of the problem. Intuitively, protection by one target lowers the return to attacking that target, but raises the relative return to investments in A and investments in attacking all other targets. Consequently, terrorists substitute away from the protecting target and towards all other forms of investment—including attacking other targets. Therefore, while private protection lowers the total resources devoted to terrorism T, it still creates negative externalities for other targets by exposing them to more terror risk.

Given rational expectations, the probability of attack faced by target i is identical to the probability of success perceived by the terrorist against the target: $p_i(s_1, ..., s_N) = \rho_i(r_i, s_i)$. Equation 3.6 implies that $\frac{\partial r_j}{\partial s_i} > 0$ and that targets face the probability of attack:

[14]Technically, the social value of deterrence depends on the nature of A. For simplicity, we assume that A has no impact on the public good, but this need not be the case. Suppose, for example, that a nonviolent propaganda campaign by a terrorist group damages the public good by diminishing national prestige. In this case, it is not clear that deterring terrorists from violence has positive benefits on balance. More broadly, our model takes a narrow view of terrorist objectives. We do not consider the merits of the terrorist cause within the context of the public good, nor the possibility of deterrence through political solutions that might be consistent with the national interest.

$$\frac{\partial p_j}{\partial s_j} = \rho_s + \rho_r \frac{\partial r_j}{\partial s_j} < 0$$
$$\frac{\partial p_j}{\partial s_i} = \rho_r \frac{\partial r_j}{\partial s_i} > 0 \tag{3.7}$$

Self-protection has private benefits ($\frac{\partial p_j}{\partial s_j} < 0$) but negative externalities ($\frac{\partial p_j}{\partial s_i} > 0$) whenever terrorists can observe protection. However, the nature of terror risk is central to this result. In the following section, we show that if protection is unobservable *and* if a terrorist group is not an extremist one, protection can sometimes involve some positive externalities.

3.1 Unobserved Protection Against Terror

It seems possible to observe physical protection against terrorist attack, such as security guards, surveillance cameras, fences, military installations, the vigilance of airport or other screeners, or the accessibility of a location. However, it may be harder to observe methods for tracing financial transactions or tracking down computer hackers. When terrorists cannot observe protection, they cannot substitute away from more fortified targets, and the displacement effect present in the case of observed protection disappears. Only the deterrence effect of protection remains. This can result in positive externalities for protection.[15]

To see this most simply, suppose terrorists or criminals know only the mean level

[15] A salient example of positive externalities in self-protection against crime is provided by Ayres and Levitt (1998), who study Lojack—a tracking device on automobiles designed to aid vehicle recovery in the event of theft—which was unobservable at the time of its introduction. In the case of a single protective device like Lojack, the best policy option would seem Pigovian subsidies for the device itself. When protection is harder for the government to identify, this option may not be available.

of self-protection, $\bar{s}$, but not the self-protection of a particular target. In this case, the syndicate's first-stage problem becomes:

$$
\begin{aligned}
D(T;\bar{s}) &= \max_{\{r_i\}} E\left[\sum_{i=1}^{N} \rho(s_i, r_i) B(L) | \bar{s}\right] \\
s.t. \sum_{i=1}^{N} r_i &\leq T
\end{aligned}
\tag{3.8}
$$

When terrorists cannot distinguish differences among targets, optimal behavior implies an even allocation of terror investments across targets. In this case, private investments in self-protection change the mean level of self-protection (and, hence, the amount of resources devoted to terrorism), but they have no effects on the syndicate's *relative* allocation of resources across targets.

For a radical group ($\nu_A < \omega$ and $A^* = 0$), self-protection has no impact on risk or terror investments: Terrorists do not alter their total investments in violent terror. In this case, there is no deterrence effect and no positive externalities. However, if the group is not "radical" in the sense that $\nu_A = \omega$, self-protection investments will lower the expected return to violent terror and thus encourage substitution toward nonviolence. This yields benefits for all—even the unprotected—in the form of terror risk reductions. In this case, there are positive externalities associated with private protection. The two key ingredients for this result are moderation in the terrorist group, and unobservable protection. We have presented the extreme case where it is impossible to observe protection. In general, however, the harder it is to observe protection, the more important is deterrence relative to displacement.

In determining the optimal policy response, the terror risk reductions must be balanced against the negative effects on the public good that may be associated with self-protection–i.e., flight from a high-risk downtown area. The effect on public goods could outweigh the positive externalities from deterrence and justify insurance subsidies even in this case. However, if the risk-reduction benefits outweigh these costs, a free-rider problem emerges. No private agent has the incentive to undertake the socially efficient level of self-protective expenditure. To address such a problem of positive externalities, the government can tax insurance and encourage greater protection.

3.2 Alternative Models of Terrorist Behavior

Of course, alternative specifications of terrorist preferences, resources, outside options or technology could yield different results for predicted behavior and the optimal policy response. For our purposes, the important point is that terrorists take self-protection choices into account when planning activity. This creates a role for public policy aimed at influencing self-protection. Although an exhaustive consideration of variations in the specification of the terrorist problem is beyond the scope of this paper, we examine one interesting alternative below.

The model presented above implies a type of "risk-aversion" in terrorist activity. Terrorists choose to spread investments across all targets because of diminishing returns. However, it is also possible that a group could find it optimal to focus all resources on a single target because of increasing returns to terrorism investments. There are a variety of possible reasons for this, some of which can be analyzed within the context of the model provided. For example, the probability of success ρ could be convex in terror investments, or the utility of terrorists could be increasing in the amount of self-protection they induce. Regardless of why returns are increasing, the focus on a single target may create instability in the market for protection in the form of an arms race among targets.

To see this, consider a situation where terrorists choose the most vulnerable target and targets understand that terrorists will hit only one of them. Evidently, there is no symmetric Nash equilibrium, where all targets spend s^* on protection. Since the terrorist group will avoid targets that are better protected than alternatives, each target faces an incentive to deviate from a symmetric equilibrium. Starting with a symmetric equilibrium, an infinitesimal increase in protective expenditures by any target reduces its probability of being attacked from $\frac{1}{N}$ to zero. Since an infinitesimal investment has a finite pay-off, every target will choose to deviate from this equilibrium. A similar argument can be used to establish that there is no asymmetric equilibrium.

The government does not have a rich set of policy choices in this situation, since partial subsidies or taxes will be ineffective. The government can end the arms race in protection only by eliminating all incentives for private self-protection. The only feasible insurance market intervention is to provide full, free insurance to all targets. Since individuals would then be fully insured and would not receive any premium reductions from self-protection, the marginal utility of self-protection would be zero (according to Equation 2.3), and the new equilibrium would involve no private self-protection by any agent. In the face of increasing returns to terrorism, the government's alternatives collapse to a discrete choice between two alternatives—an unregulated arms race in private protection, or a fully subsidized insurance market with zero investments in private protection.

4 Public Protection as a Complementary Policy

Another policy response to terrorism risk is the provision of public protection against terrorism. There are two important points to be made in the context of insurance policy: first, public protection is not necessarily a substitute for insurance market intervention, and under many circumstances it will be optimal to implement the two policies in tandem; second, public protection may often be welfare-enhancing, but it may not on its own limit the externalities associated with private protection.

Pursuing the policies in tandem may be justified when neither public protection nor insurance market intervention alone yields a first-best outcome in private protection. In general, the only way to achieve a first-best outcome in the presence of externalities in protection is through Pigovian taxes (or subsidies) on protection (financed by lump-sum taxation) that alter the margin along which private decisions are made. As we have shown, intervening in the insurance market is costly. Aside from Pigovian taxes, all policies will generally involve costs or other distortions. Therefore, no single policy will entirely eliminate the social costs associated with self-protective externalities, and there is scope for using other "second-best" policies—such as public protection—in conjunction with insurance market intervention.

Second, while collective protection may in fact be more efficient at reducing the risk of terrorism than private protection, it may or may not mitigate externalities in private protection. By lowering the risk of terrorism, public protection engenders a wealth effect that makes people feel richer and thus more inclined to spend on protection. This effect creates a tendency for public protection to promote private protection. It may be either offset or reinforced by the technological relationship between private and public protection—that is, whether public protection raises or lowers the productivity of private protection. When public protection exacerbates externalities in private protection, the complementarity between public protection and insurance market intervention is reinforced.

A prominent example of insurance market intervention and public protection working together is the case of deposit insurance and banking regulation. Both policies are aimed at stabilizing the banking system and, in particular, producing consumer confidence in banks, but neither appears adequate on its own. Regulation alone was not able to able to prevent the bank runs of the 1930's, just as deposit insurance (without appropriate regulation) was not able to prevent the thrift crisis of the 1980's. Moreover, the use of one reinforces the social value of the other. For example, if deposit insurance breeds indifference among consumers, there is a role for public regulation; likewise, the presence of regulation may reduce the costs associated with deposit insurance.

4.1 Insurance Policy in the Presence of Public Protection

In practice, governments do more to fight terrorism than just intervening in insurance markets. They use military and law enforcement strategies to protect their citizens against terror. Even in the presence of public protection, however, insurance market intervention is often called for: Public protection alone will not eliminate externalities in private protection (unless it completely crowds out private protection).

Suppose investments in public protection, σ, are available at a normalized price of 1. They lower the probability of loss, but are free from the externalities that characterize private protection. Finally, the cost of public protection is assumed to be spread equally over the N agents in the tax bill, so the government's new budget constraint is now

$$\tau_j = f(\phi, \pi_j) + \frac{\sigma}{N}.$$

Again, without loss of generality, we consider the case of radical terrorists, negative externalities in protection and corrective insurance subsidies. Include the public protection level σ in the equilibrium loss probability and premium, according to $p_j^* \equiv p_j(s_1^*, ..., s_N^*; \sigma)$, and $\pi_j^* \equiv \pi_j(s_1^*, ..., s_N^*; \alpha; \sigma)$. With equal Pareto weights, the government chooses subsidies and public protection to solve:

$$\max_{\phi,\sigma} \quad \sum_{j=1}^{N} \{p_j^* U_0(W - \tau_j - L + (1 + f(\phi, \pi_j^*) - \pi_j^*) I_j^* - ks_j^*) + ... \quad (4.1)$$
$$(1 - p_j^*) U_1(W - \tau_j - (\pi_j^* - f(\phi, \pi_j^*)) I_j^* - ks_j^*) + V(s_1^*, ..., s_N^*)\}.$$

The first order condition for insurance subsidies is essentially the same as Equation 2.10. As before, the first two lines of this expression (the marginal cost) are zero in the absence of subsidies, while the third line (the marginal benefit) is always positive when subsidies lower self-protection. Thus, if we have an interior solution in private protection (i.e., if targets protect themselves at all), the marginal utility of subsidies will be positive in an unsubsidized equilibrium. Even considering public protection as a policy option, some intervention in insurance markets is optimal, unless public protection is so efficient that its optimal level drives the level of private protection to zero. The existence of private protection against terrorism (e.g., countermeasures taken at the World Trade Center after the 1993 bombing), however, suggests that this extreme case does not obtain.

4.2 Public Protection and Externalities in Private Protection

Since neither policy can completely eliminate externalities, it makes sense to pursue them together. Of course, public protection may be optimal simply because it lowers the risk of terrorism, and even though it may exacerbate externalities in private protection. We now demonstrate how it can exacerbate these externalities, an effect which further reinforces the complementarity between public protection and insurance subsidies.

Assuming that individuals choose insurance and self-protection optimally and that the public budget remains balanced, the government's first order condition for public protection simplifies to:

$$\sum_{j=1}^{N}\left[\frac{\partial p_j^*}{\partial \sigma}(U_{0j}-U_{1j})-\frac{\partial \pi_j^*}{\partial \sigma}(1-f_\pi)I_j^*(p_j^*U_{0j}'+(1-p_j^*)U_{1j}')\right]+$$
$$\sum_{j=1}^{N}\left[(-\frac{1}{N})(p_j^*U_{0j}'+(1-p_j^*)U_{1j}')\right]+$$
$$\sum_{j=1}^{N}\sum_{m\neq j}\frac{ds_m^*}{d\sigma}\left[\frac{\partial p_j^*}{\partial s_m}(U_{0j}-U_{1j})+\frac{\partial V_j^*}{\partial s_m}-\frac{\partial \pi_j^*}{\partial s_m}(1-f_\pi)I_j^*(p_j^*U_{0j}'+(1-p_j^*)U_{1j}')\right]=0 \tag{4.2}$$

The returns to public protection depend on its productivity in reducing the probability of loss (first line), its pecuniary cost (second line), and whether it worsens or lessens externalities in private protection (third line). If public protection is sufficiently productive at limiting the probability of loss, the first two lines will be positive. If public protection worsens externalities in private protection, the third line will be negative. However, observe that it may still be a rational policy if its productivity outweighs its effect on private protection.

The impact of public protection on self-protection externalities depends on how its implementation affects the private benefits to both insurance and self-protection. On the one hand, under the standard conditions (see Appendix), insurance and public protection are substitutes because increases in public protection make it less costly to go uninsured. This in itself will tend to promote private protection. To see this, denote the marginal utility of wealth as $\lambda_j \equiv p_jU_0' + (1-p_j)U_1'$ and observe that under a balanced budget constraint,

$$\frac{\partial^2 U_j^*}{\partial I \partial \sigma} \equiv \frac{\partial p_j^*}{\partial \sigma}[(1 - \pi_j^* - p_j^* + f(\phi, \pi_j^*) + p_j^* f_\pi)(U'_{0j} - U'_{1j}) + f_\pi U'_{1j}] + \left[\frac{1}{N} + \frac{\partial \pi_j^*}{\partial \sigma} I_j^*\right] [-\frac{\partial \lambda_j}{\partial I}] \tag{4.3}$$

If the price and probability of loss are sufficiently small, the first term will be weakly negative. It embodies a substitution effect, representing the shift in the marginal benefit of insurance associated with the shift in the probability of loss induced by public protection. The second term is an income effect that could be positive or negative (or zero), depending on the productivity of public protection. CARA utility ensures that it is zero.

The technological relationship between public and private protection is another key determinant of public protection's impact on private protection. Computing the cross-partial between private and public protection reveals that

$$\begin{aligned} \frac{\partial^2 U_j^*}{\partial s \partial \sigma} \equiv & \\ & \frac{\partial^2 p_j}{\partial s_j \partial \sigma}(U_0 - U_1) - [p_j U'_0 + (1 - p_j) U'_1][\frac{\partial^2 \pi}{\partial s \partial \sigma} I_j (1 - f_\pi) + \frac{\partial \pi_j}{\partial s_j} I_j (-f_{\pi\pi} \frac{\partial \pi_j}{\partial \sigma})] \\ & - [\frac{\partial p_j}{\partial \sigma}(U'_0 - U'_1)](k + \frac{\partial \pi}{\partial s_j}(1 - f_\pi) I_j) \\ & - (\frac{1}{N} + \frac{\partial \pi_j}{\partial \sigma})[\frac{\partial p_j}{\partial s_j}(U'_0 - U'_1) - (k + \frac{\partial \pi_j}{\partial s_j}(1 - f_\pi) I_j)(p_j U''_0 + (1 - p_j) U''_1)] \end{aligned} \tag{4.4}$$

For simplicity, suppose that $f_{\pi\pi} = 0$, so that the subsidy is at most linear in the premium. The first line represents the technological relationship between public and private protection—the direct impact of public protection investment on the productivity of private protection in reducing the loss probability. The sign of this effect can be positive (if public protection makes private investments more productive at reducing the loss probability) or negative (if the reverse is true). The second line, which is positive, captures an indirect income effect; by reducing the probability of loss, public protection effectively makes the agent richer and more willing to spend on private protection. The third line represents another type of income effect. Spending on private protection affects the marginal utility of wealth, and this affects the marginal cost of spending resources on public protection. The sign of this effect depends on two factors: the form of the utility function, which determines the relationship between self-protection and the marginal utility of wealth; and whether public protection is costly or beneficial in a strictly pecuniary sense.

If public and private protection are technological complements in reducing the loss probability, the first two terms of Equation 4.4 will be positive. This could happen, for example, if public protective agencies provide information to private agents that allows them to target their own protection better. Public protection would then encourage private protection by making it more productive. The technological effect is positive and reinforced by the indirect income effect (the second line). The traditional income effect could be positive or negative, but is zero under the standard conditions used in the Appendix. The Appendix shows that in this case: $\frac{\partial s}{\partial \sigma} > 0$, and $\frac{\partial I}{\partial \sigma} < 0$.

If, on the other hand, public and private protection are *substitutes* in reducing the loss probability, the technological effect will be negative, while the indirect income effect is positive. The response of the consumer will then depend on which effect dominates. The Appendix proves that, in the case where public and private protection are perfect substitutes, $\frac{\partial s}{\partial \sigma} < 0$, and $\frac{\partial I}{\partial \sigma} = 0$.

Given its widespread adoption, it seems likely that public protection is an optimal policy strategy regardless of its impact on private protection. Its effect on private protection may be quite small compared to the gains from collectivizing protection against terrorism. However, the important point for our purposes is that it need not subsume intervention in the insurance markets: There are gains to pursuing multiple "second-best" policy options, and there are circumstances under which the gains to public protection might even come at the expense of worsening the externalities in private protection.

5 Conclusion

Government responses to insurance market problems bred by terrorism risk seem aggressive, especially relative to the corresponding responses to problems associated with natural disaster risks. This suggests an additional set of issues, beyond the usual ones in catastrophe reinsurance, related to self-protection. Accordingly, the case for or against government intervention in terror insurance markets must consider more than just the availability of capital for catastrophe reinsurance.

Cautious self-protective behavior can hurt national morale by appearing to surrender to the terrorists' demands. Self-protection against terror can also have external effects on the risks faced by other agents. As we have shown, these externalities can form a motivation for government intervention in the insurance market. If terrorists can observe self-protection, their behavior produces negative externalities in protection that can be addressed with insurance subsidies.

The U.S. government's recent enactment of federal backing of insured terrorism losses addresses these externalities in the market for self-protection; the guarantee of federal funds lowers the price of reinsurance capital and thus functions as an implicit subsidy for terror cover. Indeed, the capital market frictions emphasized in this and other catastrophe markets may be the catalyst for government action, but the distinguishing features of the terrorism insurance market—and the ones that may ultimately have driven the enactment of subsidies—are the external effects associated with self protection. A construction project foundering in Manhattan due to lack of affordable terrorism insurance coverage may be seen as a national public policy issue, while similar foundering associated with windstorm or earthquake insurance coverage is not.

This paper interprets terrorism insurance subsidies as a strategy of influencing behavior in the context of a war effort. Put differently, subsidies are intended to foster moral hazard when there are social benefits associated with moral hazard. In this light, government-sponsored terrorism insurance appears to have more in common with war-risk life insurance (government insurance provided to soldiers), deposit insurance, and propaganda (which discourages behavior likely to generate negative externalities, and vice-versa) than with programs aimed at remedying the problems in catastrophe insurance markets (such as the National Flood Insurance Program).

Although the main focus of this paper was the terrorism insurance market, the paper's ideas need not be so limited in their application. To illustrate the extension of the ideas, we have highlighted deposit insurance and banking regulation as an example of insurance subsidies and public protection working in tandem, with the former a policy specifically aimed at discouraging self-protective behavior. Going further, it seems likely that the government's policy of subsidization in other insurance markets—such as health insurance—may also have behavioral goals that have remained largely unexplored by economists. This seems a fruitful area for future research.

Mathematical Appendix

The individual chooses insurance and self-protection to maximize the objective function discussed in the text:

$$Q \equiv p_j U_0(W - L - \tau_j + (1 + f(\phi, \pi_j) - \pi_j)I_j - ks_j) + (1 - p_j)U_1(W - \tau_j + (f(\phi, \pi_j) - \pi_j)I_j - ks_j) + V_j \tag{A.1}$$

This is subject to the budget-balancing constraint:

$$\tau_j = f(\phi, \pi_j)I_j \tag{A.2}$$

The problem in Equation A.1 has the following two first order conditions:

$$Q_{s_j} = \frac{\partial p_j}{\partial s_j}[U_0 - U_1] - [p_j U_0' + (1 - p_j)U_1'](k + I_j(\frac{\partial \pi_j}{\partial s_j} - f_\pi \frac{\partial \pi_j}{\partial s_j})) + V_{s_j} = 0 \tag{A.3}$$

$$Q_{I_j} = p_j U_0'(1 + f(\phi, \pi_j) - \pi_j) - (1 - p_j)U_1'(\pi - f(\phi, \pi_j)) = 0 \tag{A.4}$$

The second derivatives are given by:

$$Q_{I_j I_j} = pU_0''(1 + f - \pi)^2 + (1 - p)U_1''(\pi - f)^2 \tag{A.5}$$

$$Q_{s_j s_j} = \frac{\partial^2 p_j}{\partial s_j \partial s_j}[U_0 - U_1] - 2\frac{\partial p_j}{\partial s_j}[U_0' - U_1']\left(k + I_j \frac{\partial \pi_j}{\partial s_j}(1 - f_\pi)\right) + V_{s_j s_j} + \left(pU_0'' + (1 - p)U_1''\right)\left(k + I_j \frac{\partial \pi_j}{\partial s_j}(1 - f_\pi)\right)^2 - \lambda_j I_j \left[\frac{\partial^2 \pi_j}{\partial s_j \partial s_j}(1 - f_\pi) - f_{\pi\pi}\left(\frac{\partial \pi_j}{\partial s_j}\right)^2\right] \tag{A.6}$$

It is also useful to calculate the cross-partial of Q.

$$Q_{s_j I_j} = \frac{\partial p_j}{\partial s_j}(U_0' - U_1')(f(\phi, \pi_j) - \pi_j) + \frac{\partial p_j}{\partial s_j}U_0' - \frac{\partial \pi_j}{\partial s_j}(1 - f_\pi)\lambda_j - (k + \frac{\partial \pi_j}{\partial s_j}(1 - f_\pi)I_j)[p_j U_0''(1 + f(\phi, \pi_j) - \pi_j) + (1 - p_j)U_1''(f(\phi, \pi_j) - \pi_j)] \tag{A.7}$$

The term $\lambda_j \equiv p_j U_0' + (1 - p_j)U_1'$ represents the marginal utility of wealth.

Before proceeding with comparative statics, we spend some time examining the second order conditions. The fulfillment of the second order conditions turns out to depend on the "curvature" of the loss probability function relative to the curvature of the utility function. The following proposition identifies some conditions sufficient for the first order conditions to represent a maximum.

Proposition 1 *Let p be decreasing and convex and U be a CARA utility function with coefficient of absolute risk aversion given by μ. Let $Z = \frac{p''}{p'}$ and $Y = \frac{p'}{p}$, with $\mu = |Y| \leq |Z|$. If, in addition, subsidized prices are weakly unfair, $f_{\pi\pi} = 0$, $V = 0$, $k = 1$, and $\pi < 0.5$, the second order conditions are satisfied.*

Proof: The SOC are satisfied if (a) $Q_{I_jI_j} < 0$, (b) $Q_{s_js_j} < 0$, and (c) $Q_{I_jI_j}Q_{s_js_j} - \left(Q_{s_jI_j}\right)^2 > 0$. Inspection of Equation A.5 reveals that (a) is satisfied. To see that (b) is satisfied, use the assumptions above, together with the fact that $\pi' = p'$, to write Equation A.6 as:

$$Q_{s_js_j} = Z\left(p_j^{'}[U_{0j} - U_{1j}] - \lambda p_j'(1 - f_\pi)I_j\right) + \mu\lambda_j\left(k + I_jp_j'(1 - f_\pi)\right)^2$$

or

$$Q_{s_js_j} = Z\lambda_jk + \mu\lambda_j\left(k + I_jp_j'(1 - f_\pi)\right)^2$$

Since $0 < (k + I_jp'(1 - f_\pi)) < k = 1$ and $-Z \geq \mu$, $Q_{s_js_j} < 0$.

To see that (c) is satisfied, use CARA utility and Equation A.4) to write Equation A.5 as

$$Q_{I_jI_j} = -\mu\lambda_j(1 + f - \pi_j)(\pi_j - f).$$

Similarly, use Equation A.4 to write Equation A.7 as

$$Q_{s_jI_j} = Y\lambda_j\left[\frac{(1 + f - \pi_j)(\pi_j - f)}{(1 - p_j)} - p_j(1 - f_\pi)\right].$$

Next, solve for $\varepsilon_j = I_jp_j'(1 - f_\pi)$. From Equation A.3,

$$\frac{p'[U_0 - U_1]}{\lambda_j} - k = \varepsilon_j,$$

which simplifies to

$$\varepsilon_j = -\frac{Y}{\mu}\left(\frac{p[U_0' - U_1']}{\lambda_j}\right) - k = \left(\frac{\pi_j - f - p_j}{(1 - p_j)}\right) - k.$$

So, we can now write:

$$Q_{I_jI_j}Q_{s_js_j} > \mu^2\lambda_j^2(\pi_j - f)(1 + f - \pi_j)\left[k - (k + \varepsilon_j)\right]^2$$

$$\left(Q_{s_jI_j}\right)^2 < \mu^2\lambda_j^2\left[\frac{(1 + f - \pi_j)(\pi_j - f)}{(1 - p_j)}\right]^2$$

Some algebra shows that

$$Q_{I_jI_j}Q_{s_js_j} - \left(Q_{s_jI_j}\right)^2 > 0 \Longleftrightarrow p < 0.5. \quad \textbf{QED}.$$

Even if the second order conditions hold, comparative statics are complicated by the interactions among the agents. We must consider these interactions when determining how a change in an underlying parameter—such as the level of the subsidy—will affect individual behavior. To do this, we study a symmetric equilibrium where all agents have the same preferences, make identical choices, and face the same prices, loss probabilities, subsidies, and taxes. Furthermore, we impose additional restrictions on the relationship between the probability of loss and agent actions. Specifically, as delineated in Proposition 3, we require that $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j} \frac{\partial p_j}{\partial s_k}\right|$ and $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$.

Are these restrictions reasonable? To shed more light on what they mean, consider the terrorist problem outlined in Equations 3.4 and 3.1. In the following proposition, we show that both restrictions are plausible in the case of a radical terrorist group. The first restriction is a direct implication of terrorist maximization in a symmetric setting under standard regularity conditions. The second restriction depends, in part, on third derivatives of the loss probability function; if these terms are sufficiently small relative to the second and first order effects, the second restriction will hold. To illustrate this, we assume that the third derivatives are zero in the following proposition.

Proposition 2 *Suppose terrorists solve Equations 3.4 and 3.1, with $P = 0$, $\rho_s < 0$, $\rho_r > 0$, $\rho_{rr} < 0$, $\rho_{ss} > 0$, $\rho_{sr} < 0$, and $s_i = s_j$ observed $\forall i, j$. Let $p_j = \rho(s_j, r_j)$. Then, $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j} \frac{\partial p_j}{\partial s_k}\right|$. Suppose, in addition, that ρ_s and p_{sr} are bounded from below, ρ_r and ρ_{ss} are bounded away from zero, and all third partials of ρ are zero. Then, for sufficiently large N, $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$.*

Proof: Rewrite the overall maximization problem as

$$\max_{\{r_i\}} \left\{ \sum_{i=1}^{N-1} \rho\left(s_i, r_i\right) + \rho\left(s_N, R - \sum_{i=1}^{N-1} r_i\right) \right\}.$$

This leads to the first order conditions

$$\rho_r^i - \rho_r^N = 0 \qquad i = 1, ..., N-1.$$

The Hessian is given by:

$$H = \begin{bmatrix} \rho_{rr}^1 + \rho_{rr}^N & \rho_{rr}^N & \cdot\ \cdot\ \cdot & \rho_{rr}^N \\ \rho_{rr}^N & \rho_{rr}^2 + \rho_{rr}^N & & \rho_{rr}^N \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \rho_{rr}^N & \rho_{rr}^N & \cdot\ \cdot\ \cdot & \rho_{rr}^{N-1} + \rho_{rr}^N \end{bmatrix},$$

where the superscripts indicate the subscript of the first argument in the probability function (i.e., $\rho^1_{rr} = \rho_{rr}(s_1,r_1)$). Evidently, for all i and j, $\rho^i_{rr} = \rho^j_{rr} = \rho_{rr} < 0$. It is then easy to verify that the second order conditions for a maximum hold.

The determinant of H can be calculated as $N(\rho_{rr})^{N-1}$. A simple exercise in comparative statics reveals that

$$\frac{\partial r_1}{\partial s_1} = \frac{1}{N(\rho_{rr})^{N-1}} \begin{vmatrix} -\rho_{sr} & \rho_{rr} & . & . & . & \rho_{rr} \\ 0 & 2\rho_{rr} & \rho_{rr} & . & . & \rho_{rr} \\ . & \rho_{rr} & 2\rho_{rr} & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & \rho_{rr} & . & . & . & 2\rho_{rr} \end{vmatrix} = \frac{-\rho_{sr}(N-1)}{\rho_{rr}N} = \frac{\partial r_i}{\partial s_i}, \quad \forall i. \tag{A.8}$$

Similarly,

$$\frac{\partial r_i}{\partial s_j} = \frac{\rho_{sr}}{\rho_{rr}N}, \forall\, i, j \neq i. \tag{A.9}$$

We can then write

$$\frac{\partial p_i}{\partial s_i} = \frac{\partial \rho^i}{\partial s_i} + \frac{\partial \rho^i}{\partial r_i}\frac{\partial r_i}{\partial s_i}, \tag{A.10}$$

$$\frac{\partial p_i}{\partial s_j} = \frac{\partial \rho^i}{\partial r_i}\frac{\partial r_i}{\partial s_j}. \tag{A.11}$$

Evidently,

$$\sum_{j\neq i} \frac{\partial \rho^i}{\partial r_i}\frac{\partial r_i}{\partial s_j} = -\frac{\partial \rho^i}{\partial r_i}\frac{\partial r_i}{\partial s_i} \Longrightarrow \left|\frac{\partial p_i}{\partial s_i}\right| > \left|\sum_{j\neq i} \frac{\partial p_i}{\partial s_j}\right|.$$

Moving on, note that

$$\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j} \frac{\partial p_j}{\partial s_k}}\right] = \frac{\frac{\partial^2 p_j}{\partial s_j \partial s_j}\sum_{k\neq j}\frac{\partial p_j}{\partial s_k} - \frac{\partial p_j}{\partial s_j}\sum_{k\neq j}\frac{\partial^2 p_j}{\partial s_j \partial s_k}}{\left(\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}\right)^2}. \tag{A.12}$$

To determine the value of this expression, we calculate

$$\frac{\partial^2 p_j}{\partial s_j \partial s_j} = \rho_{ss} + 2\rho_{sr}\frac{\partial r_j}{\partial s_j} + \frac{\partial^2 p_j}{\partial r_j \partial r_j}\left(\frac{\partial r_j}{\partial s_j}\right)^2 + \rho_r \frac{\partial^2 r_j}{\partial s_j \partial s_j} \tag{A.13}$$

$$\frac{\partial^2 p_j}{\partial s_j \partial s_k} = \rho_{sr}\frac{\partial r_j}{\partial s_k} + \frac{\partial^2 p_j}{\partial r_j \partial r_j}\left(\frac{\partial r_j}{\partial s_j}\right)\left(\frac{\partial r_j}{\partial s_k}\right) + \rho_r \frac{\partial^2 r_j}{\partial s_j \partial s_k} \tag{A.14}$$

The proposition is proven if the numerator of Equation A.12 is positive. Using Equations A.8, A.9, A.10, A.11, A.13, and A.14, we write the numerator as

$$\left[\rho_{ss} - \frac{(N+1)}{N}\frac{(N-1)}{N}\frac{(\rho_{sr})^2}{\rho_{rr}}\right]\left(\rho_r \frac{\rho_{sr}}{\rho_{rr}}\frac{(N-1)}{N}\right) - \left[\frac{1}{N}\frac{(\rho_{sr})^2}{\rho_{rr}}\frac{(N-1)}{N}\right]\left(\rho_s - \rho_r\frac{\rho_{sr}}{\rho_{rr}}\frac{(N-1)}{N}\right), \tag{A.15}$$

where we have noted that the last terms in both Equations A.13 and A.14 are zero, as they depend only on third derivatives of ρ. Furthermore, Equation A.15 > 0 reduces to

$$\rho_{ss} - \frac{(N-1)}{N}\frac{(\rho_{sr})^2}{\rho_{rr}} - \frac{1}{N}\frac{\rho_s}{\rho_r}\rho_{sr} > 0$$

Since $\left|\frac{\rho_s}{\rho_r}\rho_{sr}\right|$ is bounded and ρ_{ss} is positive and bounded away from zero, the preceding condition holds for sufficiently large N. Therefore, $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$. **QED.**

Now we are ready to study how equilibrium choices respond to changes in insurance subsidies and changes in public protection. In the analysis that follows, we adopt a set of simplifying assumptions that we refer to as the **standard conditions**:

1. V=0.
2. The second order conditions hold.
3. Utility is CARA.
4. $0.5 > \pi - f \geq p > 0$.
5. $f_{\phi\pi} \geq 0$

Proposition 3 *Assume the standard conditions,* $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}\right|$, *and* $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$. *Then* $sign(\frac{ds_j}{d\phi}) = sign(\frac{ds_j}{d\phi}|_{s_k})$, *and* $\frac{dI_j}{d\phi} > 0$.

Proof: We can differentiate the first-order conditions as follows:

$$
\begin{aligned}
&Q_{s_j\phi} + Q_{s_js_j}\frac{ds_j}{d\phi} + \sum_{k\neq j} Q_{s_js_k}\frac{ds_k}{d\phi} + Q_{s_jI_j}\frac{dI_j}{d\phi} = 0 \\
&Q_{I_j\phi} + Q_{I_js_j}\frac{ds_j}{d\phi} + \sum_{k\neq j} Q_{I_js_k}\frac{ds_k}{d\phi} + Q_{s_jI_j}\frac{dI_j}{d\phi} = 0
\end{aligned}
\tag{A.16}
$$

With identical agents in a symmetric equilibrium, $\frac{ds_j}{d\phi} = \frac{ds_k}{d\phi}, \forall j, k$. Computing $\frac{ds_j}{d\phi}$ and applying these identities yields the expression:

$$
\frac{ds_j}{d\phi}\left(1 + \frac{\sum_{k\neq j}(Q_{s_js_k}Q_{I_jI_j} - Q_{I_js_k}Q_{s_jI_j})}{D}\right) = \frac{ds_j}{d\phi}|_{s_k} \tag{A.17}
$$

D is $Q_{s_js_j}Q_{I_jI_j} - Q_{s_jI_j}Q_{s_jI_j}$ To prove $sign(\frac{ds_j}{d\phi}) = sign(\frac{ds_j}{d\phi}|_{s_k})$, it suffices to show that the term in parentheses is positive. To begin, observe the following equivalence:

$$
Q_{s_j} \equiv \left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\left(\sum_{k\neq j} Q_{s_k} - V_{s_k}\right)\right] + V_{s_j} - \lambda_j k, \tag{A.18}
$$

where λ_j is the marginal utility of wealth. Differentiating this equivalence, and observing that λ_j is independent of I_j and s_j under CARA utility and $V = 0$, we obtain:

$$
Q_{s_js_j} = \left[\frac{\partial}{\partial s_j}\left(\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right)\right]\sum_{k\neq j} Q_{s_k} + \frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\sum_{k\neq j} Q_{s_ks_j} \tag{A.19}
$$

$$
Q_{s_jI_j} = \frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\sum_{k\neq j} Q_{s_kI_j} \tag{A.20}
$$

Using the above two expressions, we can derive:

$$
\begin{aligned}
&\frac{Q_{I_jI_j}\sum_{k\neq j} Q_{s_js_k} - Q_{s_jI_j}\sum_{k\neq j} Q_{I_js_k}}{D} = \\
&\frac{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}{\frac{\partial p_j}{\partial s_j}} - \frac{Q_{I_jI_j}\sum_{k\neq j} Q_{s_k}\frac{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}{\frac{\partial p_j}{\partial s_j}}[\frac{\partial}{\partial s_j}(\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}})]}{D}
\end{aligned}
\tag{A.21}
$$

Using the conditions that $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}\right|$, and $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$, it is straightforward to show that this expression exceeds -1. This proves $sign(\frac{ds_j}{d\phi}) = sign(\frac{ds_j}{d\phi}|_{s_k})$.

Moving on,

$$\frac{dI_j}{d\phi} = \frac{-Q_{I_j\phi}\left[Q_{s_js_j} + \sum_{k\neq j} Q_{s_js_k}\right] + Q_{s_j\phi}\left[Q_{s_jI_j} + \sum_{k\neq j} Q_{I_js_k}\right]}{D + \sum_{k\neq j}(Q_{s_js_k}Q_{I_jI_j} - Q_{I_js_k}Q_{s_jI_j})}$$

Arguments made above imply that the denominator is positive. We now show that the numerator is also positive. Budget balance and the standard conditions imply that $Q_{I_j\phi} > 0$, $Q_{s_j\phi} < 0$, and $Q_{s_jI_j} < 0$. Equation A.20, together with the assumption that $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}\right|$, implies that $\left[Q_{s_jI_j} + \sum_{k\neq j} Q_{I_js_k}\right] < 0$. Since $Q_{s_j\phi} < 0$, the second term in the numerator is positive. Equation A.19, together with the assumptions that $\frac{\partial}{\partial s_j}\left[\frac{\frac{\partial p_j}{\partial s_j}}{\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}}\right] > 0$ and $\left|\frac{\partial p_j}{\partial s_j}\right| > \left|\sum_{k\neq j}\frac{\partial p_j}{\partial s_k}\right|$, implies that $\left[Q_{s_js_j} + \sum_{k\neq j} Q_{s_js_k}\right] < 0$. Since $Q_{I_j\phi} > 0$, the first term in the numerator is also positive. Since the numerator and denominator are positive, $\frac{dI_j}{d\phi} > 0$. **QED**

Corollary 3.1 *Under the conditions of Proposition 3,* $sign(\frac{ds_j}{d\sigma}) = sign(\frac{ds_j}{d\sigma}|_{s_k})$. *If, in addition,* $sign(Q_{I_j\sigma}) = -sign(Q_{s_j\sigma})$, $sign(\frac{dI_j}{d\sigma}) = sign(\frac{dI_j}{d\sigma}|_{s_k})$.

Proof. The proof of $sign(\frac{ds_j}{d\sigma}) = sign(\frac{ds_j}{d\sigma}|_{s_k})$ is identical to the one in the proposition, if we replace ϕ with σ. For the second result, observe that

$$\frac{dI_j}{d\sigma}|_{s_k} = \frac{-Q_{I_j\sigma}Q_{s_js_j} + Q_{s_j\sigma}Q_{s_jI_j}}{D}$$

$$\frac{dI_j}{d\sigma} = \frac{-Q_{I_j\sigma}\left[Q_{s_js_j} + \sum_{k\neq j} Q_{s_js_k}\right] + Q_{s_j\sigma}\left[Q_{s_jI_j} + \sum_{k\neq j} Q_{I_js_k}\right]}{D + \sum_{k\neq j}(Q_{s_js_k}Q_{I_jI_j} - Q_{I_js_k}Q_{s_jI_j})}$$

Since $Q_{s_js_j}$ and $\left(Q_{s_js_j} + \sum_{k\neq j} Q_{s_js_k}\right)$ are both negative, as are $Q_{s_jI_j}$ and $\left(Q_{s_jI_j} + \sum_{k\neq j} Q_{I_js_k}\right)$, the numerators of the two expressions share the same sign. As argued previously, both D and $D+\sum_{k\neq j}(Q_{s_js_k}Q_{I_jI_j} - Q_{I_js_k}Q_{s_jI_j})$ are positive. Hence, $sign(\frac{dI_j}{d\sigma}) = sign(\frac{dI_j}{d\sigma}|_{s_k})$. **QED**

Proposition 4 *Assume the standard conditions with* $\pi - f > p$. *Then* $\frac{ds_j}{d\phi}|_{s_k} < 0$ *and* $\frac{dI_j}{d\phi}|_{s_k} > 0$.

Proof. With $0.5 > \pi > p$, $\frac{\partial \pi}{\partial s} = \frac{\partial p_j}{\partial s}$, and CARA utility, $Q_{s_jI_j} < 0$. To see this, observe first that $\frac{\partial p_j}{\partial s_j}(U_0' - U_1')(f(\phi,\pi_j) - \pi) + \frac{\partial p_j}{\partial s_j}U_0' - \frac{\partial \pi}{\partial s_j}(1 - f_\pi)\lambda < 0$, provided that $\frac{\partial \pi}{\partial s} = \frac{\partial p_j}{\partial s}$, and $\pi + p < 1$. In the last term, $(k + \frac{\partial \pi}{\partial s_j}(1 - f_\pi)I_j) > 0$, because $U_0 < U_1$. CARA utility guarantees that the final term in parentheses (which equals $\frac{d\lambda_j}{dI_j}$) is zero.

Under a balanced budget constraint, we have the relationships $Q_{I_j\phi}|_{s_k} = f_\phi\lambda_j > 0$, and $Q_{s_j\phi}|_{s_k} = f_{\phi\pi}\frac{\partial\pi_j}{\partial s_j}\lambda_j \leq 0$. Therefore, standard comparative statics arguments imply that:

$$\frac{ds_j}{d\phi}|_{s_k} = \frac{-Q_{I_jI_j}Q_{s_j\phi} + Q_{I_j\phi}Q_{I_js_j}}{D} \tag{A.22}$$

$$\frac{dI_j}{d\phi}|_{s_k} = \frac{-Q_{I_j\phi}Q_{s_js_j} + Q_{I_js_j}Q_{s_j\phi}}{D} \tag{A.23}$$

where D is the (positive, by virtue of Proposition 1) determinant of the Hessian matrix. Evidently, $\frac{ds_j}{d\phi}|_{s_k}$ is strictly negative, and $\frac{dI_j}{d\phi}|_{s_k}$ is strictly positive. **QED**

Corollary 4.1 *Under the conditions of Propositions 2 and 4,* $\frac{ds_j}{d\phi} < 0$*, and* $\frac{dI_j}{d\phi} > 0$*.*

Proof. Under the standard conditions, Proposition 4 implies that $\frac{ds_j}{d\phi}|_{s_k} < 0$, and $\frac{dI_j}{d\phi}|_{s_k} > 0$. The strategic equilibrium conditions and Proposition 3 then imply that $\frac{ds_j}{d\phi} < 0$ and $\frac{dI_j}{d\phi} > 0$. **QED**

Proposition 5 *Assume the standard conditions (except CARA utility) with* $\pi - f = p$*. If* $f(\phi, \pi_j) = \phi\pi_j$*, then* $\frac{ds_j}{d\phi}|_{s_k} < 0$ *and* $\frac{dI_j}{d\phi}|_{s_k} > 0$*. If* $f(\phi, \pi_j) = \phi$*, then* $\frac{ds_j}{d\phi}|_{s_k} = 0$ *and* $\frac{dI_j}{d\phi}|_{s_k} > 0$*.*

Proof. With fair pricing, $Q_{s_jI_j} = \lambda f_\pi \frac{\partial p_j}{\partial s_j}$. With a multiplicative subsidy, this expression is strictly negative, and $f_{\phi\pi} > 0$. With an additive subsidy, both are zero. Equations A.22 and A.23 then imply the results**. QED**

Corollary 5.1 *Assume the conditions of Propositions 2 and 5. If* $f(\phi, \pi_j) = \phi\pi_j$*,* $\frac{ds_j}{d\phi} < 0$*, and* $\frac{dI_j}{d\phi} > 0$*. If* $f(\phi, \pi_j) = \phi$*,* $\frac{ds_j}{d\phi} = 0$*, and* $\frac{dI_j}{d\phi} > 0$*.*

Proof. The conditions of Proposition 5 imply that $\frac{ds_j}{d\phi}|_{s_k} \leq 0$ and $\frac{dI_j}{d\phi}|_{s_k} > 0$, where the former relation holds with strict inequality for a multiplicative subsidy and strict equality for an additive subsidy. Propositions 2 and 3 then imply the results. **QED**

Proposition 6 *Assume the standard conditions and* $\frac{\partial^2 p}{\partial s \partial\sigma} < 0$*. Then* $\frac{ds_j}{d\sigma}|_{s_k} > 0$ *and* $\frac{dI_j}{d\sigma}|_{s_k} < 0$*.*

Proof. In the text, it was shown that these conditions imply $Q_{I_j\sigma} < 0$ and $Q_{s_j\sigma} > 0$. The proof of Proposition 4 showed that they implied $Q_{I_js_j} < 0$. Standard comparative statics arguments imply that:

$$\frac{ds_j}{d\sigma}|_{s_k} = \frac{-Q_{I_jI_j}Q_{s_j\sigma} + Q_{I_j\sigma}Q_{I_js_j}}{D} \tag{A.24}$$

$$\frac{dI_j}{d\sigma}|_{s_k} = \frac{-Q_{I_j\sigma}Q_{s_js_j} + Q_{s_j\sigma}Q_{s_jI_j}}{D} \tag{A.25}$$

The results then follow from these expressions. **QED**

Corollary 6.1 *Under the conditions of Propositions 2 and 6,* $\frac{ds_j}{d\sigma} > 0$*, and* $\frac{dI_j}{d\sigma} < 0$*.*

Proof. Proposition 6 implies that $\frac{ds_j}{d\sigma}|_{s_k} > 0$, $Q_{I_j\sigma} < 0$, and $Q_{s_j\sigma} > 0$. Corollary 3.1 then implies that $\frac{ds_j}{d\sigma} > 0$, and $\frac{dI_j}{d\sigma} < 0$. **QED.**

Proposition 7 *Assume the standard conditions and that* σ *and* s *are perfect substitutes* ($\frac{\partial p_j}{\partial s_j} = \frac{\partial p_j}{\partial \sigma} < 0, \frac{\partial^2 p_j}{\partial s_j \partial \sigma} = \frac{\partial^2 p_j}{\partial s_j \partial \sigma} > 0$)*. Then* $\frac{ds_j}{d\sigma}|_{s_k} < 0$ *and* $\frac{dI_j}{d\sigma}|_{s_k} = 0$*.*

Proof. These conditions imply that $Q_{I_j\sigma} = Q_{I_js_j}$ and $Q_{s_j\sigma} = Q_{s_js_j}$. Comparative statics (and substitution) yields:

$$\frac{ds_j}{d\sigma}|_{s_k} = \frac{-Q_{I_jI_j}Q_{s_js_j} + Q_{I_js_j}Q_{I_js_j}}{D} = \frac{-D}{D} < 0 \tag{A.26}$$

$$\frac{dI_j}{d\sigma}|_{s_k} = \frac{-Q_{I_js_j}Q_{s_js_j} + Q_{I_js_j}Q_{s_js_j}}{D} = 0 \tag{A.27}$$

QED.

Corollary 7.1 *Under the conditions of Propositions 2 and 7,* $\frac{ds_j}{d\sigma} < 0$*.*

Proof. Proposition 7 implies that $\frac{ds_j}{d\sigma}|_{s_k} < 0$. Corollary 3.1 then implies that $\frac{ds_j}{d\sigma} > 0$. **QED.**

Reconciliation to Ehrlich and Becker (1972)

Proposition 5 shows that subsidies encourage insurance and discourage self-protection at the point of actuarially fair pricing, i.e., that insurance and self-protection are *substitutes.* This appears to contradict a result of Ehrlich and Becker (discussed on page 642 and proved on pages 646-7), in which they show that insurance and self-protection are *complements* at the point of actuarially fair pricing. In what follows, we show that there is no contradiction: The different results flow from different implicit assumptions about the relationship between self-protection and the "expense loading" (or, in our case, the magnitude of the subsidy). In particular, we show that the Ehrlich and Becker (EB) framework imparts some highly unusual properties to the expense loading, at least relative to the standard models of insurance expense loadings.

We start by translating the EB assumptions into equivalent assumptions for our framework. EB study the price of insurance defined as

$$(1+\lambda)\frac{p}{(1-p)},$$

where λ is an expense loading. The EB theorem studies a shift in the expense loading (caused by the exogenous parameter θ) that does not alter the relationship between price and self-protection s. (Note that in EB's notation, self-protection is r). This implies the following properties for λ (see EB's proof, (B7) and Footnote 49):

$$\frac{p'}{(1-p)}\lambda_\theta = -p\lambda_{s\theta} \tag{A.28}$$

$$\lambda_s = 0 \tag{A.29}$$

Putting this into our setup, we have:

$$(1+\lambda)\frac{p}{(1-p)} = \frac{p+\alpha}{1-(p+\alpha)} \Longrightarrow \alpha = \frac{\lambda p(1-p)}{1+\lambda p}$$

From Equations A.28 and A.29, we can deduce the following properties for $\alpha(s,\theta)$:

$$\alpha_s^0 = \frac{\partial\alpha}{\partial s}|_{\lambda=0} = \frac{\lambda_s p(1-p)}{1+\lambda p} = 0$$

$$\alpha_\theta^0 = \frac{\partial\alpha}{\partial\theta}|_{\lambda=0} = \lambda_\theta p(1-p) > 0$$

$$\alpha_{\theta s}^0 = 2p^2(1-p)\left[\frac{-p'}{(1-p)p} - \lambda_s\right] > 0$$

The EB result can now be reproduced within our framework. We rewrite the consumer problem as choosing s and I to maximize:

$$\begin{aligned} Q \equiv pU_0(W - L - \tau + (1-p-\alpha(s,\theta))I - ks) + \\ (1-p)U_1(W - \tau - (p+\alpha(s,\theta))I - ks) \end{aligned} \tag{A.30}$$

Evaluating all quantities at the point of actuarial fairness allows us to write the "comparative statics" equations as:

$$\begin{bmatrix} p(1-p)U_*'' & -\alpha_s^0 U_*' \\ -\alpha_s^0 U_*' & -(p_{ss}+\alpha_{ss})IU_*' \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial\theta} \\ \frac{\partial s}{\partial\theta} \end{bmatrix} = \begin{bmatrix} \alpha_\theta^0 U_*' \\ \alpha_{s\theta}^0 IU_*' \end{bmatrix} \tag{A.31}$$

Since α_s^0 is zero, while α_θ^0 and $\alpha_{\theta s}^0$ are positive, it is evident that the EB conditions lead to complementarity: $\frac{\partial I}{\partial \theta}, \frac{\partial s}{\partial \theta} < 0$.

Our analysis of subsidy changes, while we chose different notation, can obviously be translated into this setup by studying changes in α. However, our structuring of the subsidies implies different properties for α. In particular, under additive subsidies (e.g., $\alpha \equiv \theta$), $\alpha_s = 0$, and $\alpha_{s\theta} = 0$; under multiplicative subsidies (e.g., $\alpha = \theta p$), $\alpha_s < 0$, and $\alpha_{s\theta} < 0$. It is easily verified that the preceding properties yield the results of Proposition 5 when applied to Equation A.31.

Essentially, the differences arise because of differences in the assumed relationship between the premium and self-protection. In the EB analysis, an increase in the expense loading leads to a *decrease* in the productivity of self-protection: Self-protection is strictly less effective at reducing the premium. In our analysis, the opposite is true: An increase in the loading leads either to an increase or no change in the productivity of self-protection. Of course, either could be true in practice, but our assumptions match closely with the standard models of expenses—which are either fixed, proportional to expected loss, or a combination thereof.

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