

Providers' Affiliation, Insurance and Collusion*

Jean-Marc Bourgeon[†] Pierre Picard[‡] Jerome Pouyet[§]

March 21, 2007

Abstract

This paper provides a theoretical analysis of the benefits for an insurance company to develop its own network of service providers when insurance fraud is characterized by collusion between policyholders and providers. In a static framework without collusion, exclusive affiliation of providers allows insurance companies to recover some market power and to lessen competition on the insurance market. This entails a decrease in the insured's welfare. However, exclusive affiliation of providers may entail a positive effect on customers' surplus when insurers and providers are engaged in a repeated relationship. In particular, while insurers must cooperate to retaliate against a fraudulent provider under non-exclusive affiliation, no cooperation is needed under exclusive affiliation. In that case, an insurer is indeed able to reduce the profit of a malevolent provider by moving to collusion-proof contracts when collusion is detected, and this threat may act as a deterrent for fraudulent activities. This possibility may supplement an inefficient judicial system: it is thus a second-best optimal anti-fraud policy.

Journal of Economic Literature Classification Number: G22, L22.

Keywords: Insurance, Affiliation/Vertical Relationships, Fraud.

1 Introduction

The relationships between insurance companies and service providers (e.g., car repairers, hospitals...) have experienced dramatic new trends during the two last decades. Concentration in the insurance market and in the markets for related services went along with the affiliation of providers by insurance companies, creating networks of affiliated or preferred service providers. Such vertical relationships may appear at odds with the usual assumption of competitive insurance markets and they may strongly affect the efficiency of resource allocations and risk sharing. As insurance companies pay for the service provided to their clients in case of an accident, they may impose restrictions on available providers. This is often the case in the health sector, particularly in the U.S. since the so-called managed care organizations (mainly HMO and PPO systems) are prevalent nowadays.¹

*We are grateful to an anonymous referee for comments on the earlier version of this paper.

[†]INRA and Ecole Polytechnique. Address: INRA - Économie Publique, 16 rue Claude Bernard, 75231 Paris Cedex 05, FRANCE. E-mail: bourgeon@inapg.fr

[‡]Ecole Polytechnique and HEC-Paris. Address: Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, FRANCE. E-mail: pierre.picard@polytechnique.edu

[§]Ecole Polytechnique. Address: Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, FRANCE. E-mail: jerome.pouyet@polytechnique.edu

¹For a presentation of the questions raised by managed-care organizations, see Gaynor and Haas-Wilson (1999).

This is also true for other activities such as car repair by means of Direct Repair Programs (DRP).

The rise of DRP is indeed particularly striking. These programs have been around for well over twenty years in the U.S., but their actual impact did not begin to be felt, nor did the discussion regarding their legitimacy grow so heated, until the 90s. In the middle of the 90s, the estimated number of repair facilities that participated in DRP was approximately 5,000 across the U.S., with a claims volume at 8% of overall claims. Today, there are over 20,000 referral shops, thanks to the over 13,000 shops participating in State Farm's program. Allstate is next in line with approximately 5,000 facilities. In the U.S., DRP claims volume has raised regularly, reaching 20% of the total automobile claims in 1998 and 45% in 2000.

To illustrate the possible effects of DRP on market shares, Figure 1 uses data from the California Department of Insurance. It reports the market shares of the most important insurers in California from 1998 to 2004 for the private passenger auto liability line of business.

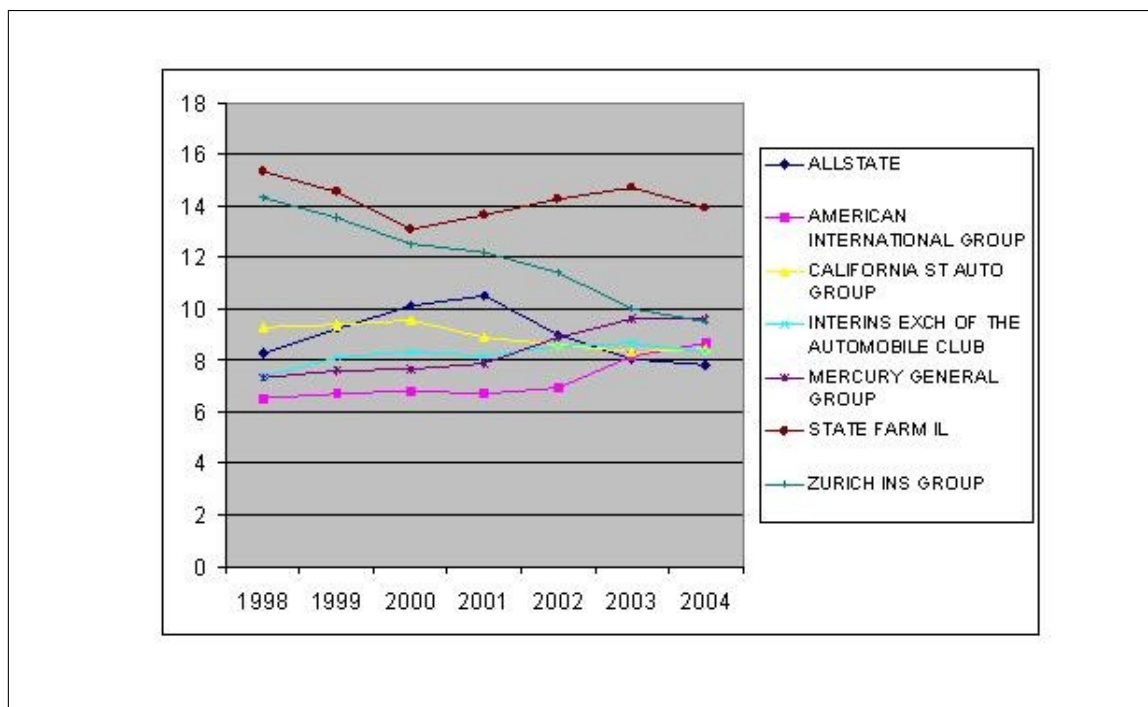


Figure 1: Market shares of the dominant insurers in California (1998-2004, passenger auto liability line of business, source: California Department of Insurance).

As it appears from the figure, up to 2000-2001, Allstate was steadily gaining market share whereas the market share of its main rival, State Farm, was declining. This might be related to the fact that Allstate was a pioneer in the implementation of Direct Repair Programs. Although the creation of DRP may have interfered with other factors, the DRP creation has provided Allstate with a strategic advantage over its competitors, until the latter finally decided to implement their own referral programs.

That trend towards more integration between insurers and service providers certainly is a world-wide phenomenon. For instance, the largest Australian automobile insurer, IAG, has recently launched a web-based quotation system known as Preferred Repairer Network System. This preference network allows claimants to benefit from services such as the timely assessment of claims, timely authorization to proceed with repairs, the opportunity

to get an early release of the repaired vehicle and payment of towing expenses.

Whether such a network of preferred providers is in the interest of insureds and whether it favors economic efficiency is still a matter of discussion. In particular, smash repairers who are not on IAG's Preferred Smash Repairers List complain they are losing business because they are being surpassed by IAG's tele-claim system. Recently, the members of the New South Wales Legislative Assembly criticized the IAG Preferred Repairers Network by emphasizing that policyholders do not have the unfettered freedom of choice provided by other major car insurers, such as Allianz.² The creation of these providers networks and the concentration trends that have followed have been criticized as a way for insurance companies to restrict competition and to increase their profits at the customers' expense.³ This has generally led regulators or competition authorities to adopt a conservative and negative stance vis-à-vis these vertical agreements.

However, more favorable explanations have also been put forward to justify the creation of DRPs, including the fact that they allow insurers (and ultimately policyholders) to benefit from economies of scale. Another reason that may stimulate the creation of referral lists lies in the alarming extent of the insurance fraud problem. Indeed, these programs may allow to better monitor the providers. Moreover, the threat of excluding deceitful providers from referral lists may act as a deterrence mechanism to dissuade providers from defrauding the insurance company they are affiliated with. This paper will indeed argue that the vertical organization between insurance companies and service providers affects the amount of fraudulent activities in the insurance sector. More specifically, the purpose of the following analysis is to appraise the consequences of the creation of service providers networks on the efficiency of resource allocation in insurance markets. Specific attention will be granted to the risk of collusion between insureds and providers in relation to insurance fraud.

How large a problem is insurance fraud is generally a difficult question to gauge as only the tip of the iceberg might be observed. According to insurance companies' specialized press, it is the second most popular white-collar crime in the U.S. Best Insurance Review estimates that 6% of claims have some element of fraud.

The importance of the relationship between insurance fraud and the behavior of service providers sometimes transpires to the public through the results of police investigations. Considering the collision repair industry only, investigations recently reveal the following striking examples among others:⁴

- In San Francisco, six people, including automobile repair shop owners and employees, were involved in more than two dozen fraudulent insurance claims. This followed the closure of 10 shops and the arrests of more than 40 people few months after a six-month undercover investigation of Santa Clara County, California, collision repair shops.⁵
- In January 2005, a New York shop owner and manager were indicted on 23 counts of insurance fraud, grand larceny and falsifying business records. According to the district attorney, the defendants fraudulently inflated insurance claims, usually by

²See New South Wales Legislative Assembly Hansard, 15 September 2005, p 49, Article 33.

³Likewise in 2003 the Texas Senate voted to pass a bill which banned ownership of repair shops by insurance companies.

⁴Many other spectacular examples of insurance fraud cases can be found at <http://www.insurancefraud.org/hallofshame/>.

⁵See Automotive Body Repair News, November, 2002.

either enhancing the damage to a vehicle or by simply failing to do the work for which they were paid.⁶

To capture the main features of the insurance fraud problem with collusion while keeping the analysis tractable, we will limit attention to a simple setup of a double vertical duopoly with two insurance companies and two service providers. Our modelling choices and main results may be summarized as follows.

Service providers (say, car repairers) compete on a horizontally differentiated market for the service they provide to customers. It is easily understood that service providers are not valued the same by customers. Indeed, both the convenience of their geographic locations and their word-of-mouth reputations may generate various patterns of demand by the users. Hence, providers have some market power due to the imperfect substitutability of their services.

By contrast, we assume that insurers are perceived as perfectly substitutable by their potential clients. They compete by offering insurance contracts which consist in an insurance premium and a net reimbursement in the event of an accident. These insurance contracts may also depend on the provider chosen by the customers.

Several affiliation structures are considered. We first perform a static (one period) analysis of the strategic interactions between insurers, service providers and policyholders. In the case of non-exclusive affiliation, customers of both insurance companies are free to choose their providers. Hence, among the customers of each insurance company, some choose to visit one provider in the event of an accident whereas others visit the other provider. In that scenario, competition between insurers is fierce as each insurance company can always undercut its rival. Therefore, insurance companies are unable to elicit any surplus from customers: the premium is actuarially fair, customers are fully insured and insurers make no profit. Providers enjoy a positive markup from their imperfect substitutability.

In the situation where insurance companies are attached to their own provider through their referral lists, the case of exclusive affiliation, we show that insurance companies are able to pull out some of the providers' market power, to the detriment of their customers. In other words, exclusive affiliation allows to transfer some market power from the differentiated providers to the undifferentiated insurers.

For the sake of completeness, we also analyze the case of common affiliation in which insurers choose the same provider as their unique referral, and the case of asymmetric affiliation choices where one insurer affiliates one single provider while the other insurer affiliates both providers.

We will carry on with this static analysis by considering the non-cooperative choice of affiliation structures by insurance companies. We show that the most likely structure that may emerge entail each insurer affiliating exclusively their own provider only.⁷ Hence, in this setting, allowing insurers to restrict access to a specific provider reduces the welfare of insureds. If the government gives more social value to the insureds' welfare (in terms on wealth certainty equivalent) than to insurers' profit, say, for equity reasons, then it should prevent insurers to restrict access to providers. Hence, in this basic setting without insurance fraud, the maximization of the policyholders' surplus could legitimately lead the government to forbid exclusive affiliation of providers by insurers.

⁶See Automotive Body Repair News, March, 2003.

⁷Both exclusive and non-exclusive affiliations are Nash equilibria of the affiliation game, but only exclusive affiliation equilibria survive the 'trembling hand' criterion.

We then consider the insurance fraud problem. To streamline the analysis, insurance fraud is modeled in a crude way. Opportunistic policyholders may file fraudulent claims when they do not have suffered any accident. Service providers (say, car repairers) facilitate fraudulent claiming by certifying that the policyholder actually needed a car repair, allowing him to receive the insurance indemnity. This insurance indemnity is the collusive stake and it is split between the two collusive actors, the policyholder and the repairer. We first develop our analysis of insurance fraud in the same static context. Insurance fraud may be deterred through auditing. If providers are risk-neutral, collusion is deterred if the expected gains obtained by providers from a successful collusive deal (i.e., the fraction of the insurance indemnity obtained by providers) is lower than the expected fine providers have to pay if audit reveals collusion.

When binding, such a collusion-proofness condition affects the features of insurance contracts offered in the market. It leads insurers to offer partial coverage policies in order to decrease the collusion stake. The lower the probability of a successful audit, the lower the insurance coverage for collusion proofness to be maintained. Likewise, the lower the fine imposed by courts on providers guilty of fraud, the lower the insurance coverage. In particular, a stronger judicial system (i.e., larger expected fines imposed on dishonest providers) leads to less distortion in the insurance market for fraud to be deterred.

In this static setting, non-exclusive providers affiliation still leads to larger policyholders' surplus than exclusive providers affiliation. More precisely, there are cases in which the collusion-proofness condition is not binding under exclusive affiliation whereas it binds under non-exclusive affiliation. The reason is that in the former case, insurance companies recover some market power and can thus reduce the amount of indemnities given to their customers, thereby reducing the collusive stake. Even in these cases, we show that non-exclusive affiliation is preferred to exclusive affiliation as the loss of surplus that results from imperfect risk-sharing under non-exclusive affiliation is smaller than the loss of surplus associated with the insurers' market power under exclusive affiliation.

Therefore, in this one-shot framework, the fraud deterrence objective does not modify our previous conclusion: The defence of the policyholders' interests may legitimately lead the government to ban insurers from excluding some providers from their networks.

Results are different when insurers and providers are engaged in a repeated relationship. Several kinds of dynamic anti-fraud strategy may be contemplated in such a setting. In particular, insurers may offer non-collusion proof contracts each period while deterring collusion by threatening to offer only collusion-proof contracts in the future to the clientele of providers who are caught colluding. Of course, a provider is deterred from defrauding the system if his loss is sufficiently large and the threat of retaliation credible.

Affiliation structures affect both the size of the loss in case of retaliation and the credibility of its occurrence. In particular, if exclusive affiliation is forbidden, retaliation against a malevolent provider is possible only if insurers agree to punish the provider simultaneously.⁸ Indeed, if there is only one insurance company (the one affected by the provider's fraud) that switches to collusion-proof contracts, insureds will elect its competitor and the profit of the malevolent provider will be unaffected.⁹

⁸Insurers may also threaten to switch to collusion-proof contracts for all providers if one of them is caught colluding. However, this threat requires an even greater level of cooperation between insurers.

⁹Several reasons make cooperation between insurers doubtful. Apart that fraud must be evidenced, observe that an insurer who do not switch to collusion-proof contracts seizes the entire clientele of the provider. More formally, considering the repeated game between insurance companies, because they earn no profit under non-exclusive affiliation, there is no threat that can force cooperation between them. Of course, it could be possible to consider situations where insurers implicitly collude and earn profit under non-exclusive affiliation. In these situations, a company could enforce cooperation against fraud by

Matters are different under exclusive affiliation since insurers offer coverage for the services offered by their provider only: cooperation between insurers is not required to retaliate against fraud. In particular, by offering collusion-proof contracts, the insurer is effectively able to reduce its provider's profit. When providers put sufficiently large a weigh on future profits, that is, when their discount factor is sufficiently large, this threat destroys the incentives to collude, even when the efficiency of audit procedures is low or when the fines imposed on spotted defrauders are low.

Hence, because exclusive affiliation forces insurers and providers to engage in an exclusive partnership, retaliation against fraud is credible in that case. This possibility may supplement an inefficient judicial system: it is thus a second-best optimal anti-fraud policy.

From a methodological standpoint, our analysis shares a common interest with papers on the organization of health care markets. For instance, Gal-Or (1997) analyzes managed-care organizations in a framework involving two insurance companies and two service providers (hospitals). Insurance companies may choose to exclude or not hospitals, depending of the agreement they can negotiate with them. It determines the agreements that are reached at equilibrium assuming that insurers are differentiated, hospitals are differentiated too and negotiation occurs according to a Nash bargaining game. The issues we focus on are similar although we are more interested in the impact of fraud, in the repeated interactions between insurers and service providers and we put the emphasis on the organization of insurance markets and on their efficiency in terms of risk-sharing properties. In a different vein, the effects of collusion between providers and insureds on insurance contracts and on the physicians' effort to deliver good care is investigated in Ma and McGuire (1997). As neither the quantity of treatment (decided by the patient) nor the physician's effort are contractible, only third-best regimes are attainable. They analyze how "professional ethics" and competition among physicians allow to relax the constraints that restrict insurance contracts.¹⁰ Ma and McGuire (2002) propose a theory of network incentives in managed health care: by imposing quantity restrictions on the providers' side, managed care allows to soften some moral-hazard problem. With respect to their work, our model details much more explicitly the pros and cons associated with a network of providers, and it puts the emphasis on the role of the network to alleviate the fraud problem. Finally, our analysis bears some resemblance with the analysis of vertical restraints in Industrial Organization, as extensively surveyed in Rey and Tirole (2006). Our contribution with respect to this literature is to account for two specificities of insurance markets, namely risk-sharing and fraud.

The paper is organized as follow. The next section sets up the model. Then, in Section 3, we analyze how the different affiliation structures impact the equilibrium on the insurance market in the absence of fraud. In Section 4, we analyze the relationship between providers affiliation and insurance fraud in a static setting where fraud may be deterred by the threat of being sentenced in courts. In Section 5, we investigate the role of exclusive affiliation as a mechanism to deter fraudulent activities in a dynamic setting with a repeated relationship between insurers and providers. Section 6 concludes.

threatening its rival to compete à la Bertrand otherwise. As we assume that competition between insurers is fierce in any case, it is natural to preclude cooperation against fraud too.

¹⁰The changes and consolidation trend in health care markets are highlighted in Newhouse (1996) and Gaynor and Haas-Wilson (1999). Several empirical studies document the consolidation trends in the health sector and investigate the benefits of managed care: see Town and Vistnes (2001), Backer and Brown (1999) and Evans Cuellar and Gertler (2005) for recent studies. Several analytical studies followed Gal-Or's (1997) investigation. See in particular Gal-Or (1999) and Lyon (1999) for a study on mergers and consolidation in health markets.

2 The Model

Our model involves three main characters: insurers, service providers and potential customers. Customers are exposed to a risk and may therefore buy an insurance policy from one of the insurance firms. In case of an accident, they may also ask for the services of one of the providers to compensate for their loss.

Providers. Two providers, denoted by $j \in \{0, 1\}$, propose services to customers who have suffered from an accident. At cost c (which, for analytical convenience, is taken to be the same across providers), a provider can fully compensate a customer from its loss. Provider j posts price p_j for its services.¹¹

Insurers. Two insurers, denoted by $i \in \{A, B\}$, offer insurance policies which consist of a premium k_i^j and a net indemnity s_i^j in the event of an accident when an insured has chosen provider j . Differently put, the premium k_i^j is paid by a customer to the insurance company in all states of nature, and, in the event of an accident, the latter provides the former with a total reimbursement or gross indemnity of $s_i^j + k_i^j$.¹²

Insureds. There is a unit mass of risk-averse potential customers, with initial wealth w , who are uniformly located on interval $[0, 1]$. Each customer may suffer from an accident with probability π . In case of an accident, the monetary loss is ℓ with $\ell > c \geq 0$. Customers have preferences described by a von Neumann-Morgenstern utility function $u(w_f)$ where w_f denotes the individual's final wealth, with $u'(\cdot) >$ and $u''(\cdot) < 0$. Visiting a provider entails a disutility which is proportional (at rate t) to the distance covered to visit the provider.

The two providers are located at the extremities of the segment, namely at $x_0 = 0$ for provider 0 and $x_1 = 1$ for provider 1. Hence, the expected utility of a customer located in $x \in [0, 1]$ is thus:

$$(1 - \pi)u(w - k_i^j) + \pi \left[u(w + s_i^j - p_j) - t|x - x_j| \right], \quad (1)$$

if that customer takes out an insurance policy from insurer i and buys the service offered by provider j . Implicit in this formulation is that customers are heterogeneous in their preferences toward the providers: to fix ideas, consider that customers have to bear different transportation costs to visit the various providers. Notice also that these transportation costs are expressed in utils. In addition to true transport disutility, these costs may correspond to the ability of each provider to offer specific services that more or less matter for each individual. For example, repairers may be specialized in some brands of car, hospitals may be more efficient for the treatment of some diseases, etc. Parameter t is thus an index of the substitutability across providers.

If a customer is not insured and chooses not to use the providers' services, then its expected utility writes as: $(1 - \pi)u(w) + \pi u(w - \ell)$. By contrast, if that uninsured customer chooses to visit, say, provider j , then its expected utility is given by: $(1 - \pi)u(w) + \pi[u(w - p) - t|x - x_j|]$. In order to focus on the most interesting scenarios, we assume that the loss

¹¹Our analysis remains qualitatively unchanged if we assume that providers offer prices which depend on the choices of insurance companies by the customers.

¹²Note that insurance policies are dependent on the choice of providers but are independent of the customers intrinsic preferences for the providers (i.e., their location x).

ℓ is sufficiently large and the transportation cost t is sufficiently small, so that uninsured customers always choose to visit a provider in the event of an accident.¹³

Providers affiliation by insurance companies. Insurers can decide to affiliate one particular provider or to affiliate both of them. If insurer i affiliates provider j only, then i 's customers are required to visit j in the case of an accident.

The game. Let us now describe the game under consideration.

1. Affiliation decisions are first made by insurance companies.
2. Insurers offer insurance policies $\{k_i^j, s_i^j\}$, $j \in \{0, 1\}$ and $i \in \{A, B\}$. Simultaneously, providers post prices p_j , $j \in \{0, 1\}$ for their services.
3. Given the observed insurance policies and providers prices, customers choose at most one insurance contract.
4. Finally, any customer who suffers from a loss decides to visit or not a provider among those who are affiliated by his insurer.

3 Optimal Affiliation Structures

In this section, we characterize the (subgame-perfect) Nash equilibrium of our game. We proceed in two steps: first, we analyze the equilibrium of the subgame starting at stage 2 in which the insurers' choice of affiliations is given. Then, in a second step, we look at the non-cooperative affiliation decisions made by insurance companies at stage 1.

3.1 Non-exclusive affiliation

In this first case, both insurance companies have affiliated both providers. Figure 2 represents the situation under consideration.

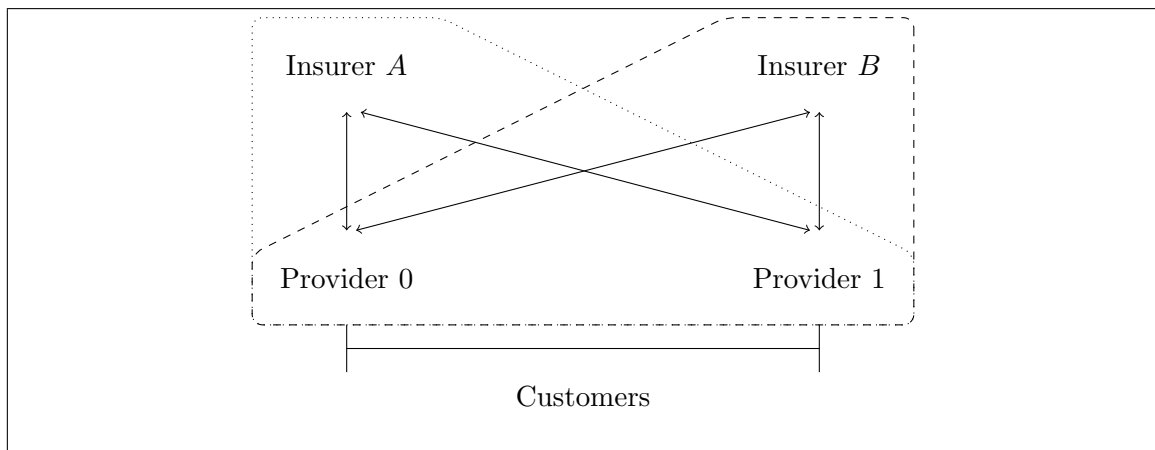


Figure 2: No affiliation.

¹³As is well-known from the IO literature, the Hotelling model is ill-suited to analyze the issue of market participation as there is a discontinuity between the case of duopoly with full market participation and the case of local monopolies when the market is not fully covered.

We focus on symmetric equilibria of that subgame in which insurers offer the same insurance policy, i.e., $k_A^j = k_B^j \equiv k^*$ and $s_A^j = s_B^j \equiv s^*$ for $j \in \{0, 1\}$, and providers offer the same price, i.e., $p_0 = p_1 \equiv p^*$. Given that decisions are taken simultaneously by the providers and the insurers, we look at the conditions under which none of the players has an incentive to deviate from that symmetric outcome.

Competition in contracts between insurers. Competition between the insurance companies leads to the following contract that is offered at equilibrium:

$$\begin{aligned} \{k^*, s^*\} \in \arg \max_{\{k, s\}} & (1 - \pi)u(w - k) + \pi u(w - p^* + s) \\ \text{s.t.} & (1 - \pi)k - \pi s = 0. \end{aligned}$$

Indeed, otherwise either insurer A or insurer B could deviate and propose another contract that would increase its profit by attracting more customers: any equilibrium contract must be such that insurance companies exactly break even and that customers' expected utility is maximized.

Straightforward manipulations show that the equilibrium insurance contract satisfies:

$$s^* = (1 - \pi)p^* \quad \text{and} \quad k^* = \pi p^*. \quad (2)$$

In words, the equilibrium insurance policies involve full insurance for the customers and actuarial premia.

The following intuition turns out to be helpful later on. When both insurers affiliate all providers, they have no possibility to discriminate between customers. Put differently, with such an affiliation structure, insurers fiercely compete to attract all potential customers, those who have a stronger preference for provider 0 as well as those who express a stronger preference for provider 1. Hence, if one insurance company, say A , enjoys a strictly positive profit, then the rival company B could destabilize such a situation by slightly undercutting A and proposing to A 's customers a more attractive insurance policy together with the possibility of choosing their preferred providers freely. As a result of the competitive process, at a symmetric equilibrium, insurance companies are led to offer a contract such that, first, they earn no profit and, second, the contract maximizes the customers' expected utility (gross of the transportation costs).

Price competition between providers. A proportion π of customers have suffered from a loss and are willing to buy the services offered by the providers. Let denote by \tilde{x} the address on $[0, 1]$ of the customer who is exactly indifferent between provider 0 and provider 1, that is:

$$u(w - p_0 + s^*) - t\tilde{x} = u(w - p_1 + s^*) - t(1 - \tilde{x}). \quad (3)$$

Hence, the marginal customer is characterized as follows (where subscript ' p ' stands for 'provider'):

$$\tilde{x}_p(p_0, p_1, s^*) = \frac{1}{2} + \frac{1}{2t} [u(w - p_0 + s^*) - u(w - p_1 + s^*)]. \quad (4)$$

Intuitively, if providers offer identical prices, then they share the demand equally since customers make their choice (of provider) on the basis on the transportation cost only. By contrast, if provider 0 offers a price lower than provider 1, it increases its market share since some customers are now willing to bear a higher transportation cost in order to

benefit from a lower price. The profit of, say, provider 0 can thus be written as:

$$\Pi_0(p_0, p_1, s^*) = \pi(p_0 - c)\tilde{x}_p(p_0, p_1, s^*). \quad (5)$$

At a symmetric equilibrium, the (necessary and sufficient) first-order condition associated with the providers' profit-maximization problem yields:

$$t - (p^* - c)u'(w - p^* + s^*) = 0. \quad (6)$$

In short, when each insurer has affiliated both providers, the providers' prices and insurers' contracts offered at equilibrium are characterized by (2) and (6). Note finally that at equilibrium all individuals actually choose to purchase insurance (as implicitly postulated) since they are risk-averse and insurance is sold at actuarial price.

Lemma 1 summarizes this first case.

Lemma 1. *At a symmetric equilibrium (p^*, s^*, k^*) with non-exclusive affiliation:*

- *insurance companies provide full coverage at actuarial price, i.e., $k^* = \pi p^*$, $s^* = (1 - \pi)p^*$*
- *providers charge a price with a margin over marginal cost such that $p^* - c = t/u'(w - p^* + s^*)$.*

3.2 Exclusive affiliation

Let us now consider the case where each insurance company has its own referral provider, a situation called 'exclusive affiliation' in the following. Hence, insurer A (resp. B) requires its customers to visit provider 0 (resp. 1) in the event they incur an accident. Accordingly, denote by $\{k_A, s_A\} = \{k_A^0, s_A^0\}$ and $\{k_B, s_B\} = \{k_B^1, s_B^1\}$ the contracts offered by the insurance companies, with $\{k_A^1, s_A^1\} = \{k_B^0, s_B^0\} = \{0, 0\}$. This situation is illustrated in Figure 3.

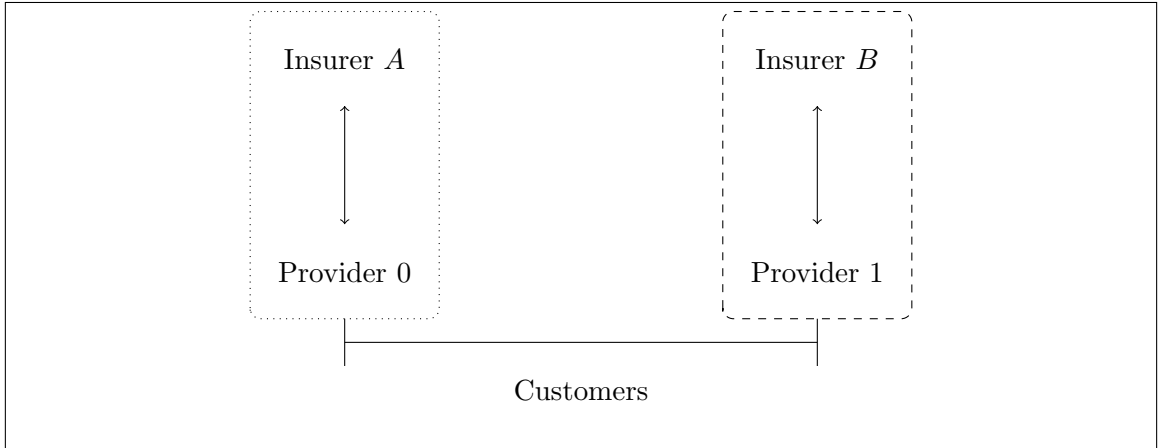


Figure 3: Exclusive affiliation.

Again we look for a symmetric equilibrium in which insurance premiums, net reimbursements and providers' prices are identical, given by $k_A = k_B \equiv \hat{k}$, $s_A = s_B \equiv \hat{s}$ and $p_0 = p_1 \equiv \hat{p}$ respectively.

Competition in contracts between insurers. In that configuration, preferences among insurers depend on the location of individuals. Let us consider the marginal customer $\tilde{x}_i(k_A, s_A, k_B, s_B, p_0, p_1)$ (where subscript ‘ i ’ stands for ‘insurer’) who is exactly indifferent between buying A ’s insurance policy (and being required to use the service of provider 0 in the event of an accident) or B ’s (which involves going to provider 1 in the event of an accident):

$$(1-\pi)u(w-k_A)+\pi [u(w-p_0+s_A)-t\tilde{x}_i] = (1-\pi)u(w-k_B)+\pi [u(w-p_1+s_B)-t(1-\tilde{x}_i)]. \quad (7)$$

Insurer A determines its policy so as to maximize its profit, or:

$$\max_{\{k_A, s_A\}} \tilde{x}_i(k_A, k_B, s_A, s_B, p_0, p_1) [(1-\pi)k_A - \pi s_A]. \quad (8)$$

At a symmetric equilibrium, the premium is then characterized as follows:

$$\frac{1-\pi}{2} + \left[(1-\pi)\hat{k} - \pi\hat{s} \right] \frac{\partial \tilde{x}_i}{\partial k_A}(\hat{k}, \hat{k}, \hat{s}, \hat{s}, \hat{p}, \hat{p}) = 0. \quad (9)$$

Hence, the optimal premium is the result of the following tradeoff: on the one hand, an increase in the premium k_A allows insurer A to raise its profit on its attached customers (a proportion 1/2 at equilibrium). On the other hand, when k_A is increased then A ’s insurance policy becomes less attractive vis-à-vis B ’s policy and A ’s market share decreases, i.e., $\partial \tilde{x}_i / \partial k_A = -u'(w - \hat{k})(1 - \pi) / (2t\pi) < 0$.

A similar tradeoff exists when the optimal net reimbursement s_A is increased. Hence, at a symmetric equilibrium, the following conditions characterize the contract offered by insurance companies:

$$\frac{(1-\pi)\hat{k} - \pi\hat{s}}{t\pi} u'(w - \hat{k}) = 1 = \frac{(1-\pi)\hat{k} - \pi\hat{s}}{t\pi} u'(w - \hat{p} + \hat{s}), \quad (10)$$

which gives $\hat{k} + \hat{s} = \hat{p}$. Simple manipulations thus yield:

$$\hat{k} = \pi\hat{p} + \frac{t\pi}{u'(w - \hat{k})}. \quad (11)$$

Equation (10) implies that customers enjoy the benefit of being fully insured (as in the non-exclusive affiliation case) but with a premium larger than the actuarial one (contrary to the non-exclusive affiliation case) since Equation (11) gives $\hat{k} > \pi\hat{p}$.

Competition between providers. Through its pricing decision, a provider contributes to the repartition of customers both across providers and across insurers. In the case of exclusive affiliation, the demand for provider 0’ services exactly coincides with the demand for insurer A ’s insurance policies. Hence, provider 0’ problem can be written as:

$$\max_{p_0} \pi(p_0 - c)\tilde{x}_i(k_A, k_B, s_A, s_B, p_0, p_1), \quad (12)$$

which gives:

$$t - (\hat{p} - c)u'(w - \hat{p} + \hat{s}) = 0 \quad (13)$$

at a symmetric equilibrium.

When the transportation cost is small, customers receive full insurance at an almost actuarially fair price; hence, all customers strictly prefer to purchase insurance provided

that the differentiation between providers is not too strong. Intuitively, this ensures that the downstream market power of providers is not too large, which, in turn, implies that neither is the upstream market power of insurers too large; henceforth, customers prefer receiving insurance at a slightly distorted premium rather than being not insured.

The next lemma summarizes these findings.

Lemma 2. *At a symmetric equilibrium $(\hat{p}, \hat{s}, \hat{k})$ where each insurer has its own referral provider:*

- *insurers provide full coverage, i.e., $\hat{s} + \hat{k} = \hat{p}$, and they charge a premium with loading $\hat{k} - \pi\hat{p} = t\pi/u'(w - \hat{k})$;*
- *providers charge a price with a margin over marginal cost such that $\hat{p} - c = t/u'(w - \hat{p} + \hat{s})$.*

Proposition 1 compares exclusive and non-exclusive affiliation systems.

Proposition 1. *As compared to non-exclusive affiliation, exclusive affiliation allows insurers to make a strictly positive profit, providers's profits are lower, and customers are worse-off.*

Proof. See the Appendix. □

Indeed, exclusive affiliation allows insurance companies to recover some ‘market power’ on the market for insurance. With exclusive affiliation of providers, the insurers’ expected gain is:

$$\frac{1}{2} \left[(1 - \pi)\hat{k} - \pi\hat{s} \right] = \frac{1}{2} \left[\hat{k} - \pi\hat{p} \right] > 0. \quad (14)$$

Differentiation between downstream providers generates market power which extends to upstream insurers have they chosen to affiliate exclusively their own provider. In other words, exclusive affiliation allows to transfer some market power from providers to insurers. Keeping this in mind, one clearly sees the impact of exclusive affiliation: even though they still receive full insurance at equilibrium, customers’ expected utility is strictly smaller with exclusive affiliation as compared to non-exclusive affiliation. In our setting, providers turn out to be worse-off too under exclusive affiliation. Insurance companies should have higher profits when we observe a trend to exclusive providers affiliation, which is a testable implication of our results.

Readers acquainted with the Industrial Organization literature may have recognized the analogy between our setting of exclusive affiliation between insurance companies and providers with vertical integration between upstream suppliers and downstream retailers: suppliers can lessen competition on the upstream market by integrating vertically with an exclusive downstream provider. There is a difference worth noticing though: vertical integration would make the insurance company perfectly internalize the choice of an insurance policy on the provider’s profit and reciprocally. Hence, the equilibrium outcome under vertical integration is likely to be less competitive than the outcome under exclusive affiliation. Consequently, in a nutshell, judging on the sole basis of customers’ welfare and absent any other considerations, non-exclusive affiliation performs better than exclusive affiliation which in turns performs better than vertical integration.

So far, our analysis has focused on two polar cases: non exclusive affiliation or exclusive affiliation. The next step to undertake is to consider the insurance companies’ incentive to create referral providers lists. This requires to study two other asymmetric situations, one

in which insurers choose to affiliate the same provider (common affiliation) and another one where one insurer affiliates one provider whereas the other insurance company affiliates all providers (asymmetric choice of affiliations).

3.3 Common affiliations

We now deal with the situation in which both insurers decide to affiliate the same provider, say, provider 0. Customers of provider 1 do not benefit from insurance coverage. This is represented in Figure 4.

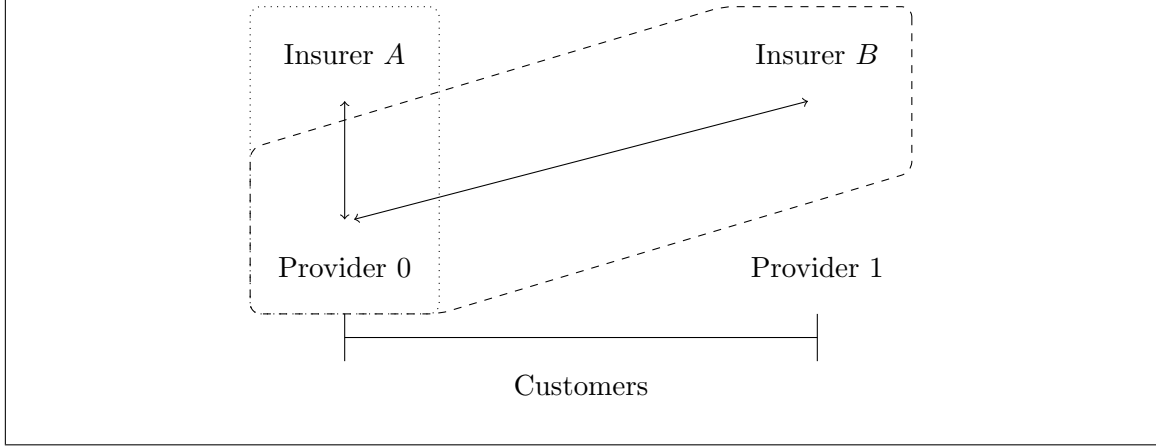


Figure 4: Common affiliation.

All insureds who suffer from a loss are required to visit provider 0; hence, from the viewpoint of the insurance companies, the common referral provider does not help in discriminating the various customers: at equilibrium, competition in contracts leads them to offer full insurance and to set an actuarial premium, given the price p_0 set by provider 0: $k = \pi p_0$ and $s = (1 - \pi)p_0$. A logic similar to that exposed in the case of non-exclusive affiliation applies.

Let us look at the customers' side. Customers must trade off, on the one hand, the possibility not to buy any insurance policy and, on the other hand, the price to pay to have access to the unique provider affiliated by the insurers. The marginal customer is now defined as follows:

$$(1 - \pi)u(w - k_0) + \pi u(w - p_0 + s_0) - \pi t \tilde{x}_i = (1 - \pi)u(w) + \pi[u(w - p_1) - t(1 - \tilde{x}_i)]. \quad (15)$$

Equation (15) expresses the fact that customers who have decided not to purchase insurance can compensate for their loss by buying provider 1' service. Profits of the providers can be written as:

$$\begin{aligned} \Pi_0 &= \pi(p_0 - c)\tilde{x}_i, \\ \Pi_1 &= \pi(p_1 - c)(1 - \tilde{x}_i). \end{aligned}$$

With common affiliation of provider 0 by both insurance companies, the rival provider 1 in fact bears a competitive disadvantage vis-à-vis provider 0: customers being risk-averse, provider 1 must set a lower price than provider 0 if it wants to capture part of the demand for repair services since its potential customers are not insured. Moreover, the fierce competition between insurers for customers that choose provider 0 tends also to reduce provider 1's market share.

Deriving the equilibrium prices set by the providers is straightforward. The next lemma summarizes this case.

Lemma 3. *If insurers have affiliated the same and unique provider, then at the equilibrium:*

- *insureds receive full insurance, the insurance premium is the actuarial one and insurance companies make no profit;*
- *the unique referral provider charges a higher price and makes a larger profit than its rival.*

Proof. See the Appendix. □

Finally, notice that when t is small a corner solution is likely to emerge: in that case, the non affiliated provider is put at such a strong disadvantage with respect to its affiliated competitor that the demand it faces (and therefore its profit) may become null.

3.4 Asymmetric affiliation choices

The last case to study involves one insurer, say A , affiliating one provider, say 0, while the other insurance company choosing to affiliate both providers. This situation is represented in Figure 5.

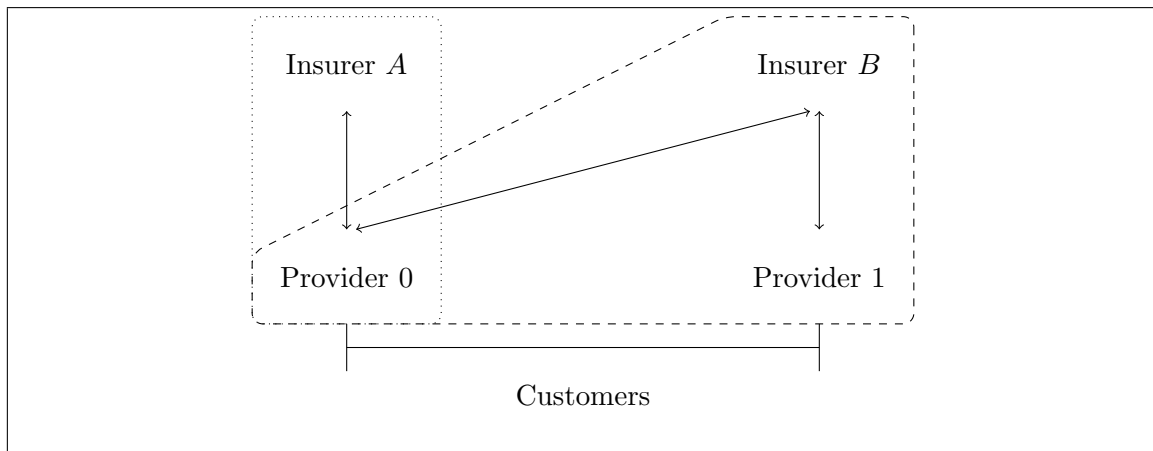


Figure 5: Asymmetric affiliations.

Then, we obtain the following lemma.¹⁴

Lemma 4. *Consider that one insurance company (say, A) affiliates with one provider (say, 0) whereas the other insurance firm (B) affiliates both providers (0 and 1). Then:*

- *insureds who have elected provider 0 receive full insurance at an actuarially fair premium whatever their choice of insurance company;*

¹⁴Customers who buy insurance and visit provider 0 in the event of an accident receive full insurance at an actuarially fair premium; hence, they always prefer to buy insurance rather than not to buy coverage thanks to the concavity of $u(\cdot)$. Customers who visit provider 1 and buy insurance from firm B receive full insurance but at a premium larger than the actuarially fair one; however, since the market is more competitive than in the exclusive affiliation scenario, the condition such that these customers prefer to buy insurance is less stringent than in the exclusive affiliations case.

- *insureds who have elected provider 1 receive full insurance but their premium is larger than the actuarial one.*

Proof. See the Appendix. □

Since insurer B doesn't impose any requirement on the customers' choice of providers, it can seize A 's market share by undercutting its offer. Competition for the insureds who have elected provider 0 is fierce, leading those customers to be fully insured at an actuarial price. Insurer B can still earn some profits from customers who have elected provider 1; however, the intense competition on the segment of customers who have chosen provider 0 tends to reduce the market share of provider 1. As a consequence, the demand that faces insurer B over which it can exert some market power is constrained by this competitive pressure. Consequently, insurer B makes a positive profit but lower than under exclusive affiliation, while insurer A makes no profit. In a nutshell, this situation lies in-between the non exclusive affiliation case (but customers who have chosen provider 1 are charged above the fair premium) and the exclusive affiliation situation (but customers who are close to provider 0 enjoy an actually fair insurance contract).

3.5 Choice of affiliation structure by non-cooperative insurers

Now that we have reviewed the possible affiliation structures, let us determine whether some of them are more likely than others to emerge. For this purpose, complement the initial insurance-provider game as follows: at the very beginning (stage 1 of the game), before insurers and providers make their offer, insurance companies choose non-cooperatively to affiliate one or both providers. Denote by \hat{R} the insurer profit under exclusive affiliation, i.e. $\hat{R} = [(1 - \pi)\hat{p} - \hat{s}]/2$. As shown in the previous subsection, the profit of the insurance company which affiliates both providers in the asymmetric affiliation case is strictly smaller than \hat{R} . To reduce the notational burden, her profit in that case is denoted by $\hat{R} - \gamma$ (with $\hat{R} > \gamma > 0$) without loss of generality.

For the sake of clarity, the payoffs for the insurers in the different configurations are summarized in Table 1.

Insurer B Insurer A	Provider 0	Provider 1	Provider 0 & 1
Provider 0	(0, 0)	(\hat{R} , \hat{R})	(0, $\hat{R} - \gamma$)
Provider 1	(\hat{R} , \hat{R})	(0, 0)	(0, $\hat{R} - \gamma$)
Provider 0 & 1	($\hat{R} - \gamma$, 0)	($\hat{R} - \gamma$, 0)	(0, 0)

Table 1: Insurers' payoffs depending on the affiliation choices.

A rapid inspection of this table shows that there are three Nash equilibria of this affiliation game: two of them correspond to exclusive affiliation ((Provider 0, Provider 1) and (Provider 1, Provider 0)), while (Provider 0 & 1, Provider 0 & 1) is the non-exclusive affiliation situation. However, affiliating both providers is a weakly dominated strategy for the insurance companies. Indeed, it is not worth for an insurer to affiliate both providers unless it is absolutely sure than its rival will do so. Should the rival deviate with a small probability from this strategy, and the insurer could be better-off affiliating one provider

only. Hence, the non-exclusive affiliation situation is not a *trembling-hand* perfect Nash equilibrium.¹⁵ We thus have the following result.

Proposition 2. *The affiliation game has three Nash equilibria: the two exclusive affiliation structures and the non-exclusive affiliation structure. However, only the two exclusive affiliation equilibria are trembling-hand (subgame) perfect.*

Proof. See the Appendix. □

When insurance company A has decided to affiliate exclusively one provider, then it is an optimal decision for the rival insurance company B to affiliate exclusively the remaining provider: indeed, choosing not to compete with the segment of customers who visit 0 in case of an accident softens the competition between providers and allows insurer B to raise her profit. Observe also that compared to the non-exclusive affiliation structure, the resulting situation of exclusive affiliation is Pareto-optimal for the insurers.

4 Affiliation and Insurance Fraud

The previous section has argued that the affiliation of providers by insurance companies is an imperfect mechanism to restore some market power at the upstream level: in particular, exclusive affiliation structures allow insurance companies to enjoy a strictly positive profit at the expense of customers (and, to some extent, of providers too). Importantly, insureds still receive full insurance from the insurers. We now look at the role of providers' affiliation choices when insureds may collude with providers.

Collusion and fraudulent claims. A simple way to introduce collusion in our setting goes as follows. We consider that insureds who did not suffer from an accident may declare a fraudulent claim to their insurance companies. Let $\theta \in [0, 1]$ be the fraction of customers that may reach a collusive deal with a provider; hence, the industry-wide number of potential collusive deals is given by $\theta(1 - \pi)$.¹⁶

Collusion in our context amounts to filing a fraudulent claim: the insured then obtains a monetary gain $-p_j + s_i^j + k_i^j$ (instead of $-k_i^j$ if it does not collude), while the provider earns p_j (instead of 0 if no collusion occurs). Therefore, the net collusive stake is $s_i^j + k_i^j$, which is assumed to be equally shared between the insured and the provider.¹⁷

Audit and fines as collusion deterrents. The threat of fraudulent claims typically leads insurance companies to implement audit strategies which allow them to detect whether a claim was justified or not. Let $q \in [0, 1]$ be the probability that the insurance company detects that the provider has engaged into fraudulent activities with a given customer.¹⁸

¹⁵For a formal definition of (normal form) trembling-hand perfect Nash equilibria, see, e.g., Mas-Colell et al. (1995) pp. 258-260.

¹⁶We assume that all policyholders are potential defrauders. A different modeling choice (see Picard, 1996), would be to assume that there are two types of policyholders, some of them being inherently honest, and other ones potentially dishonest (i.e., opportunistic).

¹⁷One could easily think about other ways to introduce collusion. What really matters for our analysis is that the stake of collusion depends on the insurance contract.

¹⁸To streamline the analysis, we assume that the audit probability is the same for both insurance companies and we neglect the cost of auditing. Since q is exogenous, in the present model, nothing substantial is lost with this simplification.

Of course, for the audit policy to be effective in deterring collusion, it must be that some kind of penalty can be implemented by the insurance companies, in addition to not providing the reimbursement $s + k$. A simple type of penalty would consist in launching a lawsuit against the provider upon a successful audit which unveils collusion. Let us therefore denote by F the monetary fine (collected by the government) that can be imposed by courts to the provider in case a fraudulent behavior has been unveiled.

Therefore, in order to prevent collusion by provider j , insurer i has to ensure that the threat of being discovered and fined is larger than the provider's share of the collusion stake. Since providers are risk-neutral, the collusion-proofness condition is written as:

$$qF \geq (1 - q) \frac{s_i^j + k_i^j}{2}. \quad (16)$$

The welfare cost entailed by the risk of collusion depends on q and F . When audits are relatively ineffective or fines imposed by courts are low (i.e., when q or F are low), then the collusion-proofness condition (16) is severe and, as we will see, it entails large adverse effects on the policyholders' welfare.

Because $p^* > \hat{p}$ and $s^* + k^* = p^*$, $\hat{s} + \hat{k} = \hat{p}$, (16) shows that collusion is more likely to occur under non-exclusive affiliation than under exclusive affiliation. More precisely, when $p^* > 2Fq/(1 - q) \geq \hat{p}$, (16) is binding in the non-exclusive affiliation case whereas it is not under exclusive affiliation; by contrast, when $\hat{p} > 2Fq/(1 - q)$, insurance fraud is a problem in both structures. Observe that when θ is low, insurers may find that distorting insurance contracts to fulfill (16) is too costly and decide to let collusion happen at equilibrium.¹⁹ The following lemma characterizes the optimal insurance contracts under exclusive and non-exclusive affiliation assuming that offering collusion-proof contracts is an equilibrium strategy. We then discuss the conditions under which collusion-proof strategies emerge at equilibrium.

Lemma 5. *The optimal collusion-proof insurance contracts and providers prices at a symmetric equilibrium are as follows when constraint (16) is binding (where subscript 'cp' stands for 'collusion-proof'):*

- *Under non-exclusive affiliation ($p^* > 2Fq/(1 - q)$): $s_{cp}^* = 2(1 - \pi)Fq/(1 - q)$, $k_{cp}^* = 2\pi Fq/(1 - q)$ and $p_{cp}^* - c = t/u'(w - p_{cp}^* + s_{cp}^*)$, such that $p_{cp}^* > s_{cp}^* + k_{cp}^*$.*
- *Under exclusive affiliation ($\hat{p} > 2Fq/(1 - q)$): $\hat{s}_{cp} < 2(1 - \pi)Fq/(1 - q) - \pi(\hat{p} - c) < s_{cp}^*$, $\hat{k}_{cp} > 2\pi Fq/(1 - q) + \pi(\hat{p} - c) > k_{cp}^*$ and $\hat{p}_{cp} - c = t/u'(w - \hat{p}_{cp} + \hat{s}_{cp})$ such that $\hat{p}_{cp} > \hat{s}_{cp} + \hat{k}_{cp}$.*
- *Moreover, when $\hat{p} > 2Fq/(1 - q)$, we have $\hat{p}_{cp} < p_{cp}^*$.*

Proof. See the Appendix. □

Lemma 5 shows that, because of the threat of collusion, insurance policies offered at equilibrium may no longer provide for full insurance whatever the affiliation structure. Insurance premia are higher under exclusive affiliation than under non exclusive affiliation whereas it is the opposite for the insurance net indemnities. We also observe that the providers' equilibrium price is lower under exclusive affiliation than under non-exclusive affiliation (the market power effect).

¹⁹Notice that allowing collusion at equilibrium might prevent a market breakdown: for instance, when $F = 0$ the only collusion-proof insurance contract which satisfies the zero-profit condition is such that $s = k = 0$, i.e., customers receive no insurance at all.

As noted above, when θ is low, insurers may prefer to let collusion happen at equilibrium rather than offering collusion-proof contracts. The presence of collusion at equilibrium under non-exclusive affiliation leads to the following zero-profit condition:²⁰

$$(1 - \pi)k - \pi s = (1 - q)\theta(1 - \pi)(k + s), \quad (17)$$

and competition between insurers leads to insurance contracts that are not actuarially fair. This is also true under exclusive-affiliation, with an optimal contract that solves: $\max_{s,k} \{[(1 - (1 - q)\theta)](1 - \pi)(k + s) - s\} \tilde{x}$, where \tilde{x} accounts for the fact that some customers will collude. In both cases, insurers offer partial coverage policies which entail a welfare loss compared to the no-fraud situation.

Whatever the affiliation structure, collusion between insureds and providers entails an *ex ante* negative effect on the insureds' welfare because half of the collusive stake is received by providers, while the total cost of collusion results in higher insurance premia. Intuitively, when θ is large and q small, this collusion cost may lead insurers to offer collusion-proof contracts. The following lemma states formally that it is indeed the case.

Lemma 6. *Under non-exclusive affiliation (resp. exclusive affiliation), there exist thresholds θ^* and q^* (resp. $\hat{\theta}$ and \hat{q}), $0 < \theta^* < 1$ and $0 < q^* < 1$, such that for all $\theta \geq \theta^*$ and for all $q \leq q^*$, there is no profitable deviation from the collusion-proof strategy (s_{cp}^*, k_{cp}^*) (resp. $(\hat{s}_{cp}, \hat{k}_{cp})$).*

Proof. See the Appendix. □

Assuming that insurance companies actually offer collusion-proof contracts at equilibrium, the following result compares the customers' expected utility under both affiliation structures.

Proposition 3. *When insurers offer collusion-proof contracts, exclusive affiliation leads to a lower customers' expected surplus than non-exclusive affiliation.*

Proof. See the Appendix. □

When $2Fq/(1 - q) > p^*$, the judicial system is efficient enough to deter fraud without incurring any efficiency loss whatever the affiliation structure. Hence, according to the analysis undertaken in the previous section, from the standpoint of customers' expected surplus non-exclusive affiliation always performs better than any other affiliation structure.

In the intermediate zone, that is, when $p^* > 2Fq/(1 - q) > \hat{p}$, the fraud deterrence constraint distorts insurance contracts in the non-exclusive affiliation regime only; in particular, insurance contracts under non-exclusive affiliation do no longer provide customers with full insurance. However, in the exclusive affiliation regime, contracts involve full insurance but with a strictly positive loading. Hence, in that case, there is a potential tradeoff between, on the one hand, the distortion associated with the market power of insurers under exclusive affiliation and, on the other hand, the imperfect risk sharing due to the collusion-proofness constraint under non-exclusive affiliation. Proposition 3 reveals that the balance always tips in favor of the non-exclusive affiliation regime.

Proposition 3 also shows that the dominance of non exclusive affiliation over exclusive affiliation remains valid when $\hat{p} \geq 2Fq/(1 - q)$, i.e., when the collusion-proofness requires to distort insurance contracts in both regimes.

²⁰Keep in mind that the fine imposed to the provider if it is caught colluding is received by the government.

Roughly speaking, in a static framework, whatever the level of expected penalty, non-exclusive affiliation is strictly preferred to exclusive affiliation from the viewpoint of customers' expected welfare. A reduction of the expected fine (weakly) increases the distortion in insurance contracts incurred by customers under both affiliation regimes. However, this loss of efficiency is never strong enough under non-exclusive affiliation to offset the distortion associated with the insurers' market power under exclusive affiliation. This will no longer be always true once the repeated relationship between insurers and providers is accounted for.

5 The Threat of Collusion: Dynamic Analysis

The repetition of the relationship between providers and insurers affects the trade-off for a provider who contemplates colluding with policyholders: the provider compares the immediate gain of collusion with the loss associated with the insurer's retaliation in future periods in the case collusion is uncovered. Assuming that collusion is deterred in such a way, insurers can neglect the collusion-proofness constraint (16) and, depending of the affiliation structure, they may offer the first-best non-exclusive or exclusive affiliation contracts characterized above.²¹

Of course, a provider is deterred from defrauding if the loss associated with the insurer's future retaliation is sufficiently large and the threat of retaliation is credible, which is affected by the affiliation structure. One way for an insurer to retaliate against a malevolent provider that it does not trust anymore (without excluding it from its referral list, if any) is to propose only collusion-proof contracts to customers who wish to visit this provider in the case of an accident since the collusion-proofness condition reduces the provider's profit. However, under non-exclusive affiliation, such a punishment against a malevolent provider requires some kind of coordination between insurers.

Indeed, suppose that only one company, say insurer A , decides to move from non-collusion proof contracts to collusion-proof contracts for the customers of a malevolent provider, say provider 0 , while insurer B does not change its contracts. Then all customers of provider 0 will elect insurer B , and the provider 0 's next period profit will be unaffected. Hence, under non-exclusive affiliation, retaliation against fraud is credible only if insurance companies punish a malevolent provider simultaneously. When insurer A finds evidence that provider 0 has participated in a fraudulent claim with one of its insureds, both insurers A and B have to offer simultaneously collusion-proof contracts to the customers of this provider (or to exclude provider 0 from their referral lists simultaneously), even if insurer B has no reason to complain against provider 0 .

Several reasons may preclude such a coordination between insurers.²² Firstly, hard information about the fraudulent behavior must be put forward by the concerned insurer. Otherwise, the joint retaliation strategy may be jeopardized by strategic manipulation of information. Secondly, in the absence of coordination between insurers, if insurer A switches to collusion-proof contracts for provider 0 , insurer B is able to seize the entire clientele of the provider and to extract some profit from this situation.²³ Coordination

²¹We assume that the competition between insurers is fierce, which precludes any kind of tacit collusion between them (which could be possible given the repeated game framework that we adopt in this section). When they have affiliated the same provider, we thus still assume that they compete 'à la Bertrand'.

²²Organizations like 'The Coalition Against Insurance Fraud' in the U.S., which unites several insurance companies, are actively acting against fraud, but mostly by alerting the legislator and promoting anti-fraud regulation. See <http://www.insurancefraud.org/>

²³More precisely, following the detection of the fraudulent behavior of provider 0 by insurer A , there exists a continuation equilibrium where provider 0 does not collude with insurer B 's policyholders and insurer B

between insurers is not needed under an exclusive affiliation structure.

To be more formal, consider an infinitely repeated game starting at date 0 with discount factor $\delta \in (0, 1)$. At each date $z \geq 1$, insurers choose their affiliation structures and providers decide either to be “opportunistic” or “honest” with each insurance company. A provider is opportunistic when it takes advantage of any profitable collusive deal with customers. A provider is honest when it declines any collusive deal. Then insurance companies offer insurance policies and providers set prices for their services simultaneously. A provider who has decided to be opportunist with insurer i reaches a collusive deal with probability θ with any policyholder of insurer i who has not suffered from a loss, while an honest provider does not collude with policyholders. As before, any collusive deal is detected with probability q .

We focus on trigger-like strategies of the following form: insurer i offers non-collusion proof contracts (s_i^j, k_i^j) as long as it has not detected collusion of provider j . If, at some date z , an insurance company detects a fraudulent claim, it offers only collusion-proof contracts from date $z + 1$ on to any individual who elects the corresponding provider. A provider’s strategy is to post a price and to be honest at each period. If the provider deviates to opportunism with company i in one period and is detected, then it remains opportunistic with this insurer in all subsequent periods.

These strategies are not part of a (subgame-perfect) equilibrium of the repeated game under non-exclusive affiliation. Indeed, under such an affiliation structure, the honesty strategy of providers is dominated by the strategy which consists in being opportunistic with (at least) one insurer every period. If the insurer detects a collusive deal and switches to collusion-proof contracts, the provider’s customers choose its competitor and the provider’s discounted expected profit is not affected. In other words, in the absence of coordination between insurers, moving to collusion-proof contracts does not deter collusion.

Matters are different under exclusive affiliation. When insurer A punishes its provider by offering collusion-proof contracts to its customers, the insurer A ’s demand reduces and insurer B adapts its strategy (along with the providers). More precisely, if provider 0 was caught defrauding by insurer A in the past, then the strategies played at all the following periods are as follows:²⁴

- Insurer A (resp. B) affiliates provider 0 (resp. 1).
- Insurer A offers the contract (k_A, s_A) which maximizes $\tilde{x}_i(k_A, k_B, s_A, s_B, p_0, p_1)[(1 - \pi)k_A - \pi s_A]$ subject to $s_A + k_A \leq 2qF/(1 - q)$.
- Insurer B offers (k_B, s_B) that maximizes $[1 - \tilde{x}_i(k_A, k_B, s_A, s_B, p_0, p_1)][(1 - \pi)k_B - \pi s_B]$.
- Provider 0 is opportunistic with insurer A and it chooses p_0 to maximize its profit.
- Provider 1 is honest with insurer B and it chooses p_1 to maximize its profit.

keeps offering non collusion-proof contracts, while insurer A switches to collusion-proof contracts. At such a continuation equilibrium, insurer A ’s (resp. B ’s) market share decreases (resp. increases).

²⁴Observe that the continuation equilibrium strategies after provider 0 has been caught defrauding are not unique: If provider 0 decides to be opportunistic for, say, only 5 periods once it has been detected, a punishment phase longer than 5 periods would be sub-optimal for insurer A . Of course, to be able to deter provider 0 to collude with customers at any time, the punishment phase must be sufficiently long. Whatever the strategy the provider may envision during the retaliation phase (and provided that insurer A ’s response is adequately adapted) the resulting behavior along the equilibrium path would be unchanged. We have thus chosen the simplest framework in order to establish that honesty may be sustained along the equilibrium path of our game.

These strategies are asymmetric: we have $(k_A, s_A) \neq (k_B, s_B)$ and $p_0 \neq p_1$, with a lower profit for insurer A than for insurer B .²⁵ They correspond to a continuation equilibrium after provider 0 has been spotted by insurer A : It is indeed optimal for insurer A to offer a collusion-proof contract, given the strategy of its provider, which is to always be opportunistic once it has been caught colluding, and provided that θ is large and q is small (as indicated in Lemma 6). Being opportunistic forever is an optimal strategy of provider 0 since insurer A will not trust it anymore.

Observe that the trigger strategies do not involve any change in the affiliation structure, and, under exclusive affiliation, acting that way is an optimal behavior of insurers. Indeed, if insurer A decides to exclude its only provider when collusion has been detected and to affiliate insurer B 's provider, the following common affiliation situation implies that both insurers make zero profit, and thus insurer A would be worse-off.

It remains to verify that under exclusive affiliation these trigger-like strategies entail an effective threat for the providers. Denote by $\hat{\Pi}_j$ the provider j 's profit under exclusive affiliation when both insurance company offer non-collusion proof-contracts, and by $\hat{\Pi}_j^{cp}$ the provider j 's profit when the insurance company it is exclusively affiliated with has chosen to offer collusion-proof contracts (while the other insurer keeps offering non-collusion proof-contracts), with $\hat{\Pi}_j^{cp} < \hat{\Pi}_j$.

The discounted profit for provider j on the equilibrium path, i.e. if no collusion occurs at any period- is given by $\sum_{z=0}^{+\infty} \delta^z \hat{\Pi}_j$. By contrast, if provider j decides to collude during the first period and then to never collude again, it expects a discounted profit given by:²⁶

$$V = \left[\hat{\Pi}_j + \tilde{x}_i(1 - \pi)\theta \left((1 - q) \frac{s_i + k_i}{2} - qF \right) \right] + \left[(1 - \tilde{q}) \sum_{z=1}^{+\infty} \delta^z \hat{\Pi}_j \right] + \left[\tilde{q} \sum_{z=1}^{+\infty} \delta^z \hat{\Pi}_j^{cp} \right].$$

where $\tilde{q} \equiv q\tilde{x}_i(1 - \pi)\theta$ is the probability that at least one collusive agreement will be unveiled. The first bracketed term is the discounted profit of engaging into fraudulent activities during the first period. On top of the per-period profit associated to insureds who suffer from a loss, the collusive provider j earns its share of the collusive stake (which depends on the fraction of customers insured by company i , i.e., \tilde{x}_i , and of the number of fraudulent customers, i.e., θ) but runs the risk of being caught and fined. The second (resp. third) term represents provider j 's discounted profit starting from period 2 on if it has not been (resp. has been) caught colluding at the end of the first period. If collusion is left undetected then it earns the profit of an exclusive provider; by contrast, if collusion is detected, it earns the profit associated to collusion proof contracts.

Provider j is deterred from colluding if $\sum_{z=0}^{+\infty} \delta^z \hat{\Pi}_j \geq V$, a condition which reduces to:

$$\hat{\Pi}_j \geq \hat{\Pi}_j^{cp} + \frac{1 - \delta}{\delta q} \left[(1 - q) \frac{s_i + k_i}{2} - qF \right]. \quad (18)$$

This condition defines a lower bound on the per-period revenue under exclusive affiliation. Intuitively, a higher audit probability q or a larger preference for the present δ both reduce the gain to collude. In the same vein, if provider j 's gain under non-exclusive affiliation by the insurance companies is not too small, then its incentives to collude remain high since the punishment if it is caught remains low. Interestingly, condition (18) is reminiscent of

²⁵When t is small, this may result in no demand for insurer A , customers being all more interested by insurer B 's offer. Note that this punishment strategy is never played along the equilibrium path.

²⁶A similar conclusion is obtained when the provider decides to collude during $T > 1$ consecutive periods and then to stick to a non fraudulent behavior.

the collusion-proofness constraint (16). Indeed, rearranging terms, we obtain

$$\frac{2q}{1-q} \left[F + \frac{\delta}{1-\delta} (\hat{\Pi}_j - \hat{\Pi}_j^{cp}) \right] \geq s_i + k_i.$$

Hence, with respect to the static case, we observe that collusion can be more easily fought by insurance companies in a dynamic context. Not only can insurers use the legal system to impose a fine F when collusion is detected but they can also use the type of contract they offer to implicitly threaten providers. Even when the legal system is quite imperfect and does not put a real threat on providers, i.e., when F is close to 0, the collusion-proofness constraint (18) can still be satisfied provided that the discount factor is sufficiently large, i.e., provided that providers put sufficiently large a weight on the loss of future profits if they are caught colluding and insurance companies change their contracts. Hence, when exclusive affiliation is allowed, insurance companies have at their disposal an additional possibility of retaliation when they discover collusion between their provider and the insureds, namely the possibility to change the contracts they offer to the provider's clientele. This puts an additional threat on the providers which reduces their incentives to engage into fraudulent activities, and this threat is the best-response to the equilibrium strategy of the providers.

Consequently, allowing exclusive affiliation may enhance the efficiency of risk-sharing through insurance markets mechanisms. This could even lead to an higher customers welfare for low F . More formally, we obtain the following proposition.

Proposition 4. *There exist $\bar{\delta} < 1$ and $\bar{F} > 0$ such that, when $F < \bar{F}$ and $\delta > \bar{\delta}$, then at a subgame perfect equilibrium of the repeated game with exclusive affiliation, collusion is deterred and the insureds' expected utility is higher than when insurers offer collusion-proof contracts under non-exclusive affiliation.*

Proof. See the Appendix. □

Under exclusive affiliation risk-sharing is efficient although customers suffer from the insurance companies' market power. However, this situation is socially preferable to a non-exclusive affiliation situation when cooperation between insurers against fraud is not possible, particularly when the extent of fraud is large and the judicial system rather ineffective in deterring collusion. Of course, should insurers being able to gather together to coordinate against fraud, such a recourse to exclusive-affiliation would not be necessary, and even detrimental. Indeed, non-exclusive affiliation (and an active competition between insurance companies) would then allow insureds to reach an higher welfare level.

6 Conclusion

This paper provides a theoretical approach to the benefits for an insurance company to develop its own network of providers when insurance fraud is characterized by collusion between opportunistic policyholders and providers. In a static framework, the exclusive affiliation of providers allows insurance companies to recover some market power and to lessen competition on the insurance market. We have demonstrated that this entails a loss of efficiency and that customers are made worse-off by this possibility. Moreover, it turns out that in a setting in which insurers choose non-cooperatively their affiliation structures, exclusive affiliation is more likely to arise at the equilibrium. We have provided anecdotal evidences for the fact that insurance companies have taken advantage in developing their own networks of providers.

However, we have also established that exclusive affiliation may entail a positive effect on customers' surplus when insurers and providers have repeated relationship. Our argument is that when insurance companies detect fraudulent claims, the mere fact that they will offer only collusion-proof contracts to the customers of the malevolent provider they do not trust anymore reduces the provider's profit. This threat acts as an additional deterrent for fraudulent activities and may supplement an inefficient judicial system: it is thus a second-best optimal anti-fraud policy.

Note finally that we have assumed that competition between insurers is fierce. However, the repeated relationship we consider in our dynamic analysis of collusion may allow insurance companies to reach higher profit levels through tacit collusion in their contract offer strategy. Analyzing the complex strategies that insurers and providers could follow in such a setup would certainly be an interesting extension of the present paper.

7 References

- Baker, L., and M. Brown (1999), "Managed Care, Consolidation among Health Care Providers, and Health Care: Evidence from Mammography", *The RAND Journal of Economics*, 30(2): 351-374.
- Evans Cuellar, A. and P. J. Gertler (2005), "Strategic integration of hospitals and physicians", *Journal of Health Economics*, in Press.
- Gal-Or, E. (1997), "Exclusionary equilibria in healthcare markets", *Journal of Economics and Management Strategy*, 6: 5-43.
- Gal-Or, E. (1999), "Mergers and Exclusionary Practices in Health Care Markets", *Journal of Economics & Management Strategy*, 8(3): 315-350.
- Gaynor, M. and D. Haas-Wilson (1999), "Change, Consolidation, and Competition in Health Care Markets", *The Journal of Economic Perspectives*, 13(1): 141-164.
- Lyon, T., (1999), "Quality Competition, Insurance, and Customer Choice in Health Care Markets", *Journal of Economics & Management Strategy*, 8(4): 545-580.
- Ma, C.-T. A., and I. Alger (2003), "Moral hazard, Insurance and some Collusion", *Journal of Economic Behavior & Organization*, 50: 225-247.
- Ma, C.-T. A., and Thomas McGuire (1997), "Optimal Health Insurance and Provider Payment", *The American Economic Review*, 87(4): 685-704.
- Ma, C.-T. A., and Thomas McGuire (2002), "Network Incentives in Managed Health Care", *Journal of Economics & Management Strategy*, 11(1): 1-35.
- Mas-Colell, A., M. Whinston and J. Green, *Microeconomic theory*, 1995, Oxford University Press.
- Newhouse, J. (1996), "Reimbursing Health Plans and Health Providers: Efficiency in Production Versus Selection", *Journal of Economic Literature*, 34(3): 1236-1263.
- Picard, P. (1996), "Auditing claims in insurance market with fraud : the credibility issue", *Journal of Public Economics*, 63(1): 27-56.
- Rey, P. and J. Tirole (2006), "A Primer on Foreclosure", working paper University of Toulouse.

Town, R. and G. Vistnes (2001), "Hospital competition in HMO networks",
Journal of Health Economics, 20(5): 733-753

A Appendix

A.1 Proof of Proposition 1

As $\hat{k} > \pi\hat{p}$, let us denote $\hat{k} = \hat{\gamma}\pi\hat{p}$ where $\hat{\gamma} > 1$ and observe that we have $k^* = \pi p^*$. Conditions (7) and (14) can be written as $\psi(\hat{p}, \gamma) = t = \psi(p^*, 1)$ where:

$$\psi(p, \gamma) = (p - c)u'(w - \gamma\pi p).$$

We have $\partial\psi(p, \gamma)/\partial p = u'(w - \gamma\pi p) - (p - c)\gamma\pi u''(w - \gamma\pi p) > 0$ and $\partial\psi(p, \gamma)/\partial\gamma = -(p - c)u''(w - \gamma\pi p) > 0$. $p^* > \hat{p}$ is derived from the fact that $\psi(\hat{p}, 1) < \psi(\hat{p}, \hat{\gamma}) = \psi(p^*, 1)$.

Since customers are fully insured, using $p^* > \hat{p}$ we have:

$$u'(w - \hat{k}) = \frac{t}{\hat{p} - c} > \frac{t}{p^* - c} = u'(w - k^*).$$

As $u(\cdot)$ is concave, we must have $w - \hat{k} < w - k^*$, hence $k^* < \hat{k}$, which proves that policyholders reach a higher expected utility under non-exclusive affiliation than under exclusive affiliation

A.2 Proof of Lemma 3

The first part of the proposition has been derived in the main text. We first demonstrate that $p_0 > p_1$. The providers choose p_0 and p_1 that solve $\max_{p_0} \pi(p_0 - c)\tilde{x}_i$ and $\max_{p_1} \pi(p_1 - c)(1 - \tilde{x}_i)$ respectively. Using $k_0 = \pi p_0$ and $s_0 = (1 - \pi)p_0$, the first-order conditions are given by:

$$\phi(p_0, \pi) = t + [u(w - \pi p_0) - (1 - \pi)u(w) - \pi u(w - p_1)]/\pi, \quad (19)$$

and:

$$\phi(p_1, 1) = t - [u(w - \pi p_0) - (1 - \pi)u(w) - \pi u(w - p_1)]/\pi, \quad (20)$$

where $\phi(p, k) = (p - c)u'(w - kp)$. Assume $p_1 \geq p_0$. As:

$$u(w - \pi p_0) - (1 - \pi)u(w) - \pi u(w - p_1) > u(w - \pi p_0) - u(w - \pi p_1) \geq 0,$$

we have $\phi(p_0, \pi) \geq \phi(p_1, 1)$. Moreover, as $\phi'_k(p, k) = -p(p - c)u''(w - kp) > 0$, we have $\phi(p_1, 1) > \phi(p_1, \pi)$ and thus $\phi(p_0, \pi) > \phi(p_1, \pi)$. But since $\phi'_p(p, \pi) = u'(w - \pi p) - \pi(p - c)u''(w - \pi p) > 0$, it implies $p_0 > p_1$, a contradiction. We thus have $p_0 > p_1$. To verify that provider 0 enjoys a larger profit than provider 1, observe that when $t = 0$ we have $p_0 > p_1 = c$. Indeed, as $u(w - \pi c) > (1 - \pi)u(w) - \pi u(w - c)$ we cannot have (19) and (20) satisfied simultaneously at $p_0 = p_1 = c$. Moreover, we have $\tilde{x}_1 = 1$ when $t = 0$, since consumers' expected utility is given by $u(w - \pi p_0)$ if they chose to be insured, while they get $(1 - \pi)u(w) + \pi u(w - c)$ if they don't: provider 0 thus sets p_0 to give consumers the certainty equivalent of being uninsured and seizes the entire market. The reader can easily verify that this is also true for $t > 0$ small.

A.3 Proof of Lemma 4

First, notice that the contracts for customers who visit provider 0 offer full insurance and actuarially fair premiums: $k_A^0 = k_B^0 = \pi p_0$ and $s_A^0 = s_B^0 = p_0(1 - \pi)$. Indeed, if it were not the case, then either insurer A or insurer B would have an incentive to undercut its rival to attract all those customers

Second, it remains to determine the contract offered by insurance company B to customers who elected provider 1. The marginal customer \tilde{x} is now defined as follows:

$$(1 - \pi)u(w - k_0) + \pi u(w - p_0 + s_0) - \pi t \tilde{x} = (1 - \pi)u(w - k_B^1) + \pi u(w - p_1 + s_B^1) - \pi t(1 - \tilde{x}).$$

The optimal insurance contract and providers' price solve:

$$\begin{aligned} & \max_{\{k_B^1, s_B^1\}} (1 - \tilde{x})[(1 - \pi)k_B^1 - \pi s_B^1], \\ & \max_{p_0} \pi \tilde{x}(p_0 - c), \\ & \max_{p_1} \pi(1 - \tilde{x})(p_1 - c). \end{aligned}$$

The comparisons with situations of exclusive affiliation and no exclusive affiliation is immediate

A.4 Proof of Proposition 2

Applying Proposition 8.F.1 of Mas-Colell et al. (1995), we have to verify that affiliating one provider only (e.g. provider 0 for insurer A and provider 1 for insurer B) is the best response to a sequence of totally mixed strategies that converges to the corresponding exclusive affiliation strategy. Consider for example the sequence of totally mixed strategies $(\sigma_A^m, \sigma_B^m) = \{(1 - 2\varepsilon/m, \varepsilon/m, \varepsilon/m), (\varepsilon/m, 1 - 2\varepsilon/m, \varepsilon/m)\}$, $m = 1, 2, \dots$, with $\varepsilon \in (0, 1/3)$, which converges to $(\sigma_A, \sigma_B) = \{(1, 0, 0), (0, 1, 0)\}$ as $m \rightarrow +\infty$, i.e. the pure strategy equilibrium (Provider 0, Provider 1). For, say insurer A , choosing provider 0 when insurer B follows strategy σ_B^m yields $(1 - 2\varepsilon/m)\hat{R} + \varepsilon/m(\hat{R} - \gamma)$, which is greater than its expected profit of choosing provider 1 (which is given by $\varepsilon/m\hat{R} + \varepsilon/m(\hat{R} - \gamma)$) or Providers 0 & 1 ($\varepsilon/m(\hat{R} - \gamma) + (1 - 2\varepsilon/m)(\hat{R} - \gamma)$).

A.5 Proof of Lemma 5

In the non-exclusive affiliation case, s_{cp}^* and k_{cp}^* are deduced from (16) which is binding and the budget constraint. Whatever the affiliation structure, the providers' programs are not directly affected by (16). Maximizing provider 0's profit leads to $\phi(p, s) = t$ at a symmetric equilibrium, where $\phi(p, s) \equiv u'(w - p + s)(p - c)$. As $\phi'_p = u'(w - p + s) - (p - c)u''(w - p + s) > 0$ and $\phi'_s = (p - c)u''(w - p + s) < 0$, we have $\phi(p_{cp}^*, s_{cp}^*) = \phi(p^*, s^*) = \phi(p^*, (1 - \pi)p^*) < \phi(p^*, s_{cp}^*)$ and thus $p_{cp}^* < p^*$. Using the implicit function theorem, solutions (p, s) of the provider's program satisfy:

$$\frac{dp}{ds} = -\frac{\phi'_s}{\phi'_p} = \frac{-tu''/u'}{u' - tu''/u'} < 1. \quad (21)$$

Since $p^* > p_{cp}^*$, we thus have:

$$p^* < p_{cp}^* + s^* - s_{cp}^*,$$

which implies:

$$p_{cp}^* > (1 - \pi)2Fq/(1 - q) + \pi p^* > 2Fq/(1 - q).$$

Under exclusive affiliation, insurer A 's program can be written as:

$$\max_{\{k_A, s_A\}} \{\tilde{x}_i [(1 - \pi)k_A - \pi s_A] : k_A + s_A \leq 2Fq/(1 - q)\},$$

where:

$$\tilde{x}_i = \frac{1}{2} + \frac{1}{2t\pi} \{(1-\pi)[u(w-k_A) - u(w-k_B)] + \pi[u(w-p_0+s_A) - u(w-p_1+s_B)]\}.$$

The corresponding Lagrangian is given by:

$$L = \tilde{x}_i [(1-\pi)k_A - \pi s_A] + \lambda [2Fq/(1-q) - (k_A + s_A)],$$

where $\lambda \geq 0$. First-order conditions are:

$$\begin{aligned} (1-\pi)\tilde{x}_i - \frac{(1-\pi)u'(w-k_A)}{2t\pi} [(1-\pi)k_A - \pi s_A] - \lambda &= 0, \\ -\pi\tilde{x}_i + \frac{u'(w-p_0+s_A)}{2t} [(1-\pi)k_A - \pi s_A] - \lambda &= 0, \end{aligned}$$

and simplify to:

$$\begin{aligned} t\pi - 2t\pi\lambda/(1-\pi) &= u'(w-\hat{k}_{cp}) [(1-\pi)\hat{k}_{cp} - \pi\hat{s}_{cp}], \\ t\pi + \lambda &= u'(w-\hat{p}_{cp} + \hat{s}_{cp}) [(1-\pi)\hat{k}_{cp} - \pi\hat{s}_{cp}], \end{aligned} \quad (22)$$

at a symmetric equilibrium. This leads to:

$$\frac{u'(w-\hat{k}_{cp})}{u'(w-\hat{p}_{cp} + \hat{s}_{cp})} = \frac{t\pi - 2t\pi\lambda/(1-\pi)}{t\pi + \lambda}.$$

Consequently, we have $u'(w-\hat{k}_{cp}) < u'(w-\hat{p}_{cp} + \hat{s}_{cp})$ when $\lambda > 0$, i.e., $\hat{p}_{cp} > \hat{s}_{cp} + \hat{k}_{cp} = 2Fq/(1-q)$. The maximization of the provider 0's program leads to condition $\phi(\hat{p}_{cp}, \hat{s}_{cp}) = t$ at a symmetric equilibrium, and thus:

$$u'(w-\hat{p}_{cp} + \hat{s}_{cp})\pi(\hat{p}_{cp} - c) = \pi t.$$

Use of (22) gives:

$$\frac{(1-\pi)\hat{k}_{cp} - \pi\hat{s}_{cp}}{\pi(\hat{p}_{cp} - c)} = \frac{t\pi + \lambda}{t\pi};$$

hence $(1-\pi)\hat{k}_{cp} - \pi\hat{s}_{cp} > \pi(\hat{p}_{cp} - c)$ when $\lambda > 0$. Consequently, \hat{s}_{cp} and \hat{k}_{cp} verify:

$$\begin{aligned} \hat{s}_{cp} + \hat{k}_{cp} &= 2Fq/(1-q), \\ (1-\pi)\hat{k}_{cp} - \pi\hat{s}_{cp} &> \pi(\hat{p}_{cp} - c), \end{aligned}$$

which gives $\hat{s}_{cp} < (1-\pi)2Fq/(1-q) - \pi(\hat{p}_{cp} - c)$ and $\hat{k}_{cp} > \pi 2Fq/(1-q) + \pi(\hat{p}_{cp} - c)$. Finally, as $\hat{s}_{cp} < s_{cp}^*$ and $\phi(p_{cp}^*, s_{cp}^*) = \phi(\hat{p}_{cp}, \hat{s}_{cp}) > \phi(\hat{p}_{cp}, s_{cp}^*)$, we have $p_{cp}^* > \hat{p}_{cp}$.

A.6 Proof of Lemma 6

Consider first the case of non-exclusive affiliation. Suppose that the providers charge prices $p_0 = p_1 = p_{cp}^*$ and that insurer B offers the collusion-proof contract (s_{cp}^*, k_{cp}^*) leading to insureds' expected utility (gross of transportation cost) EU_{cp} . Insurer A may deviate from strategy (s_{cp}^*, k_{cp}^*) by choosing a non-collusion proof contract (s, k) if:

$$(1-\pi)k - \pi s - (1-q)\theta(1-\pi)(k+s) \geq 0, \quad (23)$$

and if the insureds' expected utility with non-collusion proof contracts (gross of transportation cost), $EU_{ncp}(\theta)$, is greater than EU_{cp} , where:

$$EU_{ncp}(\theta) \equiv \pi u(w - p_{cp}^* + s) + (1 - \pi)[(1 - \theta(1 - q))u(w - k) + \theta(1 - q)u(w + (s - k)/2)]. \quad (24)$$

Obviously, if θ is close to zero and the collusion proofness constraint (16) is binding, such a strategy exists. Consider now the case $\theta = 1$. As $u(\cdot)$ is concave, we have:

$$EU_{ncp}(1) \leq \pi u(w - p_{cp}^* + s) + (1 - \pi)u(w + (1 - q)s/2 - k(1 + q)/2) \equiv E\bar{U}_{ncp}$$

The best offer that insurer A can made to customers satisfies (23) as an equality, implying:

$$k = \frac{1 - q + q\pi}{q(1 - \pi)} s,$$

which gives:

$$E\bar{U}_{ncp} \leq \psi(s) \equiv \pi u(w - p_{cp}^* + s) + (1 - \pi)u\left(w - \frac{1 - q + 2q\pi}{2q(1 - \pi)} s\right).$$

Differentiating, we get:

$$\psi'(s) = \pi u'(w - p_{cp}^* + s) - \frac{1 - q + 2q\pi}{2q} u'\left(w - \frac{1 - q + 2q\pi}{2q(1 - \pi)} s\right)$$

and $\psi''(s) < 0$. Consequently, as we have $\psi(0) < E\bar{U}_{cp}$ and $\psi'(0) < 0$ if $q < \tilde{q}$ where:

$$\tilde{q} \equiv \frac{u'(w)}{u'(w) + 2\pi[u'(w - p_{cp}^*) - u'(w)]} \in (0, 1),$$

the optimal non-collusion proof contract is $s = k = 0$ when $\theta = 1$ and $q < \tilde{q}$. By continuity, this must be also the case for θ close but smaller than 1.

In the case of exclusive affiliation, suppose that the providers charge prices $p_0 = p_1 = \hat{p}_{cp}$ and that insurer B offers the collusion-proof contract $(\hat{s}_{cp}, \hat{k}_{cp})$ leading to an insureds' expected utility (gross of transportation cost) $E\hat{U}_{cp}$. Insurer A deviates from strategy $(\hat{s}_{cp}, \hat{k}_{cp})$ by choosing a non collusion-proof contract (s, k) if:

$$\tilde{x}[(1 - \pi)k - \pi s - (1 - q)\theta(1 - \pi)(k + s)] \geq [(1 - \pi)\hat{k}_{cp} - \pi\hat{s}_{cp}]/2 > 0, \quad (25)$$

where:

$$\tilde{x} = \frac{1}{2} + \frac{1}{2t\pi}[E\hat{U}_{ncp}(\theta) - E\hat{U}_{cp}],$$

with $E\hat{U}_{ncp}(\theta)$ defined similarly to $EU_{ncp}(\theta)$ given by (24) but with \hat{p}_{cp} in place of p_{cp}^* . Obviously, if θ is close to zero and the collusion proofness constraint (16) is binding, such a strategy exists. However, when $\theta = 1$ and $q = 0$ we have

$$\tilde{x}[(1 - \pi)k - \pi s - (1 - q)\theta(1 - \pi)(k + s)] = -\tilde{x}s \leq 0 < [(1 - \pi)\hat{k}_{cp} - \pi\hat{s}_{cp}]/2$$

and insurer A has no incentive to deviate. By continuity, this must be also the case for θ close but smaller than 1 and q greater but close to 0.

A.7 Proof of Proposition 3

We know that customers are better-off without affiliation than with exclusive affiliation when $p^* < 2Fq/(1-q)$. Let us show first that this is still the case when $\hat{p} \leq 2Fq/(1-q) < p^*$. Assume the reverse, i.e., that:

$$u(w - \hat{k}) \geq (1 - \pi)u(w - k_{cp}^*) + \pi u(w - p_{cp}^* + s_{cp}^*). \quad (26)$$

As $p_{cp}^* > k_{cp}^* + s_{cp}^*$, we have:

$$(1 - \pi)u(w - k_{cp}^*) + \pi u(w - p_{cp}^* + s_{cp}^*) > u(w - p_{cp}^* + s_{cp}^*),$$

and thus (26) implies $\hat{k} < p_{cp}^* - s_{cp}^*$. Solving the provider's program we obtain that $\phi(\hat{p}, \hat{s}) = \phi(p_{cp}^*, s_{cp}^*) = t$ and, from (21), we know that the solutions (p, s) of the provider's program satisfy $dp/ds < 1$. Consequently, as $p_{cp}^* > 2Fq/(1-q) \geq \hat{p}$, we have:

$$p_{cp}^* < \hat{p} + s_{cp}^* - \hat{s}.$$

and using $\hat{p} = \hat{s} + \hat{k}$, we obtain $\hat{k} > p_{cp}^* - s_{cp}^*$, hence a contradiction.

Consider now the case $Fq/(1-q) < \hat{p}/2$. From Lemma 5, we know that $p_{cp}^* > \hat{p}_{cp}$ whatever $Fq/(1-q) < \hat{p}/2$. Again, assume that customers are better-off under exclusive affiliation than without affiliation. We have:

$$(1 - \pi)u(w - \hat{k}_{cp}) + \pi u(w - \hat{p}_{cp} + \hat{s}_{cp}) \geq (1 - \pi)u(w - k_{cp}^*) + \pi u(w - p_{cp}^* + s_{cp}^*). \quad (27)$$

Observe first that we cannot have $\hat{s}_{cp} - \hat{p}_{cp} \leq s_{cp}^* - p_{cp}^*$. Indeed, in that case (27) implies $\hat{k}_{cp} \leq k_{cp}^*$ and thus:

$$\hat{k}_{cp} + \hat{s}_{cp} - \hat{p}_{cp} \leq k_{cp}^* + s_{cp}^* - p_{cp}^*,$$

which implies $p_{cp}^* \leq \hat{p}_{cp}$ using $\hat{s}_{cp} + \hat{k}_{cp} = k_{cp}^* + s_{cp}^* = 2Fq/(1-q)$. Hence, (27) implies $p_{cp}^* > \hat{p}_{cp} + s_{cp}^* - \hat{s}_{cp}$. But since $p_{cp}^* > \hat{p}_{cp}$, we have $p_{cp}^* < \hat{p}_{cp} + s_{cp}^* - \hat{s}_{cp}$ using (21), a contradiction.

A.8 Proof of proposition 4

Define $U(F)$ as:

$$U(F) \equiv \max_{s \geq 0, k \geq 0, p \geq 0} (1 - \pi)u(w - k) + \pi u(w - p + s) \\ \text{s.t. } k + s \leq 2Fq/(1-q); (1 - \pi)k - \pi s = 0; \phi(p, s) = t$$

Hence, $U(F)$ is an upper bound of customers' expected utility under non-exclusive affiliation when insurance companies offer collusion-proof contracts. Let $\lambda \geq 0$ be the Lagrange multiplier associated to the constraint $k + s \leq 2Fq/(1-q)$. The envelope theorem gives $U'(F) = 2\lambda q/(1-q)(1-\pi) \geq 0$, with $U'(F) > 0$ for all $F < p^*(1-q)/(2q)$. Let \bar{F} denote the threshold values of F such that $U(\bar{F}) = u(w - \hat{k})$. As $u(w - \hat{k}) < u(w - k^*)$, $\bar{F} < p^*(1-q)/(2q)$. We have $u(w - \hat{k}) > U(0)$, where $U(0)$ corresponds to the expected utility of customers without insurance. Moreover, since $U'(F) > 0$ for all $F < p^*(1-q)/(2q)$, $\bar{F} > 0$ and we have $U(F) < u(w - \hat{k})$ for all $F < \bar{F}$. Denote by $\bar{\delta}$ the

threshold value of δ defined by:

$$\frac{2q}{1-q} \left[F + \frac{\bar{\delta}}{1-\bar{\delta}} (\hat{\Pi}_j - \hat{\Pi}_j^{cp}) \right] = \hat{s} + \hat{k} = \hat{p}$$

We have $0 < \bar{\delta} < 1$ and the collusion proofness constraint (18) is satisfied under exclusive affiliation for all $\delta \geq \bar{\delta}$. The proposition comes from the facts that, firstly, when $Fq/(1-q) < p^*/2$, the customer's expected utility (gross of his transportation cost) is smaller or equal to $U(F)$ when insurers offer collusion-proof contracts under non-exclusive affiliation and, secondly, that we have $u(w - \hat{k}) > U(F)$ for all $F < \bar{F}$.