Transferring the Sticky:
Individual Annuity Demand and Demographic Risk

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Abstract Demographic risk—the risk that mortality laws change in a nondeterministic way—and its implications for corporate decisions has recently been the subject of lively scientific discussion. We show that demographic risk is also a key determinant for individual annuitization decisions. Demographic risk appears to be a *sticky*, i.e., hardly transferable, risk. Whether its existence leads to a higher or lower annuity demand depends on objective factors (e.g., insurers' vulnerability to demographic shocks). Subjective factors (i.e., individuals' preferences) determine only the intensity of the annuity demand reaction to demographic risk. Our results are of significant importance not only for financial planning approaches of individual annuity buyers, but also for strategic decisions in insurance companies and for solvency regulators. Furthermore, consideration of demographic risk may both alleviate, but also intensify, the annuity puzzle.

Keywords demographic risk, annuitization, retirement decisions, annuity puzzle, intertemporal utility maximization

JEL-Classification D14, D81, D91, G11, G22, J11, J26

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Introduction

Initiated by Yaari's seminal work, the study of optimal individual annuitization decisions became of increasing interest in the economics literature. Yaari (1965) showed that, in a perfect market setting, expected utility maximizers with no utility of bequest would manage the uncertainty regarding their lifetime by annuitizing their entire wealth. Under more general assumptions on utility functions, this result was recently confirmed by Davidoff, Brown, and Diamond (2005). Empirical studies find, however, that only a small portion of private wealth is used to purchase annuities. Several theoretical and empirical posited explanations to explain this "annuity puzzle", e.g.:

- due to adverse selection and transaction costs, annuities are unfairly priced (Friedman and Warshawsky, 1988; Mitchell et al., 1999);
- constant annuity payouts in combination with borrowing and short-selling constraints may induce a suboptimal consumption profile (Brown, 2001);
- the existence of bequest motives (Yaari, 1965);
- the crowding-out effect of government pensions (Mitchell et al., 1999; Brown and Poterba, 2000);
- intra-family risk sharing (Kotlikoff and Spivak, 1981; Post, Gründl, and Schmeiser, 2006); and
- default-risk of the annuity providing insurer (Babbel and Merrill, 2006).

This paper provides a normative analysis of the impact of *demographic risk*—which is also called stochastic mortality (e.g., Cairns, Blake, and Dowd, 2006) or aggregate mortality risk (e.g., Brown and Orszag, 2006)—on individual annuity demand. Demographic risk refers to the fact that mortality laws and life tables describing lifetime uncertainty may change in a nondeterministic way (see Olivieri, 2001; Cairns, Blake, and Dowd, 2006; Brown and Orszag, 2006). Such unexpected changes in mortality may result from, e.g., medical innovations (leading to lower mortality rates, see, e.g., Hayflick, 2000; Held, 2002; Olshansky, Hayflick, and Carnes, 2002) or increased occurrence of very hot summers—comparable to the 2003 and 2006 summers in Europe—as a consequence of a global climate change (on hot summers leading to higher mortality rates, see Valleron and Boumendil, 2004; Conti et al., 2005). The importance of demographic risk for insurance company risk management decisions has been highlighted in recent publications (see, e.g., Lin and Cox, 2005; Cowley and Cummins, 2005; Cairns, Blake, and Dowd, 2006; Gründl, Post, and Schulze, 2006; Brown and Orszag, 2006). We show that demographic risk is also a key determinant in individual risk management, especially for annuitization decisions.

Our investigation utilizes an intertemporal expected utility framework with borrowing and short-selling constraints and uninsurable government pension income risk. The risk-averse individual must optimize decisions regarding consumption, saving, and how to allocate savings between a risky asset, a risk free asset, and annuities. In accordance with Olivieri (2001) and Gründl, Post, and Schulze (2006), demographic risk is modeled as a mean-preserving shock on future survival probabilities.

We show that demographic risk does not influence optimal decisions if it is stochastically independent of other sources of the individual's income risk. We make clear that this is a direct consequence of the preference assumptions under which optimal decisions are derived. This result is in accordance with Rothschild and Stiglitz (1976) in the context of informational asymmetry, and also with Franke, Schlesinger, and Stapleton (2005, 2005a) in the context of—amongst other—tax uncertainty.

However, for most individuals, demographic risk will not be independent from other sources of income risk. For example, Gründl, Post, and Schulze (2006) made clear that the solvency situation of an annuity-providing life insurer is strongly influenced by demographic risk. Depending on the structure of the life insurer's contract portfolio and asset allocation, mortality shocks can lead to an at least partial insurer default, thus linking the distribution of effective annuity payments to demographic risk.[1] Furthermore, government pension payments—especially pay-as-you-go systems—may depend on demographic developments.[2] Also, the insurer's investment in the risky asset influences its

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[1] For a general analysis of the influence of an insurer's default risk on annuity demand, see Babbel and Merrill (2006).

[2] For example, a positive shock on survival rates means that—ceteris paribus—a greater number of retirees will have to be paid out of a given amount of social security taxes. Thus, the average pension must shrink. For further analyses, see, e.g., Aaron (1966) or

solvency situation. Thus, the performance of the risky asset influences the distribution of effective annuity payments. Finally, the performance of the individual's investment in the risky asset might negatively depend on the development of future survival rates ("asset meltdown").[3]

After integrating these dependencies into our framework, the individual's optimal decisions become highly dependent on demographic risk. We show that whether or not demographic risk leads to an increase in individuals' annuity demand is determined by objective factors (such as the exposure of government pensions and/or the insurance industry to demographic shocks). Subjective factors (such as individuals' risk aversion) determine only the intensity of the annuity demand reaction with respect to demographic risk. We find that—depending on the model parameters—consideration of demographic risk may both alleviate, but also intensify, the annuity puzzle.

This paper is organized as follows. In Section 1, we first formalize our model. As the model will be analytically tractable only under very restrictive assumptions, we next calibrate the model parameters in order to perform further analyses numerically. Our results are presented in Section 2 and policy implications are derived. Section 3 summarizes.

Kotlikoff (1979). Ongoing reforms of the German government pension system provide a recent example for such a demographically driven reduction of pension payments (see, e.g., Börsch-Supan and Wilke, 2004).

[3] The asset meltdown hypothesis is motivated by the idea that an increase in survival rates results in a greater number of retirees. Besides the general argument that an "older" society is said to be less innovative and productive, increased pension obligations of business firms may have an adverse effect on their financial performances (Bakshi and Chen, 1994). That demographic variables may have an effect on asset prices, and the direction of this effect is not unequivocally accepted in the literature. For further discussion, see, e.g., Bakshi and Chen (1994); Abel (2001, 2003); Poterba (2001); Brooks (2002).

1 The Model[4]

1.1 Formalization

We consider a two-period framework. Let w_1 denote the individual's known initial wealth in period 1, and $c_i \geq 0$, $i = 1, 2$, the individual's nominal consumption in period i. Denoting by u the individual's one-period utility function with respect to a real monetary argument (period consumption), we assume that from the perspective of period 1, the individual evaluates total two-period utility additively separably by

$$u(c_1) + \beta \cdot I_{Ind} \cdot u(c_2 \cdot \delta), \tag{1}$$

where I_{Ind} is a Bernoulli-distributed random indicator variable with success probability p taking value 1 if the individual survives (so that p is the individual's survival probability), and value 0 if the individual dies before period 2 (implying that there are no bequest motives). We assume that p is representative for the whole population and subject to demographic risk. Similarly, as done by Olivieri (2001) and Gründl, Post, and Schulze (2006), we model demographic risk as a symmetrically distributed additive deviation Δp from individuals' mean survival probability p_0. Thus, the survival probability p is random and given by $p = p_0 + \Delta p$. By $\beta \in (0, 1)$ we denote the individual's subjective discount factor (expressing time preference) and $\delta > 0$ is the one-period monetary deflator.

Let s_R and s_f be the amounts of money the individual invests in a risky and a risk free asset in period 1, providing him with a risky return of $s_R \cdot R$ and a risk free return of $s_f \cdot r_f$ in period 2, respectively. We assume both s_R and s_f to be nonnegative, implying that the individual is not allowed to either short-sell the risky asset or go into debt. We further assume that the individual can decide to put some money $\pi \geq 0$ into a life annuity, providing him with an annuity payment in period 2 given he survives until period 2, i.e., if I_{Ind} realizes as 1. Total savings, i.e., $s_R + s_f + \pi$, will henceforth be denoted as S.

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[4] We summarize the model parameters and their calibrations in Table 3 at the end of this section.

Let us assume that the annuity payment in period 2 is subject to the default risk of the insurance company having sold the contract and that in the case of ruin, the individual receives only a fraction $\psi \in [0, 1]$ of every € of the agreed-upon survival benefit (depending, of course, on the premium π). We can understand ψ as the fraction of the claims against the insurer that can be recovered by some guarantee fund. Let us denote by $d(r,\varphi) \in [0, 1]$ the insurer's ruin probability *given* realizations r and φ of R and Δp. With respect to the influences of the actual outcomes of R and Δp, we require that the first derivative of d with respect to its first argument is negative, while the sign of the first derivative of d with respect to its second argument is positive. The first assumption implies that the insurer's ruin probability is the higher the poorer is the performance of the risky asset. This is plausible since risky assets usually make up a significant part of an insurer's asset portfolio. The second assumption implies that the insurer has underwritten more annuity than term-life business.[5] In case the derivative of d with respect to its second argument is equal to zero, the insurer's solvency situation appears to be immune to demographic shocks.

Note that while the ruin probability d is an indicator of the insurer's idiosyncratic risk of payment default, the default pay-off fraction ψ may be interpreted as an indication of the unprotected exposure of the whole insurance industry to systematic risks such as asset return risk and, especially, demographic risk. If ψ is very low, exposure to systematic risks is very high (which could be the case if the guarantee fund is insufficiently funded to provide adequate cover against systematic risks that hit many life insurers at the same time). On the contrary, a high ψ may mean that there is a reliable guarantor to support the insurance industry in case of a systematic default.

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[5] As discussed by Gründl, Post, and Schulze (2006), the actual dependency of the insurer's ruin probability on demographic shocks depends on the insurer's underwriting policy. For example, if the insurer holds a liability portfolio that mostly consists of annuity contracts, the relationship between d and Δp should be positive, whereas for an insurer that mostly sold term-life insurance contracts, natural hedging effects lead to a negative relationship. Furthermore, an insurer may hold mortality derivatives (see, e.g., Cowley and Cummins, 2005). In that case, the insurer's vulnerability to demographic risk depends on the degree of coverage and the basis risk of these instruments (see Blake, Cairns, and Dowd, 2006).

Let us further denote by $A \geq 0$ the actuarially fair survival payment of the annuity. Accounting for the insurer's possible ruin and assuming symmetric beliefs between the individual and the insurer regarding survival probability distributions and, as in Doherty and Schlesinger (1990), the insurer's ruin probability,[6] for a given ruin probability d and a given survival probability p, the actuarially fair survival payment A is determined via the pricing formula

$$\pi = r_f^{-1} \cdot p \cdot [(1-d) \cdot A + d \cdot \psi \cdot A]. \tag{2}$$

Thus the annuity price is the higher the lower the insurer's ruin probability and the higher the default pay-off fraction is, i.e., the annuity price increases as product quality increases (Doherty and Schlesinger, 1990). Rearranging and now accounting for the stochastic character of the arguments of d, and p itself, we achieve for the actuarially fair survival payment

$$A(\pi) = \frac{r_f \cdot \pi}{\iint (p_0 + \varphi)\left[1-(1-\psi)\cdot d(r,\varphi)\right]\cdot \mathrm{d}F(r,\varphi)} \tag{3}$$

where F denotes the joint distribution function of R and Δp. The fair survival payment A thus depends—beyond the individual's decision on π—on the default pay-off fraction ψ and on the joint distribution of the risky asset return R and the survival probability shock Δp. To implement premium loadings, let us further introduce the parameter $\lambda \in [0, 1]$. Instead of the fair amounts $A(\pi)$ and $\psi \cdot A(\pi)$ that are paid out depending on the insurer's solvency situation, only $\lambda \cdot A(\pi)$ and $\lambda \cdot \psi \cdot A(\pi)$ are paid out. A certain value for λ thus stands for a percentage premium loading of $(1-\lambda)/\lambda \cdot 100\%$.

If I_{Ins} is a Bernoulli-distributed indicator variable with success probability $1-d(r,\varphi)$ for given realizations r and φ of R and Δp (i.e., which is 0 if the insurer defaults and is 1 if it does not), we can write the effective annuity payment, let us call it $L(\pi)$, the individual receives given he survives until period 2 as

6 For an analysis of asymmetric information about the ruin probability, see Cummins and Mahul (2003).

$$L(\pi) = \lambda \cdot A(\pi) \cdot \left[I_{Ins} + (1 - I_{Ins}) \cdot \psi \right] \tag{4}$$

which is conditionally (on the realization of d) binarily distributed and purely random as soon as $\psi > 0$ and $d(r,\varphi) > 0$. *Unconditionally*, also the success probability of I_{Ins} is random, which transfers to L, too, so that $L(\pi)$ in Equation (4) actually should be written as $L(\pi \mid R = r, \Delta p = \varphi)$.

Additionally, for a given realization φ of Δp, let $Y(\varphi)$ denote the individual's income from the pay-as-you-go government pension system receivable in period 2, given the individual is alive then. Since pay-as-you-go pension systems strongly depend on the development of demographic variables (see, e.g., Aaron, 1966; Kotlikoff, 1979), we assume that $Y'(\varphi) \leq 0$, which means that an increase (decrease) in the general and individual survival probability will lead to an immediate drop (rise) or constancy in the pension payment.[7]

The individual's (from the perspective of period 1) random consumption in period 2 is thus given by

$$c_2 = s_R \cdot R + s_f \cdot r_f + L(\pi) + Y, \tag{5}$$

where s_R, s_f, and π, together with first period consumption c_1, are linked by the budget constraint $w_1 = c_1 + s_R + s_f + \pi$, and each of these four decision variables is subject to its nonnegativity constraint. The individual's evaluation of total utility is thus given by

$$u(c_1) + \beta \cdot I_{Ind} \cdot u((s_R \cdot R + s_f \cdot r_f + L(\pi) + Y) \cdot \delta) \tag{6}$$

which, of course, is also random. To determine optimal decisions on consumption, investment, and annuity purchase, we assume that the individual maximizes the expected value of total utility, which is given by

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7 See Gomes, Kotlikoff, and Viceira (2006) for a model that analyzes the welfare effects of government indecision, i.e., a case where the government tries to procrastinate the announcement of bad news on retirement benefits.

$$u(c_1)+\beta\cdot\iint(p_0+\varphi)\cdot$$
$$\cdot u\Big(\big[s_R\cdot r+s_f\cdot r_f+L\big(\pi\big|R=r,\Delta p=\varphi\big)+Y(\varphi)\big]\cdot\delta\Big)\mathrm{d}F(r,\varphi), \tag{7}$$

with respect to c_1, s_R, s_f, and π, subject to $w_1 = c_1 + s_R + s_f + \pi$ and $c_1, s_R, s_f, \pi \geq 0$.

Due to the complexity of the interrelations of influencing factors in the model and due to the no-short-selling and borrowing constraints, most of the calculations necessary to perform our analyses are not analytically tractable. We therefore determine optimal solutions numerically, making it necessary to first calibrate our model.

1.2 Calibration

In this section, we define the base values assigned to our model parameters. Section 2.3 contains a thorough analysis of the consequences different values would have.

To express the individual's *intra*temporal risk preferences, we use a utility function with constant relative risk aversion (CRRA), given by parameter $\gamma > 0$. Thus for $c > 0$, we have $u(c) = c^{(1-\gamma)} / (1-\gamma)$. The parameter of constant relative risk aversion γ is set to 3; the *inter*temporal subjective discount factor β is set to 0.95 (see, e.g., Laibson, Repetto, and Tobacman, 1998). For the individual's initial wealth, we set $w_1 = 100$.

To model demographic risk, we use three-point distributions as done in Gründl, Post, and Schulze (2006). For the state space of Δp, we set $(-0.01, 0, +0.01)$, which means that the individual's survival probability—starting from p_0—will either increase by 1 percentage point, stay constant, or drop by 1 percentage point. Onto the set of measurable subsets of this state space, we set five alternative probability measures, which we choose as done in Gründl, Post, and Schulze (2006). Table 1 contains the probability distributions used.

Table 1 Probability distributions of Δp

probability distri-bution	probability that Δp is – 0.01 (decrease in survival probability)	 0 (no demographic shock)	 + 0.01 (increase in survival probability)
1	0.000	1.000	0.000
2	0.025	0.950	0.025
3	0.050	0.900	0.050
4	0.100	0.800	0.100
5	0.200	0.600	0.200

In all distributions, the main probability mass—depending on the probability model, between 60% and 100%—is assigned to the state of nature where Δp realizes as 0, i.e., if $E(p) = p_0$ turns out to be the individual's actual survival probability. The remaining mass is then assigned equally to the "deviating" states of nature, where the individual's survival probability realizes as $p_0 - 0.01$ or $p_0 + 0.01$, respectively. For the mean survival probability we set $p_0 = 0.85$.

The one-period return of the risk free asset is set to $r_f = 1.05$, which is the sample mean of the annualized return time series of the German money market from 1950 to 2003 (see Gründl, Post, and Schulze, 2006). The one-period deflator δ is set to 0.9744, which is the inverse of the mean inflation rate given by the German consumer price index (CPI) between 1950 and 2003, which is about 1.0263.

For the return R of the risky asset we initially (Sections 2.1 and 2.2.1) assume that its conditional distribution given any realization of the demographic shock is normal with mean of 1.1567 and standard deviation of 0.3192, implying stochastical independence between R and Δp.[8] Introducing the asset meltdown hypothesis into our model (Section 2.2.2), we then drop this independence assumption, and instead assume the distribution of the risky return to be nega-

[8] These values were also taken from Gründl, Post, and Schulze (2006). They were estimated from a time series (1950 to 2003) of the German stock market index DAX.

tively correlated with the demographic shock.[9] We model this assuming, again, that conditional distributions of R given a realization of Δp are normal. Keeping conditional standard deviations constant at the level used before, i.e., at 0.3192, we now, however, model conditional expected returns to depend on the actual realization of Δp. We do this by assuming that the conditional expectation of R decreases by 2 percentage points to 1.1367 if Δp realizes as + 0.01 (i.e., improvement of the survival probability leads rather to a drop in asset returns) and increases to 1.1767 if Δp realizes as – 0.01. Table 2 summarizes the possible conditional probability distributions of R.[10]

Table 2 Probability distributions of risky return R

	distribution of R conditional on realization φ of Δp in case of	
φ	independence of R and Δp	negative dependence of R and Δp (asset meltdown)
– 0.01	N(1.1567, 0.3192)	N(1.1767, 0.3192)
0	N(1.1567, 0.3192)	N(1.1567, 0.3192)
+ 0.01	N(1.1567, 0.3192)	N(1.1367, 0.3192)

To model the insurer's ruin probability $d(r,\varphi)$ for every pair of realizations (r,φ) of $(R,\Delta p)$, we choose the log-linear functional form

$$d(r,\varphi) = exp(\alpha_{\Delta p}\cdot \varphi + \alpha_R \cdot r + const), \tag{8}$$

where $\alpha_{\Delta p}$ and α_R determine the degree of reaction of the insurer's ruin probability to the demographic scenario and the performance of the risky asset. From the partial derivative conditions above, we require $\alpha_{\Delta p}$ and α_R to be nonnegative and nonpositive, respectively. In cases where the insurer's ruin probability is not dependent on demographic risk or the performance of the risky asset, the respective reaction parameters are set to 0. Besides the natural requirement $0 \le d(r,\varphi) \le 1$, for every pair of realizations (r,φ) of $(R,\Delta p)$, we ad-

[9] See footnote 2.

[10] Dependence between capital market return and survival probability actually would require a risk adjustment in the discount factor when calculating the fair annuity premium in Formula (2). For the sake of facilitating comparative statics, however, we refrain from that.

ditionally impose the restrictions that, first, if R realizes below the 0.1%-percentile of the risky asset return distribution and at the same time Δp realizes as 0 (no survival probability shift), the insurer's ruin probability is equal to 3%; and that, second, every ruin probability induced by a realization of R increases (decreases) tenfold if Δp realized as + 0.01 (– 0.01), i.e., if there is an increase (decrease) in the survival probability. In the—with respect to the insurer's solvency situation—worst case of an asset return below about 0.126 (i.e., a loss of 87.4%) and an increase in the survival probability, the insurer will fail to meet the individual's annuity payment claims with a probability of 30%. In cases where R realizes at the other end of its support, and demographic risk realizes as a reduction in survival probability, the insurer's ruin probability is approximately 0.[11] For the unconditional (expected) ruin probability $\mathrm{E}[d(R,\Delta p)]$ (i.e., the a priori ruin probability of the insurer from the perspective of period 1), we assume that it coincides with some regulator-fixed maximum ruin probability (this is why we need the fitting variable *const* in Equation (8)). As done in Gründl, Post, and Schulze (2006), we set the insurer's maximum unconditional ruin probability to 0.01, i.e., 1%.

We set the default pay-off fraction ψ alternatively to 0 and 0.6. The loading factor λ is set to 0.9, leading to a loading of $1/9 \cdot \pi$. For the payment from the government pay-as-you-go pension system in period 2, we assume as true the functional relationship

$$Y(\varphi) = w_1 \cdot \vartheta \cdot (1 - \alpha_Y \cdot \varphi) \tag{9}$$

where the parameter $\alpha_Y \geq 0$ determines the degree of reaction of the government pension payment on a given realization φ of the demographic shock Δp. The parameter $\vartheta \geq 0$ is some pension factor determining the fraction of initial wealth the individual expects to receive as government pension in period 2.[12] A positive demographic shock on the individual's survival probability will

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[11] Of course, this calibration is to some extent arbitrary. We checked the consequences of changes in these two assumptions (3% and tenfold increase/decrease) and found robustness of the tendencies of our results.

[12] By doing this, the CRRA feature of the one-period utility function makes our results arbitrarily scaleable in the period 1 wealth-to-pension factor ratio; see Pratt (1964) and Carroll (2004).

thus lead to a proportional drop in the individual's pension payment in period 2 (starting with $E[Y(\Delta p)] = w_1 \cdot \vartheta$), while a negative demographic shock will lead to a proportional pension increase.[13] We set ϑ to 0.15. The severity of the change in the pension payment for a given demographic shock φ is determined by α_Y. Depending on whether or not Y reacts to the demographic shock, we set α_Y to 1 or 0, respectively.

Figure 1 summarizes the endogenous dependencies in the model. The variables and parameters used and their base calibration are summarized in Table 3.

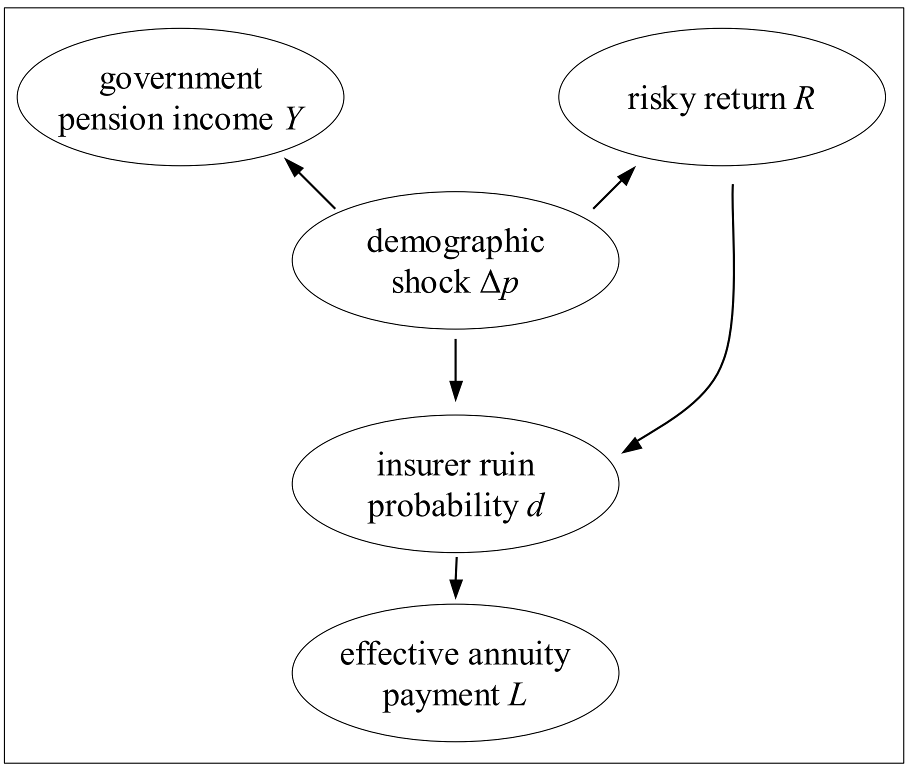

Fig. 1 Summary of model dependencies

[13] See footnote 1.

Table 3 Summary of variables and parameters, and their calibration

symbol	variable/parameter	value/specification
A	actuarially fair survival payment	see Equation (3)
α_Y	reaction parameter of government pension w.r.t. demographic shock	{0, 1}
β	subjective discount factor	0.95
c_i	consumption in period i	decision variable
d	insurer's ruin probability	$E[d(R, \Delta p)] = 0.01$
δ	one-period deflator	1/1.0263
γ	parameter of relative risk aversion	3
L	effective annuity payment	see Equation (4)
λ	annuity loading factor	0.9
Δp	demographic shock	see Table 1
p	survival probability	$p_0 + \Delta p$
p_0	mean survival probability	0.85
π	annuity premium	decision variable
ψ	default pay-off fraction	{0, 0.6}
R	risky return	see Table 2
r_f	risk free return	1.05
S	total savings	$s_R + s_f + \pi$
s_f	risk free investment	decision variable
s_R	risky investment	decision variable
ϑ	government pension factor	0.15
u	one-period utility function	CRRA
w_1	initial wealth	100
Y	government pension payment	see Equation (9)

1.3 Solving Technique

We performed the optimization of Objective (7) by using the MATHEMATICA® 5.1 implemented nonlinear optimizer `NMaximize`. To keep the optimization problem tractable, we discretized the risky return density function using Gaussian quadrature method as done, e.g., by Cocco, Gomes, and Maenhout (2005).

2 Results

As the introduction and model sections of this paper made clear, our model incorporates complex dependencies between demographic risk, government pension payments, the insurer's ruin probability, effective annuity payouts, and the performance of the risky asset. To gain insight into how demographic risk per se is perceived by an expected utility maximizer, in Section 2.1 we first analyze a situation where government pension payments, the risky return, and the insurer's ruin probability do not depend on demographic risk. After this, in Section 2.2.1, we investigate the situation including all dependencies illustrated by Figure 1, but neglecting for the moment the possibility of a dependency between demographic risk and the performance of the risky asset (asset meltdown). Since this kind of dependency has generated some controversy in the scientific literature (see, e.g., Abel, 2001, 2003; Poterba, 2001; Brooks, 2002), we will analyze that case separately in Section 2.2.2. In Section 2.3, the parameters used in our calibration are varied to generalize our results. In Section 2.4, we derive policy implications for potential insurance buyers as well as for the insurance industry and regulators.

2.1 Demographic risk as an independent multiplicative background risk

From the expected utility maximization Objective (7) it becomes apparent that a mean-preserving demographic risk does not influence the individual's decisions at all if

(i) government pension payments do not react to demographic developments (or the individual simply does not participate in such a pension scheme),
(ii) annuity providers—even if subject to default risk—appear to be immunized against demographic shocks with respect to their solvency situation, and
(iii) demographic shocks and the risky asset return are stochastically independent from each other.

If Conditions (i) to (iii) hold, the survival probability in Objective (7) can be simply evaluated separately via the expected value operator. In the case—and that is what we assumed—that $E(p) = p_0$, Objective (7) has the same form and will lead to the same optimal consumption, saving, and asset allocation decisions as it would if demographic risk did not exist at all. The individual thus behaves *risk neutrally* toward this intertemporal risk. In fact, for the individual, the risk of having used the wrong total discount factor $\beta \cdot p$ when determining optimal decisions (and thus, e.g., to have consumed too much if his survival probability turns out to go up) does not matter at all. In other words, the mere existence of demographic risk does not produce any wish for risk protection and thus does not influence the individual's annuity demand. Although at first this result seems surprising, it is, in fact, a standard implication of the individual's assumed preference structure, as analyzed in other contexts by Rothschild and Stiglitz (1976); Gollier (2001, chapter 19); Franke, Schlesinger, and Stapleton (2005, 2005a).[14]

Even if the shock was not mean preserving but only symmetric around some mean deviation (or trend), it would have the effect of a change in discounting tomorrow's consumption, rather than the effect of an (additive) background risk: the presence of a background risk generally reduces risky investment (Kimball, 1993) and increases insurance demand (Eeckhoudt, and Kimball, 1992). As, on the contrary, a change in discounting for additively separable CRRA utility evaluation does not influence asset allocation (see Gollier, 2001, chapter 19), the introduction of demographic risk in the manner considered so far does not influence investment and insurance purchasing decisions.

The individual's risk-neutral perception of demographic risk changes as soon as at least one of the three conditions (i) to (iii) above does not hold. In what follows, we will study the impacts of first dropping Conditions (i) and (ii), and then look at the effect of dropping all three conditions.

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[14] Note that for these results, it is not necessary that the insurer's ruin probability is also stochastically independent from the risky asset return R.

2.2 A first analysis of the model with dependencies

2.2.1 The insurer's ruin probability depends on demographic risk and the risky return; government pension payments depend on demographic risk

We now assume that the insurer's ruin probability depends on demographic risk and the risky return. Furthermore, government pension payments also depend on demographic risk. Here, demographic risk becomes a *sticky* risk. In states of nature where the survival probability increases due to a demographic shock, the government pension system performs poorly and the insurer's ruin probability increases, i.e., there is a higher probability that the annuity will also perform poorly. To complete the distress, exactly those cases are weighed more heavily within the expected utility evaluation (see Objective (7)). On the one hand, this occurs because for those cases the survival probability, i.e., the weight in the expected utility evaluation, is high. On the other hand, due to the concavity of u, the insurer's worsening solvency situation when survival probability increases is perceived more badly than a reduction in the insurer's ruin probability (when the survival probability decreases) is appreciated. For high values of the default pay-off fraction ψ (and low values of α_Y), these effects are somewhat alleviated, since the annuity payout (and government pension payout) reacts less to demographic risk. Thus, both parameters indicate the stickiness of demographic risk.

The annuity demand reaction function with respect to the strength of demographic risk—henceforth abbreviated by *ADRF*—is shown in Figure 2 for a default pay-off fraction of $\psi = 0.6$.

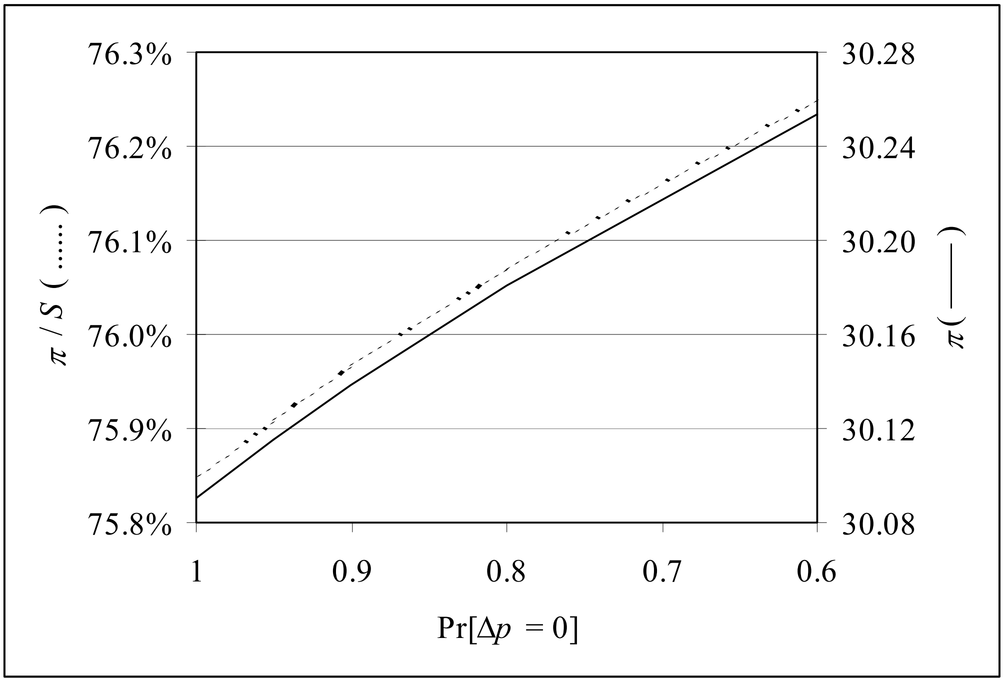

Fig. 2 Annuity demand π and annuity demand in proportion to total savings π / S dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0 (*ADRF*); default pay-off fraction $\psi = 0.6$

Figure 2 illustrates that insurance demand increases in absolute terms and also in proportion to total savings with the strength of demographic risk, i.e., with decreasing probability that the demographic shock is 0.[15] This increase in insurance demand as demographic risk increases is the reaction one would intuitively expect: higher risk is accompanied by higher insurance demand. The actual causal mechanisms leading to such positively sloped *ADRF*, however, are rather complex.

As demonstrated in Section 2.1, the individual has no incentive to protect himself against demographic risk per se, for example, by buying annuities. Furthermore, due to the additional dependency between the insurer's ruin prob-

[15] Total savings decrease slightly with increasing demographic risk. This is the normal (prudent) reaction of a CRRA investor to an increase in background risk (see Deaton, 1991; Elmendorf and Kimball, 2000). The tendency of annuity demand reaction w.r.t demographic risk is in absolute terms and relative terms equal. In the remainder of this paper, we show only relative values, i.e., portfolio shares.

ability, annuity default, and demographic risk, an annuity cannot be used to fully hedge against this risk. Things have to be different. Government pension payments negatively depend on the realization of the demographic shock. Demographic risk thus increases the background risk stemming from the government pension system and therefore increases the individual's wish to create a safer asset allocation (cp. Kimball, 1993; Elmendorf and Kimball, 2000). To do this, the individual shifts savings out of the risky asset, as illustrated in Figure 3. In deciding where the assets should be shifted to, the individual compares the specific payment characteristics of the risk free asset and the annuity. If the annuity is risk free itself (and not loaded too heavily), it will clearly dominate the risk free asset due to the inherent mortality premium on top of the risk free return (see, e.g., Brown, 2001). However, if the annuity becomes risky, the choice now depends on a comparison between the situation-dependent certainty equivalent of the annuity and the risk free investment. If the default pay-off fraction ψ is high, this comparison will favor the annuity leading to a zero demand for the risk free investment.

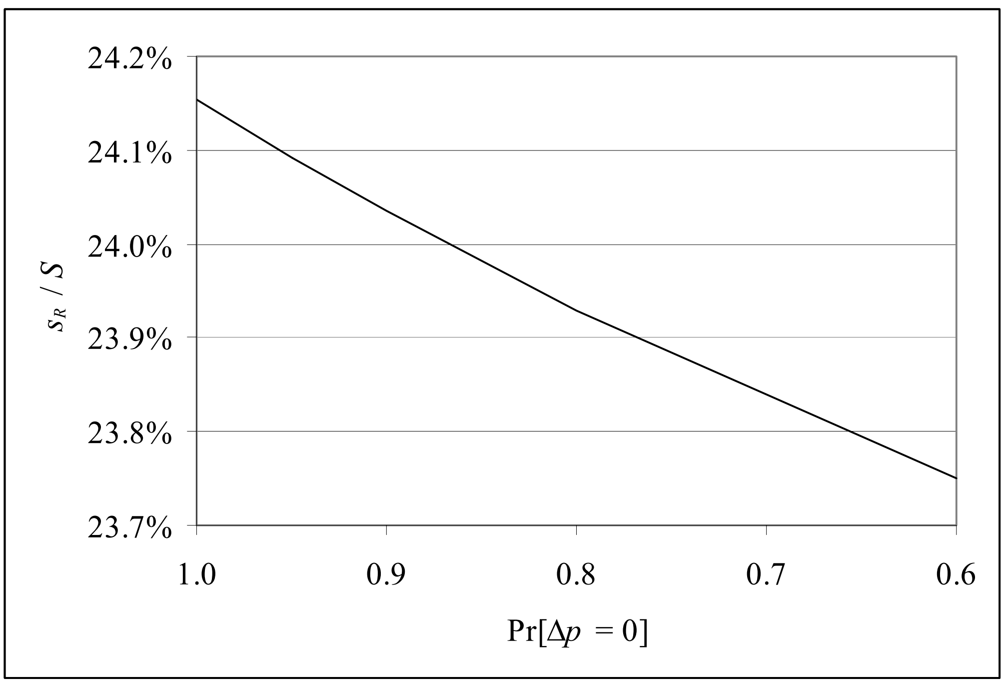

Fig. 3 Risky investment in proportion to total savings s_R / S dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0; default pay-off fraction $\psi = 0.6$

If the default pay-off fraction ψ of the annuity shrinks, the risk free investment becomes comparatively more attractive, and finally, it will be used for saving. This is the case in Figure 4, where ψ is 0; i.e., in the case of the insurer's ruin, the individual receives no payment at all from the annuity.[16]

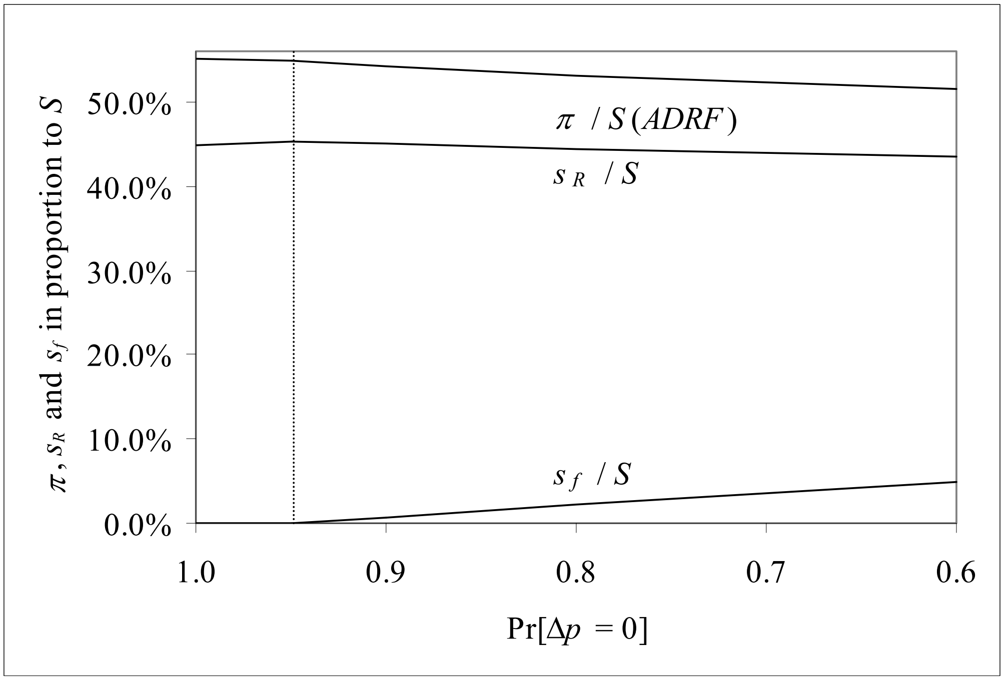

Fig. 4 Annuity, risky, and risk free investment in proportion to total savings S dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0; default pay-off fraction $\psi = 0$

ADRF now *decreases* with increasing demographic risk.[17] Moving from $\Pr[\Delta p = 0] = 1$ to $\Pr[\Delta p = 0] = 0.95$, i.e., introducing demographic risk, the emerging (government pension) background risk induces the individual to reduce annuity demand and to shift savings into the risky asset, which now, for $\psi = 0$, appears to be the less risky investment opportunity (for $\psi = 0.6$, this shift did not take place; see Figure 2). In this demographic probability model,

[16] This extreme case is what Kahneman and Tversky (1979) and Wakker, Thaler, and Tversky (1997) called probabilistic insurance.

[17] This reaction by the individual to demographic risk can be observed for $\psi < 0.2$. Inside the interval $0.2 \leq \psi \leq 0.3$, the slope of *ADRF* reverses by changing its shape from a decreasing function to a parabola, and from there ($0.3 < \psi$) to an increasing function.

the risk free investment is still not attractive enough (or, in other words, the background risk is not severe enough) to induce risk free savings. In fact, the individual would even go into debt, if the borrowing constraint did not keep him from doing so.

For lower values of $\Pr[\Delta p = 0]$, higher demographic risk or, to be more precise, the increased background risk, now creates a situation where any portfolio of annuity and the risky asset alone is too risky for the individual. He begins to shift savings away from annuities and the risky investment into the risk free investment, which means that, due to the high systematicness of demographic risk, *ADRF* now decreases. Note that this decrease is lower in the region where the borrowing constraint is binding, which can be explained as follows. The slope of the *ADRF* in the region where the borrowing constraint is nonbinding can be seen as the individual's "normal" reaction to demographic risk. If there was no borrowing constraint, for $\Pr[\Delta p = 0] \geq 0.95$ the individual would go into debt. However, as this is not possible, the individual seeks another way to get some risk into his portfolio. One way is to become more tolerant toward demographic risk, leading to a less strong reaction to demographic risk in the constrained regions.

Comparing the annuity demand shown in Figures 2 and 4 it is apparent that a reduction in the default pay-off fraction ψ does not only reverse the portfolio decisions for different strengths of demographic risk. Also, given a certain value of $\Pr[\Delta p = 0]$, a decrease in the default pay-off fraction ψ leads to a reduction in annuity demand, which is the normal reaction: a reduction of ψ neither changes the expected value of the return from the annuity nor its actuarial fairness (see Equations (2) and (3)). However, it makes the return more volatile, increases the interval of possible effective payments, and increases the annuity's exposure to demographic risk (thus also increasing the positive dependence on the government pension payment; see Equation (4)). Due to risk aversion and the prudence feature of u, this results in a lower annuity demand (cp. Kimball, 1993; Elmendorf and Kimball, 2000).

2.2.2 The insurer's ruin probability depends on demographic risk and the risky return; government pension payments depend on demographic risk; the risky return depends on demographic risk (asset meltdown)

We now introduce the asset meltdown hypothesis into the model. So far, the two systematic risks (demographic and asset return risk) have been assumed to be stochastically independent from each other. The asset meltdown hypothesis now links demographic risk and the return of the risky asset directly, in a way that a combination of both risks becomes more undesirable to the individual (cp. Table 2). Furthermore, if the individual decides to hold the risky asset, government pension payments also become riskier because the dependency between demographic risk and the return of the risky asset implicitly links government pension payments to the performance of the risky asset (see Figure 1). Finally, an asset meltdown increases the stickiness of demographic risk, since the states of nature that are weighed more heavily in the expected utility evaluation (increases in the survival probability) are accompanied by rather poor returns from the risky asset.

Consequently, the tendencies shown in Figures 2 and 4 in Section 2.2.1 become more strongly pronounced when the asset meltdown hypothesis is introduced. Figure 5 shows *ADRF* for the case of a default pay-off fraction $\psi = 0.6$.

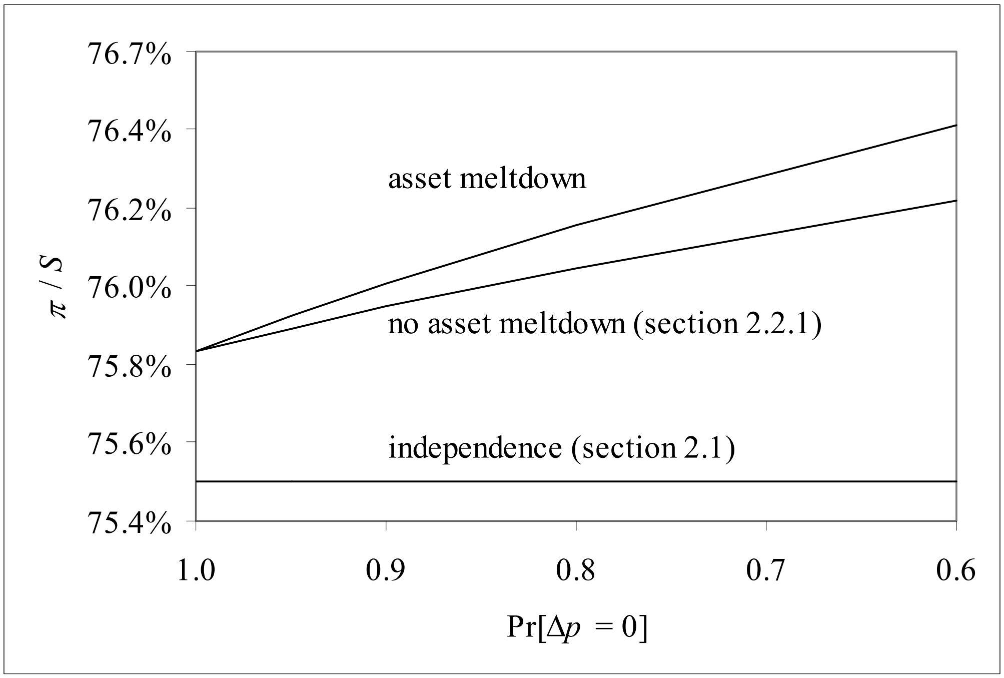

Fig. 5 Annuity demand in proportion to total savings S dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0 (*ADRF*); default pay-off fraction $\psi = 0.6$

The additional dependence of both the risky asset and the government pension payments on the performance of the risky asset return leads—when ψ is high—to a higher demand for the comparatively safe risky asset: the annuity (cp. Viceira, 2001; Cocco, Gomes, and Maenhout, 2005). This occurs because the just introduced dependence in the first instance disturbs the pay-off characteristics of the risky asset. As the annuity was also previously (in Section 2.2.1) linked to both demographic and risky return risk, the new dependence affects the (for ψ high, in any case rather safe) annuity only slightly.

If the annuity becomes increasingly risky as ψ decreases, *ADRF* goes down (shown in Figure 6) and, as before (see Figure 4), the individual shifts some of his wealth into the risk free asset (not shown here). As above, the effects, having led to a decreasing *ADRF* as shown in Figure 4, are intensified by introducing the asset meltdown.

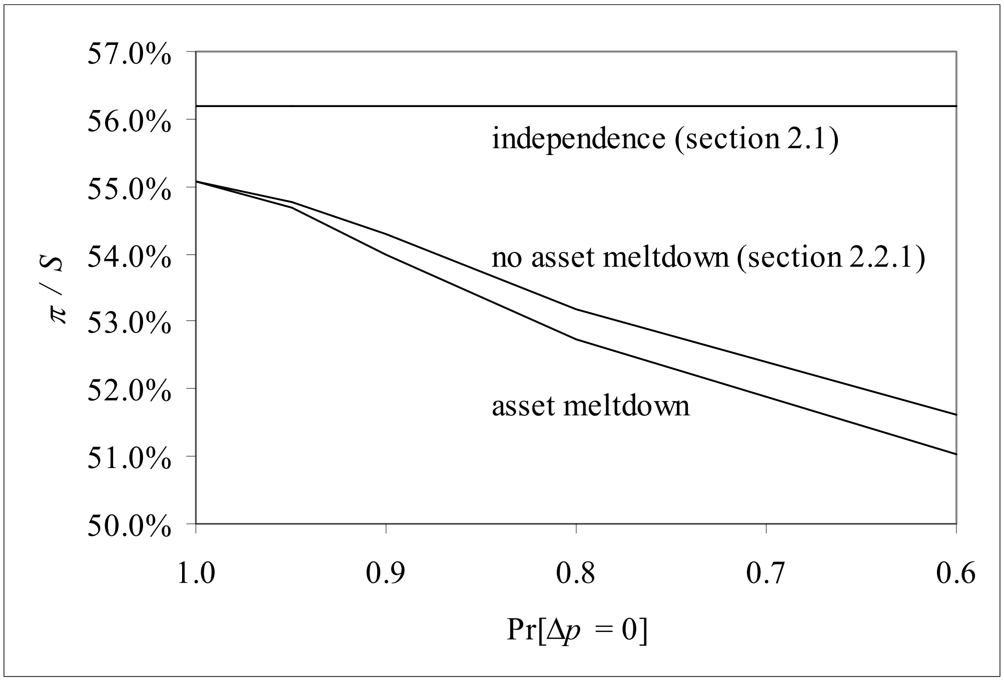

Fig. 6 Annuity demand in proportion to total savings S dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0 (*ADRF*); default pay-off fraction $\psi = 0$

Figures 5 and 6 also contain the independence case, discussed in Section 2.1, where annuity demand did not react at all to demographic risk. The fact that—compared to this case—annuity demand increases (decreases) for higher (lower) values of the default pay-off fraction ψ underlines the arguments made above. For higher values of ψ (Figure 5), the individual reacts to the increase in risk stemming from the introduction of dependencies by shifting savings from the risky asset to the comparatively less risky annuity. For lower values of ψ, again, the reaction is the opposite—*ADRF* becomes negatively sloped (see Figure 6).

2.3 A deeper analysis of the impacts of model parameters on *ADRF*

The numerical examples set out in Section 2.2 revealed that there are several influences that must be considered regarding an individual's annuity demand reaction w.r.t. demographic risk (*ADRF*). In particular, we observed that

ADRF may have a negative or positive slope, i.e., in some situations the individual reacts to increasing demographic risk by increasing, in other cases by decreasing, the portfolio share of annuities. The sign of the slope of *ADRF* appears to be controlled by the default fraction pay-off parameter ψ. In what follows we provide a deeper analysis of the mechanisms driving our model.

When we vary the parameters γ, β, λ, p_0, α_Y, the strength of the demographic shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ, we observe fundamental differences in parameter influence: some parameters (γ and β) control only the *intensity* of the individual's reaction to demographic risk, i.e., the magnitude of the slope of *ADRF*, whereas other parameters (ψ, λ, p_0, α_Y, the strength of the demographic shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ) also control the *direction* of the individual's reaction, i.e., the sign of the slope of *ADRF*. Moreover, emphasizing our results of Section 2.2, it turns out that whether or not the borrowing constraint, $s_f \geq 0$, is binding is also a factor that influences the magnitude of the slope (but not the sign) of *ADRF*.

In the following, we discuss these influences in detail. We start in Section 2.3.1 with parameters that do not influence the sign of the slope of *ADRF*, next, in Section 2.3.2, we examine other parameters that are able to change the sign of the slope of *ADRF*. Section 2.3.3 contains general conclusions from the preceding sections.

2.3.1 Parameters influencing only the magnitude of the slope of *ADRF*

An increase in the relative risk aversion parameter γ generally leads to a safer asset allocation (see Gollier, 2001, chapter 4). From the discussion of Figures 2 and 4 in Section 2.2.1, we know that the most important factors influencing how the individual will achieve this increased safety are, first, whether the annuity is rather safe (ψ high) or not (ψ low) and, if ψ is low, second, whether the borrowing constraint is binding or not.

If ψ is rather high (increasing *ADRF*) and thus the annuity appears to be more attractive than the risk free asset, increased desire for a safe asset allocation will lead to an even stronger shift out of the risky asset into the (safer) annuity than for a lower γ. The increasing *ADRF* becomes steeper. In the case where ψ is rather low (decreasing *ADRF*) and the borrowing constraint is binding, our discussion of Figure 4 showed that the individual develops some risk tolerance toward demographic risk. As risk aversion goes up, however, the individual's level of additional risk tolerance becomes much more sensitive to an increase in demographic risk. Thus, the individual will react more strongly to increasing demographic risk: the falling *ADRF* becomes steeper.

Only when the borrowing constraint is nonbinding, and the individual is permitted to act freely in the face of increasing demographic risk, we find a decrease in reaction intensity. For a higher γ, the individual invests a greater amount in the risk free asset and chooses a lower exposure to demographic risk (via purchasing annuities) than if γ is low. Due to his lower exposure, the individual's adjustments to the annuity share of the portfolio will be less pronounced than for lower γ. Thus *ADRF* becomes flatter.

The influence of the subjective discount factor β on the optimal decisions depends on whether the individual receives government pension income. Generally, an increase in the subjective discount factor β leads to higher total savings since the individual puts a larger weight on future consumption. If government pension income is 0 ($\vartheta = 0$), the individual's asset allocation is independent of β for CRRA utility.[18] If the individual does receive pension income ($\vartheta > 0$), the pension income works to some extent as a substitute for risk free savings (see, e.g., Cocco, Gomes, and Maenhout, 2005). Higher total savings due to an increase in β naturally lead to an increase in the relation of total savings to pension income. This relative reduction of a risk free position induces the individual to choose a safer asset allocation for his savings or, in other words, the individual acts more risk averse. Thus, the effects of an increase in β on the slope of *ADRF* are the same as in the case of an increase in γ. Table 4 summarizes the results of this subsection.

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[18] Here, asset allocation is shown to be myopic (see, e.g., Gollier, 2001, chapter 19).

Table 4 Influence of variation of model input parameters on *ADRF*

	magnitude of absolute slope of *ADRF* when borrowing constraint	
influence of increase in	does not bind	binds
relative risk aversion γ	↓	↑
subjective discount factor β (for $\vartheta > 0$)	↓	↑

2.3.2 Parameters influencing the sign of the slope of *ADRF*

Whereas γ and β control *behavior* in a given risk situation, the parameters ψ, λ, p_0, α_Y, the strength of the demographic shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ control the risk situation *itself*, thus leading to changes in the way the individual reacts to demographic risk, i.e., the sign of the slope of *ADRF*. At this, it turns out that only for variations in ϑ, the strength of the mean return shift in case of the asset meltdown, and ψ it is of importance, whether the borrowing constraint is binding or not.

A decrease in the loading factor λ, i.e., an increase in the premium loading, leads to a proportional drop in annuity payments for a given annuity premium, which makes the annuity—compared to alternative investment opportunities—less attractive in all states of nature. Annuity demand shrinks (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). An increase in the volatility of effective annuity payouts with increasing demographic risk is now penalized more heavily. If the slope of *ADRF* already has a negative sign (compare Figure 2), a decrease in λ makes *ADRF* even steeper. The slope of a positive *ADRF* (compare Figure 4) becomes flatter; with increasing demographic risk, the individual more reluctantly shifts savings to the now less attractive annuity. If the annuity actuarially becomes too unfair, the slope of *ADRF* will even reverse and become negative. Thus a decrease in λ turns *ADRF* clockwise.

Varying the mean survival probability p_0 creates two effects: on the one hand, from Objective (7) it follows that an increase in p_0 is equivalent to an increase in the mean "overall" discount rate $\beta \cdot p_0$, which leads to the same tendencies as in the case of an increase in β. On the other hand, from Equation (3) it follows that an increase in p_0 leads to a drop in the mortality credit of the annuity. Thus the expected return from the annuity shrinks in a way similar to the case of a decrease in λ. Again, in all states of nature the annuity becomes less attractive compared to alternative investment opportunities, and annuity demand decreases (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). In our calibration, the second effect dominates the first, and thus an increase of p_0 leads to the same (although more moderate) tendencies as a decrease in λ.

Note that, as discussed in Section 2.1, if the demographic shock instead of being mean-preserving is only symmetric around some mean shift in survival probability, this can be decomposed into a deterministic increase in p_0 and a mean preserving shock. An additional assumption of a trend in changes in survival probability would thus have the same effect as an increase in the mean survival probability p_0.

An increase in the reaction parameter of government pensions w.r.t. demographic shocks, α_Y, makes government pension payments more volatile, i.e., background risk and the stickiness of demographic risk increase. Since effective annuity payouts are positively correlated with government pension payments, the annuity becomes less attractive relative to other (uncorrelated) assets, similar to what happens in the case of a decrease in λ. Consequently, the tendency of change in the slope of annuity demand curves is the same as in the case of a decrease of λ.

An increase in the strength of the demographic shock (in the base case it was set to $\pm$ 0.01) also leads to an increase in background risk and makes the stickiness of demographic risk more pronounced, since bad states of nature are weighed even more heavily in the expected utility evaluation. Thus an increase in the strength of the demographic shock leads to the same tendencies as an increase in α_Y, although they will be more pronounced.

Increases in the strength of the mean return shift in the case of an asset meltdown make the link between the risky asset and demographic risk even more risky to the individual. Here, the effect on the *ADRF* depends on whether the borrowing constraint is binding. If the borrowing constraint binds, i.e., if the individual perceives the annuity as a rather favorable, safe investment, the individual reacts to stronger demographic risk by reducing the risky position in favor of the annuity; *ADRF* turns counterclockwise (compare Figure 5). If the borrowing constraint does not bind, the individual increases his risk free position in disfavor of the annuity. The stronger the demographic risk and mean return shift, the more likely he will be to transfer assets in this manner; *ADRF* turns clockwise (compare Figure 6).[19]

As discussed in Section 2.2, also the default pay-off fraction ψ is important for the sign of the slope of *ADRF*. This parameter's influence on *ADRF* depends on whether or not the borrowing constraint binds. For low ψ, the annuity is a rather risky asset. The risk free asset is attractive enough so that the borrowing constraint does not bind and positive risk free savings are induced. *ADRF* was shown to have a negative slope. Increasing ψ here leads to a steeper *ADRF*; it turns clockwise. Although the risk return tradeoff of the annuity improves, the individual is less willing to hold this asset as demographic risk increases because where ψ is low, demographic risk—when it occurs—has a stronger impact on the individual. When increasing ψ at a low level, the risk exposure of the annuity remains high. Nevertheless, especially for $\Pr[\Delta p = 0] = 1$, the individual increases the amount in the annuity with increasing ψ due to the better risk return tradeoff. However, since the individual is now more invested in the annuity in the first place,[20] he reacts more sensitively to the introduction of demographic risk. If demographic risk is introduced, i.e., $\Pr[\Delta p = 0] < 1$, the higher but nevertheless still low ψ is not enough to overcome the individual's fear of demographic risk. This changes as soon as ψ takes values high enough

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[19] The *ADRF* turning tendencies observed here do also hold for "asset meltups", i.e., for the case of a positive dependency between Δp and R. Such dependency may result from changes in saving behavior due to demographic shocks. For example, if survival probability rises, younger investors will save more to cope with their now increased expected life-span, whereas older investors will decumulate their assets more slowly. Both effects may lead to increases in asset prices, see, e.g. Poterba (2001).

[20] An increase in annuity demand due to an increasing default pay-off fraction is consistent with the results of Babbel and Merrill (2006).

to dominate the risk free asset, i.e., to make the borrowing restriction bind. With increasing ψ, *ADRF* turns counterclockwise until, eventually, the sign of its slope becomes positive (see Figure 2).

An increase in the government pension income factor ϑ (see Equation (9)) means, on the one hand, an increase in the individual's exposure to demographic risk. On the other hand, it also means an increase in the individual's risk free income position. As a consequence, the effects of variations in the parameter ϑ on *ADRF* are less clear-cut. Here, the value of the default-payoff fraction parameter ψ, i.e., the systematic exposure of the annuity to demographic risk, and the binding or nonbinding nature of the borrowing constraint, are of great importance. For high ψ, we observed the following general tendency: an increase in government pension factor ϑ and the thus induced increase in the individual's exposure to demographic risk turns the positively sloped *ADRF* clockwise until it finally becomes negatively sloped. For low ψ, *ADRF* stays negatively sloped, but the complex interplay of the factors mentioned above leads to different turning directions depending on the specific value of ϑ. A thorough analysis of this interplay is provided in the Appendix.

The influence on *ADRF* of variations in model input parameters as discussed in this section is summarized in Table 4.[21]

[21] We tested the validity of our results by using several thousand numerical examples.

Table 4 Influence on *ADRF* of variation in model input parameters

influence of increase in	turning direction of *ADRF* when borrowing constraint: does not bind	turning direction of *ADRF* when borrowing constraint: binds
loading factor λ	↺	↺
mean survival probability p_0	↻	↻
reaction of government pensions w.r.t. demographic shocks α_Y	↻	↻
strength of demographic shock	↻	↻
strength of mean return shift in the case of an asset meltdown	↻	↺
default pay-off fraction ψ	↻	↺
government pension income factor ϑ given a low ψ	↻ ⇨ ↺	↺ ⇨ ↻
government pension income factor ϑ given a high ψ	↻	↻

2.3.3 Conclusions from section results

The analyses provided in Sections 2.3.1 and 2.3.2 showed that model parameters can be classified into two groups: those parameters influencing the individual's risk perception (γ and β) and those that influence the risk situation itself (ψ, λ, p_0, α_Y, the strength of the demographic shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ). While the parameters from the first group, which are all *subjective* (preference) parameters, determine only the intensity of the individual's reaction to demographic risk, the parameters from the second group, which are all *objective* parameters, also influence the direction of the annuity demand reaction to demographic risk. In the numerical examples presented in Section 2.2, we demonstrated this by varying ψ in such a way that in one case ($\psi = 0.6$), the annuity was perceived as being rather safe, while in the other case ($\psi = 0$), the annuity was perceived as rather risky due to its exposure to the systematic risks. As we have seen, the sign of the slopes of the individual's respective *ADRF* changed. From the analyses in Section 2.3.2, we now know that this result also would occur if we vary one (or more) of the other parameters of the second group (i.e., parameters influencing the risk situation) such that the individual's exposure to systematic risks in general, and to demographic risk in particular, is increased. Any constellation of objective parameters thus create a risk situation determining the sign of the slope of the individual's *ADRF*. The intensity of reaction within this risk setting is then only fine-tuned by the individual's subjective risk perception parameters. That means that even if ψ is rather high, a high value of the government pension reaction parameter α_Y can lead to a negatively sloped *ADRF*. Such a turn in *ADRF* cannot be achieved by increases in γ or β within their domains.

2.4 Policy implications

A first and important implication of our results is that demographic risk, per se, does not increase the demand for annuities. It rather depends on the riskiness of the whole situation (or more exactly: the stochastic dependencies with demographic shocks) in which potential insurance buyers decide how much of their wealth they are willing to annuitize. Parameters influencing this risk situation can be:

- individual specific (e.g., claims on the government pension system, or the expected survival probability);
- valid for all potential insurance buyers (e.g., the extent to which systematic insurance defaults can be absorbed, or in how far asset returns and/or the stability of the government pension system is exposed to demographic shocks); and
- controllable by the insurer (e.g., exposure to systematic risks or choice of premium loadings).

All the objective parameters together determine whether a potential insurance buyer will react with an increased or decreased demand for annuities when new information on the characteristics of demographic risk becomes available, e.g., by the publication of a new study in this field. Regarding this, it is not only the expected trend of future development in mortality, but also the *precision* of the survival probability forecasts (with respect to both the extent and the probability of possible deviations) that are of importance.

People whose (future) government pension income is very high compared to present wealth already have a high exposure to demographic risk. For these people, annuities become less attractive with increasing demographic risk, even if the guarantee fund is working quite well and the insurer itself provides a robust solvency situation. For other potential insurance buyers who have a rather low or no government pension income, also at lower degrees of safety of the whole risk situation, higher uncertainty about future mortality can even result in an increase in annuity demand.

Since we have seen that implementing the consideration of demographic risk into individual decision making may result in both decreases and increases in annuity demand, our results may both alleviate, but also intensify, the annuity puzzle. In any case, potential insurance buyers should check their demographic risk exposure when making an annuity purchasing decision. For such a decision, the direction of their demand reaction does not depend on their subjective risk perception characteristics.

When considering the specific market situation (e.g., the characteristics of potential insurance buyers) and additionally the costs that will accompany the actions taken, our model can help the insurance industry answer the following questions:

- How can strategic decisions with respect to premium policy, or measures that aim at decreasing idiosyncratic (via asset liability management) or systematic risk exposure (via financing guarantee funds), stimulate annuity demand in dependence of the severity of demographic risk?
- How will increased research activity with respect to the actual changes in mortality rates and public awareness of demographic risk lead to an increase or a decrease in annuity demand?
- What will be the consequences for the annuity business if the asset meltdown hypothesis turns out to be true?
- Can combinations of annuities with mortality derivatives aimed at reducing the annuities' vulnerability to demographic risk stimulate annuity demand?

For the field of insurance regulation, our model illustrates that the ruin probability alone is not sufficient to describe and control an insurer's solvency situation (see also Butsic, 1994; Wang, 1998). Additional measures, such as expected policyholder deficit or the tail value at risk, should be considered.[22]

3 Summary and Directions of Further Research

In this contribution, we studied the impact of uncertainty about future mortality developments (demographic risk) on individuals' annuity demand. Within an intertemporal expected utility maximization framework, we showed that demographic risk does not influence annuity purchasing decisions if it is stochastically independent from all other sources of income risk. However, for most potential insurance buyers, this condition does not hold. Rather, demographic risk appears to be a very *sticky* risk, i.e., it is hardly transferable. We showed that whether the existence of demographic risk will lead to an increase or a decrease in individuals' annuity demand is determined by objective factors (such as the exposure of the government pension system and/or the insurance industry with respect to demographic shocks) and not by subjective factors (such as individual risk aversion). Subjective factors determine only the intensity of the annuity demand reaction to demographic risk. Our results show that consideration of demographic risk may both alleviate, but also intensify, the annuity puzzle.

As a model extension, bequest motives together with the additional possibility to buy term life insurance contracts could be incorporated into the model leading to a certain hedging opportunity for the individual. Due to the systematicness of demographic risk, however, this hedging opportunity at the same time may increase the individual's (and his heirs') exposure to demographic risk as, depending on the vulnerability of the insurer to demographic risk, annuities and term life contracts may default at the same time, once more emphasizing the stickiness of demographic risk.

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[22] The Swiss solvency regulation already uses such measures, see, e.g., Keller, Luder, and Stober (2005).

In a further step, the individual annuity demand model may be combined with a corporate model such as of Gründl, Post, and Schulze (2006). Within the arising game situation, the shareholder value maximizing insurer then would choose its optimal exposure to demographic and capital market risk. The resulting solvency situation of the insurer is then incorporated into the decision calculus of expected utility maximizing individuals deciding on their demand for annuities and/or term-insurances which again is an input parameter for the calculus of the insurer. The resulting equilibrium solution could provide valuable input for policymakers concerned with insurance regulation and the consequences of the privatization of retirement systems.

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Appendix: Variations in the government pension income factor ϑ and their impact on *ADRF*

For variations in the government pension income factor ϑ, the influence on *ADRF* is less clear-cut. Figure 7 contains the individual's annuity demand reaction to the strength of demographic risk for given values of ϑ. It plots the difference of the annuity share in the individual's savings between the case where the probability of a zero demographic shock $\Pr[\Delta p = 0]$ is 0.6 and where it is 1 against ϑ.[23] Negative values in Figure 7 thus indicate a negative sloped *ADRF*, and vice versa.

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[23] Thus we look on the left and right boundary of the abscissa of Figures 2 and 4. Other pairs of $\Pr[\Delta p = 0]$ delivered the same tendencies.

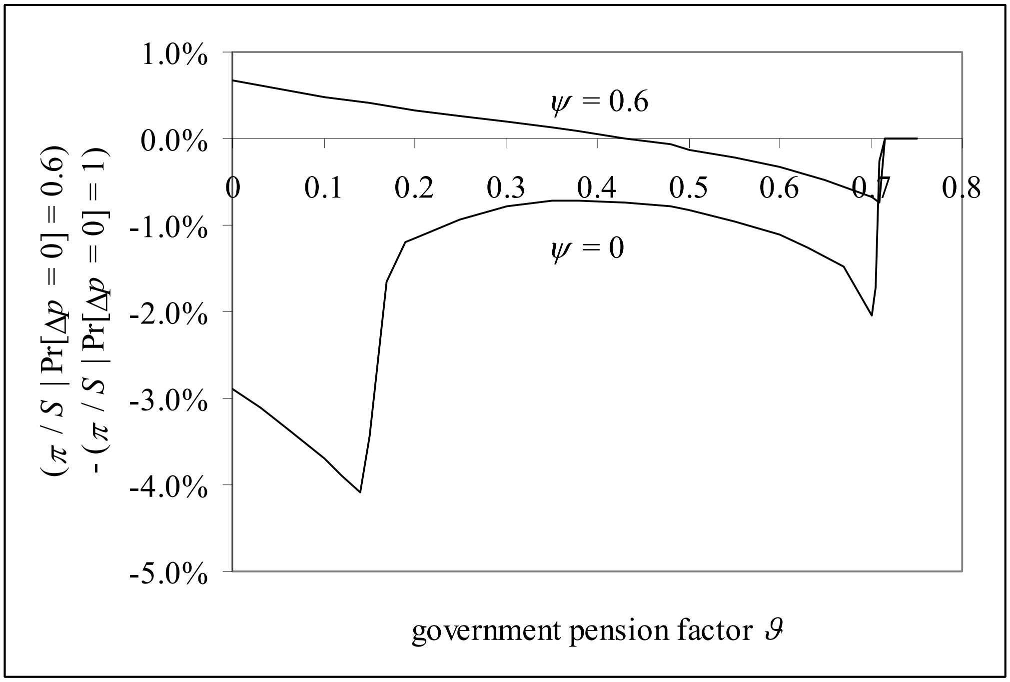

Fig. 7 Demographic risk induced changes in the annuity demand proportion of total savings *S* depending on the government pension income factor ϑ

Here, the value of the default pay-off fraction parameter ψ is of great importance. Looking at the base calibration government pension income factor $\vartheta = 0.15$ and the curve where $\psi = 0.6$, we see that the sign of the slope of *ADRF* is positive (the curve in Figure 7 takes positive values), which refers to the case shown in Figure 2. When moving to the right, i.e., increasing government pension income, the sign of the slope of *ADRF* reverses at about $\vartheta = 0.43$ (the curve for $\psi = 0.6$ in Figure 7 takes negative values). That means, for higher ϑ (opposite to Figure 2), the higher the demographic risk, the less the individual invests in the annuity. For $\vartheta > 0.71$, the curve is equal to 0. Here, the individual in both probability models does not buy any annuities (therefore, the difference in the portfolio shares is also 0). The large pension income crowds out private annuity demand (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). The reason for the overall negative slope of the curve $\psi = 0.6$ in Figure 7 is the individual's increasing exposure to demographic risk as ϑ increases. The positive stochastic dependence of the annuity and government pension income makes annuities less attractive the higher the individual's government pension income is. At $\vartheta = 0.43$, this effect becomes so strong that the total risk situation—which so far was perceived by the individual so safe that he reacted to demographic risk by an increase in annuity demand—now turns into a rather risky situation where demographic risk reduces annuity demand. *ADRF* turns clockwise.

For low values of the default pay-off fraction ψ, e.g., for $\psi = 0$, in Figure 7, the sign of the slope of *ADRF* stays negative but the impact of an increase in government pension income is more complex. To understand the effects, in Figure 8, we additionally plotted the individual's risk free investment.

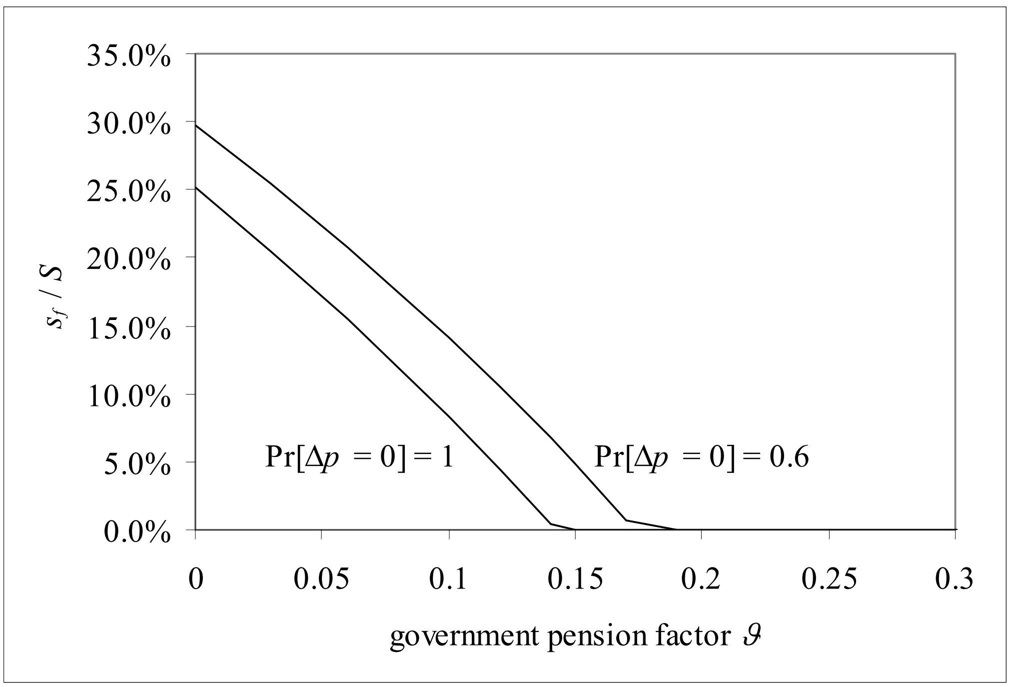

Fig. 8 Risk free investment in proportion to total savings S dependent on the government pension factor ϑ; default pay-off fraction $\psi = 0$

For very low values of ϑ (between 0 and about 0.15), increasing demographic risk leads to a sharp reduction in the annuity portfolio share, a reduction that becomes even more pronounced when ϑ increases (see Figure 7). The reason for this is the same as that given above: the annuity payment for $\psi = 0$ reacts very strongly to demographic risk and thus is strongly positively correlated with government pension income (this explains the highly negative value of the curve for very low values of ϑ). As ϑ increases (in between the interval

[0, 0.15]), the individual, on the one hand, reacts by reducing annuity demand more strongly with increasing demographic risk (which is due to the described higher exposure to demographic risk). This is the same effect we observed for increases in γ and β. On the other hand, the individual reduces the risk free investment (due to the higher absolute risk free income part in the government pension income payment). At one point, however, due to the borrowing constraint, no further reduction of the risk free investment is possible (see ϑ around 0.15 to 0.20 in Figures 7 and 8). This means that although the individual becomes less risk averse—due to the higher risk free part of the pension income—he cannot react by simply reducing his risk free investment. Instead, he wants to increase the riskiness of the remaining portfolio. In doing so, he becomes less sensitive to the positive correlation between annuity and pension income, leading to a weaker decrease in his demand curve with respect to demographic risk (the curve in Figure 7 becomes less negative; cp. also Section 2.2.1). This is what we previously observed for decreasing γ and β. This effect, however, is again caught up by the effect of the increasing exposure to demographic risk as ϑ increases. This again leads to a reversion of the curve in Figure 7, meaning that the individual again becomes more sensitive to the increasing positive correlation between annuity and pension income as demographic risk rises. For $\vartheta > 0.705$, the individual reduces annuity demand to 0 for both demographic models.