

Optimal Sharing of Labor Productivity Risks and Mix of Pay-As-You-Go and Savings

The quadratic case

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Abstract

This paper investigates two related issues. One is the optimal design of Pay-As-You-Go (PAYG) social security for the intergenerational sharing of labor productivity risks. It is shown that partial contingency of the benefits on the stochastic labor productivity is ex ante optimal, from a lifetime perspective of the risk exposure, when intergenerational transfers are costly. The other issue is the ex ante optimal combination of PAYG and savings in the small open economy. We obtained that the government may induce a higher saving rate in the "Stackelberg" equilibrium, due to the lower expected PAYG return and given the household's desire of smoothing consumption over time. An interpretation of this result is that a funded pension pillar is ex ante optimal next to the PAYG scheme.

JEL classification: H55, H21, D91.

Keywords: intergenerational risk sharing, PAYG social security, household savings.

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Possible errors are mine, further comments and suggestions are welcome.

Introduction

This paper addresses two related issues: the optimal intergenerational sharing of labor productivity risks, through a Pay-As-You-Go (PAYG)¹ social security with wage indexation²; and the optimal combination of PAYG scheme and funded savings for the retirement provision.

The recent U.S. debate on social security illustrates the relevance of these topics. One official reform proposal was the indexation of social security benefits to prices instead of wages; and partial funding, i.e. a decrease in PAYG financing by diverting part of the payroll contribution to individual accounts and investing these funds in stocks and bonds.³

The main argument in favor of abolishing the wage indexation is the increasing costs for workers of sharing labor productivity gains when the ratio of retirees to workers is increasing. On the other side, one can argue that the indexation of social security benefits to wages keeps the relative living standards of the elderly in exchange for sharing risks with the young. According to Shiller (1999), risk sharing is the central aim of old-age social security schemes.⁴ We argue further that the goal of PAYG schemes should be no other than the provision of intergenerational risk sharing, because intergenerational transfers are costly (see Box "The expected PAYG premium"), and more suitable instruments exist or may be devised for sharing risks at the intragenerational level; or for pursuing other goals than risk sharing, like poverty alleviation at the old age.

The paper relates to the above arguments, by measuring the long-run costs and benefits of providing intergenerational risk sharing, in the context of a small open economy.

In the design of the optimal scheme, we assume PAYG financing and full indexation of benefits to the "efficiency" wages, following Thøgersen (1998), who showed that both features are necessary for the optimal pooling of labor productivity risks, based on the analysis in Gordon and Varian (1988). The methodological novelty is that not only the *base* of the PAYG contribution is contingent on the stochastic labor productivity, but also the *rate* of the PAYG contribution itself.

Although the net effect of the optimal social security is the shift of labor productivity risks from the young to the elderly, we found that the "ex ante" optimal rates of PAYG contribution vary negatively with the stochastic growth rate of the economy, which implies partial contingency of the PAYG transfers. Partial social protection to the elderly is not only "ex ante" optimal, but also a necessary condition for positive PAYG transfers

The model also determines analytically the "ex ante" optimal retirement provision. The numerical analysis shows the "ex ante" optimal rates of saving and PAYG contribution in the long-run equilibrium, in alternative scenarios of the exogenous parameters (interest rate, labor force, average productivity growth, etc.).

There is a consensus that PAYG schemes reduce aggregate household savings, but the empirical evidence is not conclusive about the size of this effect. Moreover, the comparison of

¹PAYG denotes schemes financed by intergenerational transfers from workers to retirees.

²The paper abstracts from monetary issues by modeling a real economy.

³See Munnell (2004) for a summary of the main proposal presented in 2001, by "President Bush's Commission to Strengthen Social Security", and for other reform proposals that were in discussion.

⁴The current U.S. debate on social security dates from the middle of the 1990's, and it gained new impulse during the two Bush administrations, who announced the social security reform as one of the national priorities. We do not present here all competing views and nor the controversy about funding.

the empirical literature on the effects of pension schemes on household savings is challenging because of the use of different saving definitions, the differences in institutional setups, and other methodological issues. Nevertheless, it seems that there is robust evidence of a partial offset, based on the limited substitutability between the schemes and liquidity constraints.⁵

For our purposes of analysis, we use the term "funded" savings in opposite to the "PAYG" retirement provision. In the model, funded savings are at the discretion of households, but they may also be induced by the government, because it anticipates household decisions when setting the social security policy (Stackelberg equilibrium).

According to the best response of a representative household, funded savings may decrease due to the social security contribution and the anticipation of benefits. However, the degree of substitutability between the schemes is fundamental for determining the savings in equilibrium. For instance, precautionary savings may be induced by the contingent PAYG scheme. However, in this paper, the government assumes an optimal savings response according to quadratic preferences, which do not generate precautionary savings.

In the "ex ante" optimal equilibrium with quadratic preferences, we found that the government may increase the saving rate in equilibrium, in comparison to the laissez-faire economy, based on the expected positive PAYG premium; i.e. given that the expected growth rate in the economy (the indexed PAYG return) is lower than the open-economy interest rate (the return on funded savings). Since the model abstracts from a second pension pillar, one interpretation for the higher saving rate in equilibrium is that a funded pension pillar, next to the PAYG social security, is "ex ante" optimal because of the difference in return rates and the desire of smoothing consumption over time.⁶

Review of the literature

The paper extends two previous contributions on the optimal intergenerational sharing of labor productivity risks: Enders and Lapan (1982) and Bohn (2003). The three models share a common set of features: the life cycle is represented in two periods and two generations always overlap (2x2 OLG model); households are not altruistic; in the laissez-faire economy the only source of uncertainty is the stochastic labor productivity; the stochastic disturbances are independent and identically distributed with mean zero.

Enders and Lapan (1982) assumed a closed economy *without* capital, with labor, and money as the only storage-value device. Workers sell part of their production to the elderly in exchange for money, and prices adjust to the supply of goods and to the existent monetary balances. Prices vary if the supply of goods (the savings of the young) is not constant in consequence of productivity shocks. In this economy, the "ex ante" optimal intergenerational risk sharing is attained when the saving rate is constant, because then prices and consumption are constant on average (*i.i.d.*, zero-mean shocks). The laissez-faire economy is "ex ante" optimal when preferences are logarithmic. Otherwise, a compulsory PAYG scheme with a

⁵We refer to an offset, measured in marginal terms, as the decrease in household savings due to a \$1 increase in pension contributions. See Kohl and O'Brien (1998) for a survey of the applied econometric literature, and Bernheim (2002).

⁶Due to the lower PAYG return, the expected return on the *total* retirement provision (funded savings and PAYG contribution) is lower than in the laissez-faire economy. Thus, consumption at the old age becomes relatively more expensive in the presence of the PAYG, and the rate of funded savings may actually increase.

constant rate of contribution is "ex ante" optimal and, in the PAYG economy, household savings are zero. Enders and Lapan (1982) showed, in the *stochastic* economy without capital, that *PAYG social security dominates money as a storage-value device*⁷, because in the PAYG economy the government prevents inflation.⁸

Bohn (2003) assumed a closed economy *with* capital and labor. The young and the elderly generations are *simultaneously* exposed to labor productivity shocks as in Enders and Lapan (1982), but now because the interest rate is correlated to wages (closed economy and Cobb-Douglas production). There is *asymmetry in risk exposure* due to less-than-complete capital depreciation. The non-depreciated capital is not a function of the current productivity shock, and it represents a safe asset to be consumed by the elderly. The central planner solution for the "ex ante" optimal risk sharing involves the correction of the asymmetry in risk exposure between the overlapping generations, through the equalization of the elasticities of consumption with respect to a simultaneous shock. The "ex ante" optimal risk sharing may be obtained in a decentralized economy, with government, by transferring risk from the young to the elderly, through intergenerational transfers that are fully indexed to the labor productivity.

Bohn (2003) wondered why actual fiscal policy instruments (e.g. government bonds and the PAYG social security) are not, or not completely, indexed to the stochastic labor productivity when the young are more risk exposed than the elderly. He investigated alternative hypotheses (endogenous labor supply and stochastic capital depreciation), but they could not provide an explanation. He concluded that, unless assuming that the elderly are more risk averse (by including habits in consumption), his model cannot provide a rationale for partial or no contingency of these fiscal instruments as they shift risk in the wrong direction, from the elderly to the young.⁹

In contrast, in this paper, partial contingency of the social security benefits may be "ex ante" optimal without assuming that the elderly are more risk averse. The government takes a lifetime perspective on risk exposure, and it takes the PAYG premium into account.

Bohn (2003) showed, in the *stochastic* economy with capital, that *PAYG social security and government debt are not equivalent in the allocation of productivity risks* (Borgmann 2005, p. 52; see also Kaul 2001 on a previous version, Bohn 1998).¹⁰

Before closing this section, we review some contributions in the literature on efficiency in

⁷The contribution of the young to the social security is not stored or invested, but it is transferred to the elderly (PAYG). However, the mandatory character of the scheme assures the young that the next generation will comply as well, and that bails the promise of the future benefit.

⁸Samuelson (1958) showed in a *deterministic* pure-exchange economy that *money is equivalent to PAYG social security*, when traded at positive prices, as a device for attaining efficiency in intertemporal allocation in an OLG economy.

⁹Bohn (2003) addresses the U.S. social security debate. According to his model, abolishing the wage indexation and investing part of the social security contributions in government bonds would worsen the intergenerational allocation of labor productivity risks, because the young would pay a constant return, out of a stochastic income, on the implicit (social security) and the explicit (government bonds) public debt.

¹⁰Diamond (1965), in the extension of Samuelson (1958) to capital and labor, showed the possibility of dynamic inefficiency (over accumulation of capital) in the laissez-faire economy, and the effects of public debt on capital accumulation. Later, it was shown that *PAYG social security and internal public debt are equivalent instruments for restoring dynamic efficiency* in deterministic OLG closed economies, as they reduce the rate of capital accumulation.

OLG economies, in order to place the normative approach adopted in the paper.

Rangel and Zeckhauser (2001) drew attention to an open debate on efficiency in stochastic economies. According to them, while in deterministic economies an intervention is efficient according to an atemporal and single definition of "Pareto optimum" (it is welfare improving for someone without being adverse for someone else), in stochastic economies the time dimension is relevant for assessing the welfare because agents may change identity upon the resolution of the uncertainty. Consequently, the single atemporal definition evolved into three debatable criteria of Pareto optimum.

Under the "ex ante" criterion, the agent's identity is defined by the birth date only, the "interim" criterion adds to the birth date the realization of the birth-date event, whereas the "ex post" criterion adds the realization of all stochastic events (birth and after birth). Since the "ex post" criterion excludes the possibility of insurance, the literature has adopted the "ex ante" and the "interim" criteria.¹¹

The interim criterion is weaker than the ex ante, because if an allocation is ex ante optimal it is also interim optimal (from an ex ante perspective), but an allocation that is interim optimal may be not ex ante optimal.¹² Nevertheless, interventions that are interim Pareto improving are harder to attain than ex ante ones: "(...) In the ex ante view, a Pareto improvement can occur when a generation wins in some incarnations, loses in others, but is better off on average. This is not interim Pareto improving, however, because it hurts some incarnations (...)" (Rangel and Zeckhauser 2001, p. 120). The ex ante approach is associated to the Rawlsian principle of "ignorance veil"¹³, according to it, the infinite-horizon agent that sets the policy (the government) should ignore any realization of the stochastic events in consideration, including the realization of initial conditions that are relevant under the interim criterion.

Demange (2002) ruled out the possibility of interim Pareto improvement through PAYG social security, in a "dynamic efficient" economy (positive PAYG premium), if all "after-birth" events can be insured (sequentially complete markets), or if productive land can be traded without short sales (an infinitely-lived asset that is traded at positive prices in any state of the world).¹⁴

However, sequentially complete markets or productive land are not substitutes for the natural missing market in stochastic OLG economies: the market of ex ante insurance for

¹¹According to Rangel and Zeckhauser (2001), the debate on efficiency in stochastic economies arised in the monetary economics literature, with the *ex post* criterion of Lucas, in 1972. Pointing out the insurance problem, in 1977, Muench considered the *ex ante* criterion, showing the differences in results between the two approaches. In 1982, Peled defended the *interim* criterion, arguing that it involves a better representation of the information set that is available to agents when they make decisions.

¹²In 1977, Lucas noticed that ex ante insurance would prevent many allocations that are (interim) Pareto optimum (Sargent, 2001).

¹³Rawls, John. 1971. *A Theory of Justice*. Cambridge, Massachusetts: Belknap Press of Harvard University Press. Revised edition, 1999.

¹⁴In deterministic OLG economies with capital, McCallum (1987) and Homburg (1991) ruled out the possibility of over-accumulation of capital in the presence of land (a fixed, non-reproduceable production factor). Richter (1993) extended Homburg (1991) to an economy with uncertain labor income and, in a less general setup than Demange (2002), he also ruled out an interim Pareto improving role for the PAYG social security when productive land is available.

birth-date events.¹⁵ The reasons for this missing market are related to the sequential structure of trade that also characterizes deterministic OLG economies. In these infinite horizon economies, in every period a generation of finitely-lived agents enters the market; coexistent generations may trade, but not the unborn, since alive generations do not act on their behalf unless they are altruist. An assessment of altruism is thus necessary for judging the need of governmental intervention and the effectiveness of policy.

Blanchard and Fisher (1987, p. 105) provide an example of altruistic preferences, where parents include the discounted expected utility of their offsprings in their utility function, such that the resulting objective function is equivalent to the one of an infinitely-lived agent (Ramsey model), where the market allocation is efficient and there is no need of intervention. Weil (1987) showed that the neutrality of public debt, the Barro-Ricardo equivalence (the proposition that the time path of taxes has no effects on the economic activity), fails to hold in the absence of altruism (bequests). The empirical evidence does not support the Barro-Ricardo equivalence (Bernheim, 1987). In addition, there is mixed evidence on the relevance of bequests for wealth accumulation, and there are also alternative explanations for the bequest motive that are not related to altruism, e.g. accidental bequests.

This paper assumes that households are not altruistic, and the PAYG social security may be ex ante Pareto improving due to the lack of ex ante insurance for the stochastic labor productivity that is revealed at "birth", i.e. when the young generation joins the labor market. The government adopts the ex ante approach and its intervention is justified by risk averse households, who would like to have ex ante insurance for the "birth-date" events, and by the market failure in the provision of such insurance in the absence of altruism.

Muench was doubting after formulating the ex ante criterion (or ET-PO, "equal treatment Pareto-optimal"): "I am not asserting that the ET-PO criterion for optimality is the appropriate one. It seems to imply that all future generations agree before history begins. How can this be democratically achieved? (...)" (Muench 1977, cited in Sargent 2001).

Rangel and Zeckhauser (2001) investigated voting institutions. They assumed a 2x2 OLG model, where agents are subject to a stochastic, after-birth event at the old age (a collapse or a boom in the stock markets). They argued that the ex ante optimal, intergenerational, risk sharing implies the share of losses or gains with the young generation, according to the stochastic outcomes. They modelled an infinitely repeated game, where nature chooses the state and the median voter chooses the policy. They showed that if the median voter is elderly, the outcome is always a policy that favors income redistribution over risk sharing, i.e. the elderly median voter always chooses to tax the young, which means "expropriation" in the case of positive stock-market performance. When the median voter is young, he votes for income redistribution (taxation of the elderly) in any event ("expropriation" of the elderly in the case of collapse), if he expects that the median voter in the next period will be the elderly, or if he believes that the choice of the unborn generation is independent of his current choice. The young median voter chooses risk sharing if, and only if, he expects that the young will be the median voter in the next period; the history of previous interactions shows intergenerational

¹⁵According to Kaul (2001), Dutta and Polemarchakis pointed out, in 1990, that the maximum degree of market completeness in stochastic OLG economies is "sequentially complete markets". Weil (forthcoming) credits Chattopadhyay and Gottardi for claiming, in 1999, that the maximum efficiency that markets may attain is interim optimality.

solidarity when the young is in power; and the expected payoff from choosing risk sharing is higher. In addition, ruling out the possibility of expropriation would drop the probability of the ex ante risk-sharing policy to zero, because the young chooses risk sharing only if he fears "retaliation" from the next generation.

The electoral game above illustrates the case where the ex ante optimal, intergenerational, risk sharing may be attained through voting institutions only if it is also interim optimal from an interim perspective. This is a remote possibility because, as noticed before, many interim optimal allocations are not supported from an ex ante perspective.

Rangel and Zeckhauser (2001) also investigated market institutions: "voluntary Ponzi" schemes¹⁶, besides money and contingent markets. They showed that the three instruments may fail to attain intergenerational risk sharing, even under the weaker concept of interim Pareto optimum ("after birth" event), because the voluntary contributions to the Ponzi scheme may be zero and money and contingent assets may not be traded. The reasons for these failures were already investigated in deterministic OLG economies.

Geanakoplos (1987), in a survey of the earlier OLG literature dealing with sub-optimality and indeterminacy, provided an explanation for these OLG "irregularities", in comparison with Arrow-Debreu economies. Both frameworks are based on "agent optimization", "rational expectations" and "market clearing". However, Arrow-Debreu are *finite* horizon economies, composed of a *finite* number of finitely-lived agents. All agents meet before the trade starts and prices are set in advance, at the beginning, and all markets clear. Money has never value in Arrow-Debreu economies, because it does not have storage or consumption value in the last period and, by backward induction, in none of the periods. The competitive market allocation is Pareto optimal.

Geanakoplos (1987) argued that is the *combination* of the *infinitely* succession of finitely-lived agents and the *infinite* horizon of the economy that explains sub-optimality and indeterminacy of equilibrium in OLG economies. It is not the overlapping of generations per se, because altruism can provide the link to an infinitely-lived agent (Ramsey type) model, as argued before, which has the same equilibrium properties of the Arrow-Debreu model. It is not market incompleteness, because also when markets are (sequentially) complete, trade may not occur. It is also not the infinite horizon of the economy alone, since Bewley showed, in 1972, an infinite-horizon economy with a finite number of agents where the equilibria are Pareto optimal (Geanakoplos, 1987).

The OLG demographic structure (the infinitely succession of finitely-lived agents that are not linked by altruism) imposes a *sequential structure of trade* that can lead to a lack of "compatible desires and resources" (Geanakoplos, 1987), which is the explanation for sub-optimality, for instance, in the deterministic pure-exchange economy in Samuelson (1958), and also in the stochastic (contingent markets) economy in Rangel and Zeckhauser (2001).¹⁷ In addition, the "lack of market clearing at infinity" (Geanakoplos, 1987), due to the sequential structure of trade and the infinite horizon of the economy, explains both sub-optimality and indeterminacy. In the deterministic economy in Samuelson (1958), non monetary (sub-optimal)

¹⁶"Ponzi" schemes resemble PAYG financing, because past obligations are met by contracting new debt.

¹⁷Rangel and Zeckhauser (2001) emphasized that the constraints of sequential trade are relaxed as the number of overlapping generations increases (multi-period OLG models), such that market institutions may attain interim Pareto optimality but not ex ante.

and monetary (optimal) equilibria are both possible outcomes (indeterminacy), which depend on "believes" or "expectations". In the stochastic economy in Rangel and Zeckhauser (2001), it is not the lack of "compatible desires and resources", but the *indeterminacy* of the equilibrium and "self-fulfilling" expectations that drive rational agents to the sub-optimal equilibrium where money is not traded or voluntary intergenerational schemes are not implemented.

Hence, since voluntary PAYG schemes may never arise in OLG economies, and since the *ex ante* optimal intergenerational risk-sharing policy may not be optimal from an interim perspective; there is an implementation problem. In this paper, similarly to the literature on pensions and to actual social security schemes, the government solves this problem by determining that the participation in the PAYG social security is *mandatory*.¹⁸

Further, because the rates of PAYG contribution are contingent on the stochastic labor productivity, the government "anchors" expectations by announcing a constant covariance between the rate of PAYG contribution and the stochastic growth rate of the economy, in order to avoid unnecessary uncertainty and to determine an unique *ex ante* equilibrium.

The model

The model is described in six sections; the conclusions, appendix and references follow after. Section 1 contains the assumptions about the labor force, technology, income and risk exposure. Section 2 describes the preferences and determines the best response of households. Section 3 defines the government problem and its approach. Section 4 contains the equilibrium between *ex ante* rates of PAYG contribution and savings, and the *ex ante* welfare for additive quadratic preferences. Section 5 further characterizes the optimal risk-sharing policy. Section 6 illustrates the model with the numerical analysis.

1 Preliminary assumptions

1.1 Labor force

The labor supply is given exogenously, and the labor force evolves according to:

$$L_t = (1 + n_t) L_{t-1}; \tag{1}$$

where, L_t is the size of the young generation alive at time t .

1.2 Technology

Labor productivity is augmented by A_t , the number of "efficiency" units per worker, in the Cobb-Douglas production function: $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$.

¹⁸We do not refer to a problem of "dynamic inconsistency" of the social security policy, because the *ex ante* optimal policy is also interim optimal when seen from an *ex ante* perspective. Gordon and Varian (1988) referred to the problem of engaging the young generation in the PAYG scheme as one of "dynamic inconsistency", but they provided some economic incentives for adhering to the policy when mentioning possible default costs: explicit default and financial disruption, or implicit default and the costs of unexpected inflation (Gordon and Varian 1988, p. 195).

The labor-augmenting technology is given by the following stochastic process:

$$A_{t+1} = (1 + g_{t+1}) A_t = (1 + g + \epsilon_{t+1}) A_t ; g > 0 \text{ and } 1 + g + \epsilon_{t+1} > 0 ; \quad (2)$$

where, $1 + g$ is the average growth rate in the long run; and ϵ_{t+1} is the stochastic disturbance.

The stochastic disturbances are independent and identically distributed (*i.i.d.*) with mean zero. The productivity shocks represent random variations in the human capital, but young generations are supposed to be more productive. The labor efficiency is revealed at the "birth" date, when the young generation joins the labor force.

The growth rate of the labor force in "efficiency" units is:

$$1 + \nu_t = (1 + n_t)(1 + g_t). \quad (3)$$

Definition 1 *In the stochastic steady state, i.e. when the growth rate of the labor force ($1+n$), the interest rate ($1+r$) and the capital intensity ($K_t/A_tL_t \equiv k$) are constant; the aggregate income (Y_t) grows by $1 + \nu_t$ whereas the marginal labor productivity grows by $1 + g_t$.¹⁹*

1.3 Production and factor prices

We assume a small open economy, with free capital flows and perfect competition. Therefore, the interest rate is given and normal profits are zero. The open economies share the same technology, but the labor productivity shocks are independent across countries, and thus the interest rate is risk free. In addition, firms do not face uncertainty because investments are made after the labor productivity is known.

Firms choose the capital intensity, in order to maximize profits:

$$\max_{k_t} \frac{\Pi_t}{A_t L_t} = \max_{k_t} [k_t^\alpha - (\delta + r_t) k_t - w_t];$$

where: $\Pi_t = K_t^\alpha (A_t L_t)^{1-\alpha} - (\delta + r_t) K_t - w_t A_t L_t$ is the profit function of the representative firm; K_t is the capital stock; L_t is the no. of workers; δ is the rate of capital depreciation; r_t is the rate of interest; w_t is the competitive wage.

The problem of the representative firm is standard. The first-order condition for a maximum determines that the marginal productivity of capital equals the rates of capital depreciation and the interest: $\alpha k_t^{\alpha-1} = \delta + r_t$. The competitive wage is obtained as a residual by assuming perfect competition: $\Pi_t = 0 \Rightarrow w_t = k_t^\alpha - (\delta + r_t) k_t$; and since w_t is equal across the open economies, there is no labor mobility.

For simplicity, we assume complete capital depreciation after one period ($\delta = 1$).

¹⁹The marginal labor productivity: $(1 - \alpha) k_t^\alpha A_t$ grows at the same rate as labor efficiency does, when the capital intensity is constant.

1.4 Risk exposure and dynamic efficiency

Since the rate of interest is given by the open economy equilibrium and it is risk free, the elderly generation is not exposed to the productivity shock that hits the young generation. Thus, the assumption of less-than-complete capital depreciation ($\delta < 1$), which drove the asymmetry in risk exposure in Bohn (2003), is not relevant here. However, the qualitative results of the model would perhaps not change if the rate of interest would be stochastic and $\delta < 1$ as in Bohn (2003). The relevant assumption here, which is also assumed in Bohn (2003), is that the young are more risk exposed to current (interim) productivity shocks; because some asymmetry in risk exposure is necessary for risk sharing. In this respect, we take the assumption of interim asymmetry to the extreme, as the elderly are not exposed at all to current productivity shocks.

From the ex ante perspective ($-\infty$), however, both generations are risk exposed. Although the interest rate is safe, the elderly are risk exposed because assets (the accumulated savings) are a function of the stochastic "efficiency" wage received when young (wA_{t-1}), which is only revealed at time $t - 1$. The young are risk exposed because, although the competitive wage (w_t) is safe, the "efficiency" wage (w_tA_t) is stochastic and it is revealed at time t .

In sum, in the laissez-faire economy, the elderly and the young are asymmetrically risk exposed because they are not exposed to a simultaneous shock.

In addition, since the shocks are *i.i.d.* and, in order to abstract from other potential risks (e.g., demography, interest rate and average productivity), we assume that the economy is in the stochastic steady state; all generations are *equally* risk exposed along the lifetime.

Hence, when looking at the ex ante problem, the government will not aim to equalize the risk exposure between the coexistent generations, as the central planner in Bohn (2003) does, but rather it will use that interim asymmetry in order to attain a better risk sharing from a lifetime perspective.

Further, the economy is expected to be strictly *dynamic efficient*, i.e. we assume that the expected rate of economic growth (the indexed PAYG return) is below the rate of interest (the return on funded savings). Thus, intergenerational transfers are costly, and the government is able to provide a second-best solution only. Enders and Lapan (1982) and Bohn (2003) abstracted from the PAYG premium; the first by assuming an economy without capital, and the latter by solving the problem with a central planner.²⁰

2 Households

This section describes household preferences and the intertemporal allocation of consumption, with the labor supply and the net income given exogenously.

²⁰In the appendix, Box "The expected PAYG premium" summarizes main results in the literature about the costs of intergenerational transfers in different institutional setups: central planned economy; closed and small-open dynamic efficient economies.

2.1 The household problem

Households make decisions when they are young. The young have an *interim* perspective, i.e. they know their own efficiency (A_t) and their contingent rate of PAYG contribution (θ_t), but they ignore that of the next generation (A_{t+1} and θ_{t+1}).

Households are not altruistic and they do not leave bequests. The young representative household maximizes the expected lifetime utility from consumption:

$$E_t [U_t(c_{1,t}; c_{2,t+1})] = u(c_{1,t}) + \frac{T_2}{T_1} \rho E_t [u(c_{2,t+1})]; \quad (4)$$

where, $u(\cdot)$ is the instantaneous utility from consumption; $1/\rho \geq 1$ is the rate of time preference (present over future); $0 < T_2/T_1 \leq 1$ is the *subjective* expected time spent in retirement relatively to working years.

The young weight the current and the expected felicities with the (expected) length of the correspondent period, normalized by T_1 . The standard 2x2 OLG model falls in the special case of $T_2/T_1 = 1$.

We believe that allowing for $T_2/T_1 < 1$ is relevant for the calibration of the 2x2 OLG model during the numerical analysis, because savings are likely to depend on the relative time that households expect to spend in retirement. The parameter T_2/T_1 would be unnecessary in a multi-period model, where the number of working and retirement periods may differ explicitly. In contrast, in the two-period model, ignoring it is likely to bias savings. In addition, although we do not model retirement decisions and the interactions with savings, as pointed out in Feldstein (1974); variations in T_2/T_1 allows for a static comparative analysis of the equilibrium in light of exogenous changes in labor supply patterns and/or expected longevity.

Savings are a positive function of T_2/T_1 , as shown in the Euler equation (7). However, if subjectively the young expects a shorter retirement period, from an objective point of view there are still two periods of equal length in the life cycle. The social security provides insurance for longevity risks, and the government ignores T_2/T_1 when defining its objective function.

The budget constraint faced by households, during the working period, is:

$$c_{1,t} = [(1 - \theta_t) w_t - s_t] A_t; \quad (5)$$

where, $c_{1,t}$ is the consumption; θ_t is the *contingent* rate of PAYG contribution at time t ; and $s_t A_t$ are the household savings.

During the retirement,

$$c_{2,t+1} = [\theta_{t+1} (1 + \nu_{t+1}) w_{t+1} + (1 + r_{t+1}) s_t] A_t; \quad (6)$$

where, $c_{2,t+1}$ is the consumption; $\theta_{t+1} (1 + \nu_{t+1}) w_{t+1} A_t$ is the *contingent* PAYG benefit at time $t + 1$; where the growth rate of the economy: $1 + \nu_{t+1} = (1 + n) (1 + g_{t+1})$ represents the PAYG return indexed to the stochastic labor productivity growth.

The PAYG transfers are contingent because of the indexation of benefits $(1 + g_{t+1})$, and the contingency of the bases $(w_t A_t$ and $w_{t+1} A_{t+1})$ and rates $(\theta_t$ and $\theta_{t+1})$ of PAYG contribution to the stochastic labor efficiency $(A_t$ and $A_{t+1})$.

The representative young household chooses the optimal saving rate (s_t) that maximizes the expected lifetime utility in (4), given the budget constraints in equations (5)-(6), which yields the optimal consumption path or Euler equation:

$$\frac{T_2}{T_1} \rho (1 + r_{t+1}) E_t [u' (c_{2,t+1})] = u' (c_{1,t}). \quad (7)$$

Along the optimal path, a ceteris paribus (de)increase in T_2/T_1 implies that savings should (de)increase, such that the expected marginal utility from consumption in the second period (in)decreases.

2.2 Quadratic preferences

This subsection determines the "best response", i.e. the interim optimal savings of the representative young household, given the social security policy. Section 4 determines the interim optimal savings in equilibrium.

Since some additional assumptions about preferences are necessary, beyond the general additive utility function in (4), we analyze the quadratic case. Thus, the instantaneous utility is:

$$u (c_{i,\tau}) = c_{i,\tau} - \frac{a_t}{2} c_{i,\tau}^2 \quad ; \quad c_{i,\tau} \in \{c_{1,t} ; c_{2,t+1}\} \quad \text{and} \quad a_t > 0. \quad (8)$$

The quadratic function provides tractability but at the cost of some well-known drawbacks: the marginal utility (u') may become negative as consumption increases, contrary to rational choice; and stationary preferences $(a_t = a)$ display increasing risk aversion, contrary to the empirical evidence of constant relative risk aversion.

Notice that younger generations are supposed to be wealthier, because the labor productivity grows in the "stochastic" steady state $(g > 0$ and $1 + g + \epsilon_{t+1} > 0)$. Nevertheless, a proper choice of the parameter a_t allows for different attitudes towards risk and, as shown at the end of this subsection, the choice made below implies quasi-constant relative risk aversion *across generations* and it bounds u' to non-negative values.

Definition 2 *The free parameter in the quadratic function is non stationary, and it is calibrated as the inverse of the "efficiency" wage: $a_t \equiv 1/w_t A_t$.*

2.2.1 The best response

Substituting the budget constraints (5)-(6) into the Euler equation (7) yields the interim optimal savings (see technical appendix)²¹:

$$s_t A_t = \left[mps_t (1 - \theta_t) w_t + \frac{mpc_t - (1 + r_{t+1}) mps_t}{(1 + r_{t+1}) a_t A_t} - mpc_t \frac{E_t [\theta_{t+1} (1 + \nu_{t+1})] w_t}{1 + r_{t+1}} \right] A_t;$$

²¹The technical appendix is available under request.

where, $m_{ps}_t \equiv \frac{1}{1 + \frac{T_2}{T_1} \rho (1+r_{t+1})^2}$ is the marginal propensity to save; $m_{pc}_t \equiv 1 - m_{ps}_t$ is the marginal propensity to consume.

Substituting for $a_t \equiv 1/w_t A_t$, and assuming that the economy is in the stochastic steady state, it yields the best-response function:

$$s_t A_t = \left[\begin{array}{c} m_{ps} (1 - \theta_t) + \frac{m_{pc} - (1+r)m_{ps}}{1+r} \\ -m_{pc} \left(\frac{\bar{\theta}(1+\nu) + Cov[\theta_{t+1}, (1+\nu_{t+1})]}{1+r} \right) \end{array} \right] w A_t; \quad (9)$$

where, $\bar{\theta}$ is the expected rate of PAYG contribution; $Cov[\theta_{t+1}, (1 + \nu_{t+1})]$ is the covariance between the rate of PAYG contribution and the growth rate in the next period.

The first term in (9) is the marginal propensity to save out of the disposable income. The term $m_{pc} - (1 + r) m_{ps}$ is peculiar to quadratic preferences and we call it the "self-insurance". It is not a function of the contingent social security because also in the laissez-faire economy ($\theta_t = 0 \ \forall t$) there is self-insurance.²² The last term is the anticipation of the expected PAYG benefit.

Proposition 1 *Households, alive at any time t , know the following policy variables: the contingent rate of PAYG contribution revealed at time t (θ_t); the expected contribution rate of the next generation ($\bar{\theta}$), since the government announces it ex ante ($-\infty$). In addition, although the rate of growth in the next period (ν_{t+1}) and the rate of PAYG contribution paid by the next generation (θ_{t+1}) are unknown at time t ; households know the covariance between them, since the government "anchors" expectations by announcing ex ante a constant covariance.*

Thus, it is assumed that,

$$E_t [\theta_{t+1}] = E_{-\infty} [\theta_{t+1}] \equiv \bar{\theta} \quad \forall t;$$

and,

$$Cov[\theta_{t+1}, (1 + \nu_{t+1})] = Cov[\theta_t, (1 + \nu_t)] \quad \forall t. \quad (10)$$

Based on the definition of $1 + \nu_{t+1}$ in (3), define:

$$Cov[\theta_{t+1}, (1 + \nu_{t+1})] \equiv (1 + v) \frac{Cov[\theta_{t+1}, A_{t+1}]}{E_{-\infty}[A_{t+1}]}, \quad (10a)$$

and, from the assumption in (10), it follows that:

$$\frac{Cov[\theta_{t+1}, A_{t+1}]}{E_{-\infty}[A_{t+1}]} = \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} \quad \forall t. \quad (10b)$$

²²Blanchard and Fisher (1989), in chapter 6, show that among the HARA class of felicities: quadratic, CARA and CRRA; the quadratic is the only function that does not generate precautionary savings.

The solution of the government problem, in Section 4, yields $\bar{\theta}$ and $\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}$. The assumptions made in Proposition 1 avoid unnecessary uncertainty, and they are necessary for the existence of an unique ex ante equilibrium.

The consumption rates are obtained by substituting the interim optimal savings in (9) into the budget constraints (5)-(6):

$$\frac{c_{1,t}}{wA_t} = \left\{ \begin{array}{l} mpc \left[(1 - \theta_t) + \frac{\bar{\theta}(1+\nu) + Cov[\theta_{t+1}, (1+\nu_{t+1})]}{1+r} \right] \\ - \frac{mpc - (1+r)mps}{1+r} \end{array} \right\} \equiv [\cdot]_1, \quad (11)$$

$$\frac{c_{2,t+1}}{wA_t} = \left\{ \begin{array}{l} (1+r)mps(1 - \theta_t) + \theta_{t+1}(1 + \nu_{t+1}) \\ - mpc [\bar{\theta}(1 + \nu) + Cov[\theta_{t+1}, (1 + \nu_{t+1})]] + mpc - (1+r)mps \end{array} \right\}. \quad (12)$$

Taking expectations of (12) at time t yields the (interim) expected optimal consumption in the second period:

$$\frac{E_t[c_{2,t+1}]}{wA_t} = \left\{ \begin{array}{l} (1+r)mps \left[(1 - \theta_t) + \frac{\bar{\theta}(1+\nu) + Cov[\theta_{t+1}, (1+\nu_{t+1})]}{1+r} \right] \\ + mpc - (1+r)mps \end{array} \right\} \equiv [\cdot]_2. \quad (13)$$

2.2.2 Risk aversion degrees and rational choice with non-stationary preferences

Before closing this section, we discuss the implications of calibrating the free quadratic parameter, in terms of risk attitudes and rational choice.

Given that $a_t \equiv 1/wA_t$, the quadratic function is now: $u(c_{i,\tau}) = c_{i,\tau} - \frac{1}{2} \frac{c_{i,\tau}^2}{wA_t}$; $c_{i,\tau} \in \{c_{1,t} ; c_{2,t+1}\}$. The coefficient of *absolute* risk aversion is $\frac{1}{\{1-[\cdot]_i\}wA_t}$, where $[\cdot]_{i=1,2}$ are consumption rates, e.g. the ones computed in equations (11)-(13). The coefficient of *relative* risk aversion is $\frac{[\cdot]_i}{1-[\cdot]_i}$. Hence, $a_t \equiv 1/wA_t$ implies *decreasing absolute* risk aversion *across generations*, since $A_{t+1} > A_t \quad \forall t$; and quasi-constant *relative* risk aversion, because $[\cdot]_i$ is constant, except by θ_t (the contingent rate of PAYG contribution).

Further, the marginal utility is positive in the laissez-faire economy ($\theta_t = 0 \quad \forall t$), according to equations (11) and (13): $u'(c_{1,t}) = \frac{mpc}{1+r}$ and $E_t[u'(c_{2,t+1})] = mps$. Thus, we can conclude that $u'(\cdot) > 0$ also in the PAYG economy ($\theta_t > 0 \quad \forall t$), since current and (interim) expected consumptions are lower than those in the laissez-faire economy, because of the expected lower PAYG return.

The appendix contains a summary of household behavior with quadratic preferences, according to different values assigned to a_t . We carry the assumption of $a_t \equiv 1/wA_t$ throughout the analysis, but there we indicate the implications of alternative values. The coefficients of absolute and relative risk aversion are: a) increasing, when $a_t \equiv a$ (stationary preferences); b) decreasing and quasi-constant respectively, when $a_t \equiv 1/wA_t$; c) decreasing when $a_t \equiv 1/wA_t^2$; d) increasing when $a_t \equiv 1/w$. In concerning rational choice, except for $a_t \equiv 1/wA_t$, the sign of $u'(c_{1,t})$ depends on a and/or A_t and they should be bounded for yielding $u'(\cdot) > 0$.

3 The government

The reason for governmental intervention is the existence of an uninsurable risk, namely the stochastic labor efficiency revealed at "birth" date. At that time, when the young generation joins the labor market, insurance is not available because the labor efficiency is common knowledge. Ex ante insurance is also not available because the unborn cannot trade with existing generations, since households are not altruistic.

The government adopts an *ex ante* perspective of the risk-sharing problem, under the assumption that risk averse households would like to have *ex ante* insurance for birth events. The government intervenes under the principle of the "ignorance veil". Subsection 1.4 defined the risk exposure in the laissez-faire economy: the elderly assets are a function of the stochastic labor income revealed at time $t - 1$, whereas the stochastic income of the young generation is revealed at time t . From an ex ante perspective ($-\infty$), both events are not revealed yet, and thus the young and the elderly are exposed to risk.

3.1 The government problem

The government is considering to implement a PAYG social security scheme in period t . Provided that the rate of PAYG contribution is positive, the first elderly generation is always better off because of "windfall benefits", since no prior contributions were made to the scheme. Thus, the government leaves this generation out of the analysis, but it cares that the rates of PAYG contribution are not negative.

From an ex ante view, the first young and all future generations are unborn, and they are equally risk exposed along the lifetime because the productivity shocks are *i.i.d.* and the economy is in the stochastic steady state. In addition, because $a_t \equiv 1/wA_t$, they are also (quasi) equally relative-risk averse as discussed before. Consequently, the government assigns equal weights to these generations, and its objective function reduces to the ex ante lifetime utility of a representative generation.²³

The government chooses a path of PAYG contribution rates that solves the following problem:

$$\begin{aligned} \text{Max}_{\theta_t, \theta_{t+1}} \quad & E_{-\infty} \left[u(c_{1,t}^*) + \rho E_t \left[u(c_{2,t+1}^*) \right] \right] \\ \text{s.t.} \quad & 0 \leq \theta_t, \theta_{t+1} \leq 1; \end{aligned} \tag{14}$$

where, $c_{1,t}^*$ and $c_{2,t+1}^*$ are the consumption functions in equations (11)-(12), and $E_t \left[u(c_{2,t+1}^*) \right]$ is the interim expected utility in the second period.

The PAYG budget is balanced in each period, i.e. the aggregate benefits are equal to the aggregate contributions: $(1 + \nu_t) \theta_t w A_{t-1} = (1 + n) \theta_t w A_t$, according to the budget constraints of households, defined in equations (5)-(6). Thus, the government problem is solved as an unconstrained maximization problem, but the contribution rates are bounded as indicated in (14).

²³In appendix, Box "Social welfare weights and dynamic consistency of the policy" works out the reduction of the government problem, from a sum of discounted lifetime utilities to the lifetime utility of a representative generation. I thank Roel Beetsma for raising these concerns.

Further, when discussing the parameter T_2/T_1 , we argued that the social security provides coverage for longevity risks, thus the government ignores T_2/T_1 when weighting the lifetime welfare in (14).²⁴ Nevertheless, the government takes into account the consumption functions of households, which are functions of T_2/T_1 through the marginal propensities to save and consume.

4 Ex ante equilibrium and welfare

The timing of the decisions and the interaction between the decision makers can be described as a "Stackelberg leadership" game. The government has advantage by setting the PAYG social security before households determine funded savings. Households are informed about the government policy and its commitment to it in the long run, according to the assumptions made in Proposition 1. The government knows households expectations and their best response function, and it ultimately determines the combination of PAYG and funded savings in the equilibrium because it can induce the household response. Households have no means of deviating because the participation in the PAYG scheme is mandatory.

Proposition 2 *The government intervenes once for all. It announces ex ante the optimal policy and its commitment to it in the long run. The time evolving per se does not change the optimal policy, and thus it is dynamically consistent. In addition, there is credibility. These conditions, together with the mandatory character of the PAYG social security, are the necessary and sufficient conditions for the stability of the "Stackelberg" equilibrium.*

The next subsections determine the ex ante optimal policy functions, the saving and consumption rates in equilibrium, and the ex ante lifetime welfare. Equations (18), (17a) and (9a) show below that the ex ante equilibrium is uniquely determined by the exogenous parameters.

4.1 The first-order conditions of the government

Differentiating the government objective function (14) with respect to θ_t , using the Euler equation (7) and the consumption functions in (11) and (12) yields:²⁵

$$E_{-\infty} [E_t [u' (c_{2,t+1})] A_t] = 0. \quad (15)$$

Differentiating (14) with respect to θ_{t+1} , using the consumption functions in (11) and (12), and the constant-covariance assumption in (10), it yields another first-order condition:

$$E_{-\infty} [E_t [u' (c_{2,t+1})] A_{t+1}] = 0. \quad (16)$$

²⁴Leaving T_2/T_1 in equation (14) would not alterate the first-order conditions of the government problem.

²⁵See, in the Box "Social welfare weights (...)", the derivation of the first-order conditions from an alternative specification of the government objective function and the reduction to the conditions in (15)-(16).

4.2 The expected optimal rate of PAYG contribution

Substituting the quadratic function and $a_t \equiv 1/wA_t$ into the first-order condition (15); substituting for $E_t[c_{2,t+1}]$ from equation (13); using the assumptions of constant covariance in (10) and *i.i.d.* zero-mean shocks (see technical appendix); it yields the expected optimal rate of PAYG contribution as a function of the covariance:

$$\bar{\theta} = \frac{1}{1 - \frac{1+\nu}{1+r}} \left[1 - \frac{1 - [mpc - (1+r)mps]}{(1+r)mps} \right] - \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}, \quad (17)$$

$$\Rightarrow \bar{\theta} = -\frac{1}{1 - \frac{1+\nu}{1+r}} \frac{1}{1+r} - \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]},$$

where, $1 - \frac{1+\nu}{1+r}$ is the "golden rule" gap (see Box "The expected PAYG premium"); $(1+r)mps$ is the gross return on the marginal propensity to save; $Cov[\theta_t, A_t]$ is the covariance between the contingent rate of PAYG contribution and the labor efficiency.

4.2.1 The optimal covariance

The optimal covariance is obtained in a similar way, but using the second first-order condition (16), substituting for $c_{2,t+1}$ from equation (12) and for $\bar{\theta}$ from (17) (see technical appendix):

$$\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} = -\frac{1}{1 - \frac{1+\nu}{1+r}} \frac{1}{1+r} \frac{n(1+g)}{(1+r)mps - (1+g)} \quad \forall t. \quad (18)$$

4.2.2 The conditions for an interior solution

Substituting (18) into (17) yields the expected optimal rate of PAYG contribution:

$$\bar{\theta} = \frac{1}{1 - \frac{1+\nu}{1+r}} \frac{1}{1+r} \left[\frac{n(1+g)}{(1+r)mps - (1+g)} - 1 \right]. \quad (17a)$$

From equation (17), $\bar{\theta}$ is increasing in the rate of "self-insurance": $mpc - (1+r)mps$; i.e. the government accounts for a measure of household's desire of smoothing consumption. From equation (17a), $\bar{\theta}$ is decreasing in the average rate of PAYG premium: $1 - \frac{1+\nu}{1+r}$. In addition, the ex ante optimal policy functions in (17)-(18) depend on preference parameters through the marginal propensity to save only.

The second line in equation (17) shows a trade-off between dynamic efficiency and risk sharing. Since the average rate of PAYG premium is positive ($1 - \frac{1+\nu}{1+r} > 0$), a positive expected rate of PAYG contribution implies a negative covariance: $\bar{\theta} > 0 \Rightarrow \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} < 0$; i.e. the ex ante optimal PAYG scheme also shifts risk *from the elderly to the young*. Notice that, by assumption, the PAYG benefits are fully indexed to the labor efficiency and this shifts risk *from the young to the elderly*; but the negative covariance partially offsets the indexation effect and it provides *partial* social protection to the elderly.²⁶

²⁶I am grateful to Eytan Sheshinski for the suggestion of allowing for state-contingent rates of PAYG contribution, which turned to be fundamental for finding an interior solution for the risk-sharing problem.

Notice that, from an ex ante perspective, the young benefit from making a positive transfer to the elderly because it allows to anticipate consumption on the account of the expected PAYG benefit. The elderly benefit from pooling risks with the young, as the elderly are also risk exposed from an ex ante view. In other words, the social security reduces the volatility of ex ante consumption in both periods.

Nevertheless, equation (17) shows that, if the covariance of the rate of PAYG contribution is not negative, or the magnitude is not large enough, a negative rate of PAYG contribution is optimal, which means that dynamic efficiency dominates the risk-sharing motive. A negative PAYG transfer would make the first young and all future generations better off when $v < r$ because instead of incurring the PAYG premium, they would earn the difference in return rates. However, it is not ex ante Pareto optimal, because it would reduce the welfare of the first elderly generation.

From equation (17a), the boundaries for yielding an expected positive rate of PAYG contribution are: $n > 0$ and $1 + g < (1 + r)mps < 1 + v$, given that $v < r$. Thus, the rate of growth of the labor force should be positive. In addition, in the second condition, the right side means that the gross return on the marginal propensity to save should be lower than the average indexed return of the PAYG scheme.²⁷ Both conditions seem to assure that the cost of the social security is not prohibitive for households. The left side of the second inequality relates to risk exposure, as argued in the proposition below.

Proposition 3 *In the stochastic steady state, at any time t , the ex ante accumulated assets of the elderly are: $(1 + r) E_{-\infty} [s_{t-1}A_{t-1}]$, and they are a function of the stochastic "efficiency" wage received when young (wA_{t-1}), according to equation (9); while the ex ante "efficiency" wage of the young generation is: $E_{-\infty} [A_t] = (1 + g) E_{-\infty} [A_{t-1}]$, according to equation (2) and given that the shocks are independent.*

Thus, $(1 + r)mps$, the gross return on the marginal propensity to save; and $1 + g$, the average growth rate of labor efficiency; they are ex ante ($-\infty$) rates of marginal risk exposure, of the elderly and the young generations respectively, to the same stochastic perturbation (ϵ_{t-1}).

The main results of this subsection are summarized below.

Lemma 1 *Consider a wage-indexed PAYG social security, in the presence of labor productivity risks, with contingent rates of PAYG contribution. Given, (i) the small open economy is strictly dynamic efficient on average ($v < r$); (ii) preferences are additive quadratic and non stationary: $u(c_{i,\tau}) = c_{i,\tau} - \frac{\alpha_t}{2}c_{i,\tau}^2$; $c_{i,\tau} \in \{c_{1,t}; c_{2,t+1}\}$ and $a_t \equiv 1/w_tA_t$.*

The necessary and sufficient conditions for a positive expected optimal rate of PAYG contribution ($\bar{\theta} > 0$), in the "stochastic" steady state, are: a) the rate of growth of the labor force should be positive ($n > 0$); b) the gross return on the marginal propensity to save ($(1 + r)mps$) should be lower than the average indexed return of the PAYG scheme ($1 + v$); c) the elderly generation should be relatively more risk exposed than the young to a common shock, according to the difference in the ex ante marginal rates of risk exposure: $(1 + r)mps - (1 + g) > 0$.

Proof. See equation (17a). ■

²⁷From equation (9), $(1 + r)mps * \theta_t$ is the gross return on forgone marginal savings due to the PAYG contribution.

Theorem 1 *Given, (i) the PAYG social security implies an expected positive premium ($1 - \frac{1+\nu}{1+r} > 0$); (ii) the PAYG benefits are positively indexed to the "efficiency" wages, due to the higher risk exposure of the young generation to current labor productivity shocks.*

Unless there are ex ante welfare gains also from providing partial social protection at the old age, because of higher ex ante risk exposure of the elderly to a common labor productivity shock, such that a negative covariance between the rates of PAYG contribution and the stochastic labor efficiency is ex ante optimal; dynamic efficiency dominates the risk-sharing motive, i.e. the expected optimal rate of PAYG contribution is non positive.

Proof. See equations (17)-(18) and Lemma 1. ■

4.3 The optimal saving and consumption rates

Section 2 determined the best response and the consumption functions, given the policy variables. This subsection adds the knowledge of the optimal policy functions and it determines the household intertemporal allocation in the (ex ante) equilibrium. For comparison purposes, we also compute the equilibrium in the laissez-faire economy.

Setting $\theta_t = 0 \forall t$ (and denoting laissez-faire equilibrium values by the superscript lf), in equation (9), it yields the optimal saving rate in the laissez-faire economy:

$$\frac{s_t^{lf} A_t}{wA_t} = \frac{mpc}{1+r}; \quad (19)$$

which is a function of the marginal propensity to consume because of the self-insurance component.

Given that $mps \equiv \frac{1}{1 + \frac{T_2}{T_1} \rho(1+r)^2}$ and $mpc \equiv 1 - mps$, it follows that $\partial mpc / \partial (T_2/T_1) > 0$.

Substituting (19) into the budget constraints (5)-(6), and setting $\theta_t = 0 \forall t$, yields the optimal consumption rates in the laissez-faire economy:

$$\frac{c_{1,t}^{lf}}{wA_t} = 1 - \frac{mpc}{1+r} \quad \text{and} \quad \frac{c_{2,t+1}^{lf}}{wA_t} = mpc. \quad (20)$$

Substituting for $\bar{\theta}$ from equation (17) into the interim optimal savings in (9), and using from equations (10a)-(10b) that $Cov[\theta_{t+1}, (1 + \nu_{t+1})] = (1 + \nu) \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}$, it yields the optimal saving rate in the PAYG equilibrium:

$$\frac{s_t A_t}{wA_t} = -mps * \theta_t + \frac{mpc}{1+r} \left(\frac{1}{1 - \frac{1+\nu}{1+r}} \right). \quad (9a)$$

The difference in saving rates is:

$$\frac{s_t A_t}{wA_t} - \frac{s_t^{lf} A_t}{wA_t} = \frac{mpc}{1+r} \left[\frac{1}{1 - \frac{1+\nu}{1+r}} - 1 \right] - mps * \theta_t. \quad (21)$$

Lemma 2 Given, (i) the small open economy is strictly dynamic efficient on average ($v < r$); (ii) preferences are additive quadratic and non stationary: $u(c_{i,\tau}) = c_{i,\tau} - \frac{a_t}{2}c_{i,\tau}^2$; $c_{i,\tau} \in \{c_{1,t}; c_{2,t+1}\}$ and $a_t \equiv 1/w_t A_t$.

In the PAYG equilibrium the saving rate is higher, in comparison to the laissez-faire economy, when the rate of "self-insurance" is positive: $mpc - (1+r)mpr > 0$; and when the contingent rate of PAYG contribution satisfies: $\theta_t < \frac{1}{1-\frac{1+v}{1+r}} - 1$, where $1 - \frac{1+v}{1+r}$ is the expected rate of PAYG premium.

Proof. See equation (21). ■

The numerical analysis shows that the above conditions are satisfied, by the ex ante rate $\bar{\theta}$, in the baseline and in most of the alternative scenarios yielding an interior solution. In our interpretation, since the model abstracts from a second pension pillar, the higher saving rate may represent a contribution to a funded pillar in addition to the PAYG, which is ex ante optimal, because of the difference in expected return rates and the wish of smoothing consumption over time represented by the rate of self-insurance.

The expected PAYG premium is an objective measure of the substitutability between the PAYG and funded savings. However, under quadratic preferences, the uncertainty related to the contingent social security does not affect the household behavior, in the sense that the self-insurance does not depend on the PAYG scheme. Under other preference assumptions, we could expect precautionary saving driven by the contingent social security, which is another indication of the substitutability between the schemes.

In addition, the ability of controlling for risk aversion is limited. As shown before, $a_t \equiv 1/w_t A_t$ is a convenient choice because it implies (quasi) constant relative risk aversion across generations and rational choice. However, given the choice of the free parameter, the coefficient of risk aversion (the *degree* of risk aversion) is determined endogenously as a function of consumption. Hence, the government cannot test alternative hypotheses of risk-aversion degree. The coefficients of absolute and relative risk aversion are: $\frac{1}{\{1-[\cdot]_i\}wA_t}$ and $\frac{[\cdot]_i}{1-[\cdot]_i}$, respectively, as discussed in subsection (2.2.2), where $[\cdot]_i$ are consumption rates. Notice that the coefficients of risk aversion vary along the life cycle, but $\frac{[\cdot]_i}{1-[\cdot]_i}$ does not vary across generations because of $a_t \equiv 1/w_t A_t$.

The relevant coefficients for policy analysis seem to be the coefficients of relative risk aversion obtained in the laissez-faire equilibrium, where $[\cdot]_i$ is defined according to (20). For reasonable assumptions about the exogenous parameters, they decline at the old age. Hence, our finding that providing partial social protection to the elderly may be ex ante optimal, besides being a necessary condition for positive PAYG transfers, is not related to higher risk aversion at the old age as suggested in Bohn (2003).

The (interim and ex post) consumption rates in the PAYG equilibrium are obtained by substituting the saving rate in equilibrium, from equation (9a), into the budget constraints (5)-(6):

$$\frac{c_{1,t}}{wA_t} = 1 - \frac{mpc}{1+r} \left[(1+r)\theta_t + \frac{1}{1-\frac{1+v}{1+r}} \right], \quad (12a)$$

$$\frac{c_{2,t+1}}{wA_t} = \theta_{t+1} (1 + \nu_{t+1}) - (1 + r) mps * \theta_t + \frac{mpc}{1 - \frac{1+\nu}{1+r}}. \quad (13a)$$

The ex ante consumption rates in the PAYG equilibrium, used in the welfare analysis, are obtained using equations (10a)-(10b) and (17) (technical appendix). They reflect ex ante welfare gains from risk sharing that are not visible from interim or ex post:

$$\frac{E_{-\infty} [c_{1,t}]}{wE_{-\infty} [A_t]} = \frac{E_{-\infty} [c_{2,t+1}]}{wE_{-\infty} [A_t]} = 1. \quad (22)$$

4.4 The ex ante lifetime welfare

Substituting for the quadratic function and for $a_t = 1/wA_t$ into the ex ante lifetime utility of a representative generation:

$$E_{-\infty} [u(c_{1,t}^*) + \rho u(c_{2,t+1}^*)] = E_{-\infty} \left[\left(1 - \frac{1}{2} \frac{c_{1,t}^*}{wA_t}\right) c_{1,t}^* + \rho \left(1 - \frac{1}{2} \frac{c_{2,t+1}^*}{wA_t}\right) c_{2,t+1}^* \right].$$

Notice that we do not take interim expectations of the utility in the second period: $E_t [u(c_{2,t+1}^*)]$, as we did in the objective function in (14), because here we want to evaluate the ex post welfare as seen from ex ante.²⁸

Substituting for the optimal consumption rates in (20), taking ex ante expectations and dividing both sides by $wE_{-\infty} [A_t]$, it yields the ex ante lifetime welfare in the laissez-faire economy:

$$\frac{E_{-\infty} \left\{ u(c_{1,t}^{lf}) + \rho [u(c_{2,t+1}^{lf})] \right\}}{wE_{-\infty} [A_t]} = \frac{1}{2} \left[1 - \left(\frac{mpc}{1+r} \right)^2 \right] + \rho \left[1 - \frac{1}{2} mpc \right] mpc. \quad (23)$$

Substituting above for the consumption rates in the PAYG equilibrium, in equations (12a) and (13a); taking expectations; dividing both sides by $wE_{-\infty} [A_t]$; and substituting for the ex ante optimal consumption rates, $\frac{E_{-\infty} [c_{i,\tau}]}{wE_{-\infty} [A_t]} = 1$ from (22):

$$\frac{E_{-\infty} \{u(c_{1,t}) + \rho [u(c_{2,t+1})]\}}{wE_{-\infty} [A_t]} = \left\{ \begin{array}{c} \frac{1}{2} \left(1 + \frac{mpc}{1+r} \frac{1}{1 - \frac{1+\nu}{1+r}} \right) + \frac{1}{2} mpc \frac{E_{-\infty} [c_{1,t} \theta_t]}{wE_{-\infty} [A_t]} \\ + \rho \left[1 - \frac{1}{2} \left(\frac{mpc}{1 - \frac{1+\nu}{1+r}} \right) \right] \\ - \frac{\rho}{2} \left[\frac{E_{-\infty} [c_{2,t+1} \theta_{t+1} (1 + \nu_{t+1})]}{wE_{-\infty} [A_t]} - (1 + r) mps \frac{E_{-\infty} [c_{2,t+1} \theta_t]}{wE_{-\infty} [A_t]} \right] \end{array} \right\}.$$

In what follows, we derive the terms in expectations above, which provide the covariances between consumption and PAYG contribution and replacement rates.

²⁸The objective function of the government in (14) captures all relevant information about household behavior, including the expectations.

Given that $Cov[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[xy] - \bar{x}\bar{y}$, and substituting for $\frac{E_{-\infty}[c_{i,\tau}]}{wE_{-\infty}[A_t]} = 1$ from equation (22):

$$\frac{E_{-\infty}[c_{i,\tau}\theta_t]}{wE_{-\infty}[A_t]} = \bar{\theta} + E_{-\infty}\left[\left(\frac{c_{i,\tau}}{wE_{-\infty}[A_t]} - 1\right)(\theta_t - \bar{\theta})\right] \quad ; \quad c_{i,\tau} \in \{c_{1,t}; c_{2,t+1}\}.$$

Substituting for $\frac{c_{1,t}}{wE_{-\infty}[A_t]}$ from equation (12a), and given that $Var[x] = E[(x)^2] - (\bar{x})^2$:

$$\frac{E_{-\infty}[c_{1,t}\theta_t]}{wE_{-\infty}[A_t]} = \bar{\theta} - mpc * Var[\theta_t]. \quad (24)$$

Substituting for $\frac{c_{2,t+1}}{wA_t}$ from equation (13a), and given that $\theta_{t+1}(1 + \nu_{t+1})$ and $(\theta_t - \bar{\theta})$ are uncorrelated:

$$\frac{E_{-\infty}[c_{2,t+1}\theta_t]}{wE_{-\infty}[A_t]} = \bar{\theta} - (1+r)mpr * Var[\theta_t]. \quad (25)$$

Given that $Cov[x, y] = E[xy] - \bar{x}\bar{y}$, and substituting for $\frac{E_{-\infty}[c_{2,t+1}]}{wE_{-\infty}[A_t]} = 1$:

$$\frac{E_{-\infty}[c_{2,t+1}\theta_{t+1}(1 + \nu_{t+1})]}{wE_{-\infty}[A_t]} = E_{-\infty}[\theta_{t+1}(1 + \nu_{t+1})] + \frac{Cov[c_{2,t+1}, \theta_{t+1}(1 + \nu_{t+1})]}{wE_{-\infty}[A_t]}.$$

Substituting for $Cov[\theta_{t+1}, (1 + \nu_{t+1})]$ from (10a)-(10b), and for $\bar{\theta}$ from (17) into $E_{-\infty}[\theta_{t+1}(1 + \nu_{t+1})]$, and for $\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}$ from (18), it yields the ex ante replacement rate:

$$E_{-\infty}[\theta_{t+1}(1 + \nu_{t+1})] = (1+v)\left[\bar{\theta} + \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}\right] = -\frac{\frac{1+\nu}{1+r}}{1 - \frac{1+\nu}{1+r}}. \quad (26)$$

Given that $Cov[x, y] = E[(x - \bar{x})(y - \bar{y})]$, and substituting for $\frac{E_{-\infty}[c_{2,t+1}]}{wE_{-\infty}[A_t]} = 1$:

$$\frac{Cov[c_{2,t+1}, \theta_{t+1}(1 + \nu_{t+1})]}{wE_{-\infty}[A_t]} = E_{-\infty}\left[\left(\frac{c_{2,t+1}}{wE_{-\infty}[A_t]} - 1\right)(\theta_{t+1}(1 + \nu_{t+1}) - E_{-\infty}[\theta_{t+1}(1 + \nu_{t+1})])\right].$$

Finally, substituting for $\frac{c_{2,t+1}}{wA_t}$ from equation (13a), and given that $\theta_{t+1}(1 + \nu_{t+1})$ and θ_t are uncorrelated:

$$\frac{Cov[c_{2,t+1}, \theta_{t+1}(1 + \nu_{t+1})]}{wE_{-\infty}[A_t]} = Var[\theta_{t+1}(1 + \nu_{t+1})]. \quad (27)$$

Substituting (24)-(27) above and simplifying, it yields the ex ante lifetime welfare in the PAYG economy:

$$\begin{aligned} & \frac{E_{-\infty}[u(c_{1,t}) + \rho[u(c_{2,t+1})]]}{wE_{-\infty}[A_t]} \\ &= \left\{ \begin{array}{l} \frac{1}{2} \left[1 - mpc \left(\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} + mpc * Var[\theta_t] \right) \right] \\ + \rho \frac{1}{2} \left[\begin{array}{l} 1 - Var[\theta_{t+1}(1 + \nu_{t+1})] \\ - (1+r)mpr \left(\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} + (1+r)mpr * Var[\theta_t] \right) \end{array} \right] \end{array} \right\}. \quad (28) \end{aligned}$$

The conditions for ex ante welfare improving in the PAYG equilibrium are:

$$\frac{E_{-\infty} \left[u(c_{1,t}) - u(c_{1,t}^{lf}) \right]}{wE_{-\infty} [A_t]} > 0 \Rightarrow Var [\theta_t] < \left(\frac{1}{1+r} \right)^2 - \frac{1}{mpc} \frac{Cov [\theta_t, A_t]}{E_{-\infty} [A_t]}, \quad (29)$$

$$\frac{E_{-\infty} \left[u(c_{2,t+1}) - u(c_{2,t+1}^{lf}) \right]}{wE_{-\infty} [A_t]} > 0 \quad (30)$$

$$\Rightarrow Var [\theta_{t+1} (1 + \nu_{t+1})] < 1 - [2 - mpc] mpc - (1 + r) mps \left(\begin{array}{c} \frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]} \\ + (1 + r) mps * Var [\theta_t] \end{array} \right).$$

In sum, when solving the government problem, we found the expected optimal rate of PAYG contribution and the optimal covariance in equations (17a)-(18), and from the assumptions made in (10)-(10b), we know the optimal covariance with the growth rate. The welfare analysis characterizes further the distribution of the contingent rates of PAYG contribution: equation (29) determines the upper bound for the variance of the contingent contribution rate; and equation (30) does the same for the variance of the contingent replacement rate. These boundaries assure that the PAYG scheme is ex ante welfare improving in each of the life-cycle periods.

5 The contingent rates of PAYG contribution

A further characterization of the contingent rates of PAYG contribution is possible through the analysis of the ex post PAYG benefit.²⁹

The deviation of the PAYG benefit from the average, in period $t + 1$, is:

$$B_{t+1} - \bar{B} = [\theta_{t+1} (1 + \nu_{t+1}) - \bar{\theta} (1 + \nu)] wA_t ;$$

where, $B_{t+1} = \theta_{t+1} (1 + \nu_{t+1}) wA_t$ is the PAYG benefit received by a representative elderly household of generation t .

Since $1 + \nu_{t+1} = (1 + n) (1 + g + \epsilon_{t+1})$, according to the assumptions in (2)-(3); rewrite $B_{t+1} - \bar{B}$ as:

$$B_{t+1} - \bar{B} = (1 + n) [\theta_{t+1} \epsilon_{t+1} + (1 + g) (\theta_{t+1} - \bar{\theta})] wA_t.$$

The deviation of the PAYG benefit from the average can be decomposed into two sources. One is the indexation of benefits to the stochastic productivity: $(1 + n) \theta_{t+1} \epsilon_{t+1}$; and the other is the contingency of the rate of PAYG contribution: $(1 + n) (\theta_{t+1} - \bar{\theta})$. The deviation due to the indexation has the same sign of the shock (ϵ_{t+1}), whereas $(\theta_{t+1} - \bar{\theta})$ has the opposite sign, according to the ex ante optimal covariance.

²⁹I thank John Marshall for the question about the direction of change of the total PAYG benefit, given the realization of the stochastic productivity.

We assume that the indexation effect is dominant, and thus $B_{t+1} - \bar{B}$ has the same sign of ϵ_{t+1} . Notice that if $B_{t+1} - \bar{B}$ would have the opposite sign, it would be contrary to the policy of shifting current productivity risks from the young to the elderly and inconsistent with partial social protection at the old age.

Below, we consider the three possible outcomes: $\epsilon_{t+1} \leq 0$, and we determine the respective intervals for θ_{t+1} for the three cases that are consistent with the ex ante optimal risk-sharing policy.

The first case is:

$$\epsilon_{t+1} = 0 \text{ and } B_{t+1} - \bar{B} = 0 \Rightarrow \theta_{t+1} = \bar{\theta}; \quad (31a)$$

where, $\bar{\theta}$ is given by (17a).

The second case is:

$$\epsilon_{t+1} > 0 \text{ and } B_{t+1} - \bar{B} > 0 \Rightarrow \theta_{t+1} < \bar{\theta} \text{ and } \theta_{t+1}\epsilon_{t+1} > -(1+g)(\theta_{t+1} - \bar{\theta}).$$

where, a positive shock implies a rate of PAYG contribution that is below the average rate and, together with the condition of a dominant indexation effect, it provides the following interval for θ_{t+1} :

$$\frac{1+g}{1+g+\epsilon_{t+1}}\bar{\theta} < \theta_{t+1} < \bar{\theta}. \quad (31b)$$

The third case, $\epsilon_{t+1} < 0$ and $B_{t+1} - \bar{B} < 0$, implies that:

$$\bar{\theta} < \theta_{t+1} < \frac{1+g}{1+g+\epsilon_{t+1}}\bar{\theta}; \quad |\epsilon_{t+1}| < 1+g. \quad (31c)$$

Provided that the disturbances are smaller than $1+g$ in magnitude, otherwise an adverse shock would reduce the human capital; the above intervals are well defined and they contain the optimal state-contingent rates of PAYG contributions for the three possible kinds of events.

6 Numerical analysis

In this section, we analyze the ex ante equilibrium between the rates of PAYG contribution ($\bar{\theta}$) and savings ($E_{-\infty} \left[\frac{s_t A_t}{w A_t} \right]$), determined in equations (17) and (18) and by taking ex ante expectations of equation (9a).

The interest and growth rates are expressed per period ($T_1 \equiv 35$ years) and in an annual basis. In the baseline scenario, the annual values are: the open economy rate of interest (r) is 3,5 percent, the rate of growth of the labor force (n) is 0,5 percent, and the expected rate of growth of labor productivity (g) is also 0,5 percent. The preference parameters are: the rate of time preference, future over present (ρ), and the subjective-expected relative retirement length (T_2/T_1). The parameter ρ is 0,3 in the baseline, which corresponds to a subjective discount rate of about 3,3, which is slightly below the interest rate in the period. T_2/T_1 is 15/35.

Table 1a (appendix) shows the equilibrium and the sensitivity analysis for the interest and growth rates. In the baseline, the expected rate of PAYG premium is 57,5 percent, the

expected optimal rate of PAYG contribution is 13 percent and the ex ante saving rate is 25 percent, while the saving rate in the laissez-faire economy is 18 percent.

First, we varied the productivity growth rate and found a tiny interval for the annual rate (from 0,45 to 0,60 percent) for an interior solution, given the other baseline values. When $g = 0,40$ percent, the expected growth rate $(1 + \nu)$ falls below the the gross return on the marginal propensity to save: $(1 + r)mps$, which violates the condition for an interior solution. When $g = 0,65$ percent, the absolute value of the optimal covariance is above one, which is out of the range for an interior solution.³⁰

Second, we varied the rate of labor force growth, which has about the same effect of varying the productivity, since both affect the PAYG premium but not the marginal propensity to save. The baseline is very sensitive to the rate of PAYG premium: a ceteris paribus increase of the premium of about 12 percent decreased the optimal PAYG contribution by more than 60 percent (from 13 to 5 percent). Similarly, a ceteris paribus decrease of the premium by circa of 13 percent increased the optimal PAYG contribution by approximately 77 percent (from 13 to 23 percent).

Third, we varied the open economy interest rate, and thus the PAYG premium and the marginal propensity to save varied, since they are ceteris paribus increasing and decreasing functions of the interest rate, respectively. When the interest rate falls (from 3,5 to 3 percent per year), even though the PAYG premium drops by more than 13 percent, the expected PAYG contribution falls by more than 38 percent (from 13 to 8 percent) because of the increase in the marginal propensity to save (from 0,41 to 0,5). The fall of the interest rate means that future consumption is relatively more expensive and households need to save more. Equation (17a) shows that the expected contribution is decreasing in the marginal propensity to save. An increase in the interest rate (from 3,5 to 4 percent per year) also has two opposite effects on the ex ante optimal PAYG contribution: even though the PAYG premium increases, the marginal propensity to save falls; and the expected PAYG contribution increases (from 13 to 33 percent), showing that the effect of the savings response is dominant upon the PAYG premium effect.

Hence, when the marginal propensity to save rises because future consumption is relatively more expensive when the interest rate declines, there is less room for risk sharing regardless of the lower PAYG premium (vice-versa).

Fourth, we varied the interest and the productivity growth rates simultaneously, but the attempt to find other equilibrium points failed.

Table 1b (in appendix) contains the sensitivity analysis for the rate of time preference and the expected retirement length. None of the deviations from the baseline provided an interior solution. In all cases, the condition for an interior solution that was violated is: $1 + g < (1 + r)mps$ (see $(1 + r)mps$ in the last column).

³⁰Notice that $Cov[\theta_t, A_t]$ is normalized by $E_{-\infty}[A_t]$, and thus $Cov[\theta_t, A_t]/E_{-\infty}[A_t]$ is expressed in the same units of θ_t , and it should be within the interval $(0, 1)$ for an interior solution.

Conclusions

The paper extended previous analyses of the optimal intergenerational sharing of labor productivity risk to a small open and dynamic efficient economy. When departing from Bohn (2003), and assuming that: a) intergenerational transfers are costly; b) the overlapping generations are not exposed to simultaneous shocks; c) there are equal risk exposure and quasi-constant relative risk aversion across generations; the risk-sharing problem changed in a fundamental way. Consequently, the approach changed from an analysis of the interim asymmetry in risk exposure between the overlapping generations, to an ex ante lifetime perspective of the risk exposure.

We found theoretical support for providing *partial* social protection at the old age, through positive intergenerational transfers (from the young to the elderly) that are *partially* contingent on the stochastic labor productivity, i.e. full indexation of PAYG benefits and contribution bases to the stochastic labor productivity growth (by assumption), but negative (optimal) covariance between the rates of PAYG contribution and the labor efficiency. The indexation shifts risk from the young to the elderly, whereas the negative covariance of the contribution rates shifts risk in the opposite direction. The net effect is *partial contingency* of the PAYG benefits on the stochastic labor productivity. According to the model, partial social protection at the old age is optimal, without assuming that the elderly generation is more risk averse than the young. Further, it is a necessary condition for positive PAYG transfers when the economy the PAYG premium is positive.

The conditions for an interior solution show that, unless: households are ex ante relatively more risk exposed at the old age, the rate of growth of the labor force is positive and the indexed PAYG return is higher than the gross marginal rate of forgone savings due to the social security contribution; there is no interior solution for the expected optimal PAYG contribution. Moreover, the expected optimal rate of PAYG contribution is ceteris paribus decreasing in the expected PAYG premium, increasing in the rate of self-insurance and decreasing in the marginal propensity to save. The numerical analysis shows that the mps effect prevails upon the PAYG premium as both move in opposite direction as the interest rate varies.

We found also that the government may increase the saving rate in the equilibrium, based on an objective indication of the substitutability between the schemes (the difference in return rates), and the household desire of smoothing consumption over time (the positive rate of self-insurance). Since we abstracted from a funded, second-pillar, pension scheme; we interpret this result as an indication that such scheme is ex ante optimal next to the PAYG scheme, under additive quadratic preferences.

This paper solved the model for additive, non-stationary quadratic preferences, after controlling for common drawbacks of the quadratic function, in order to display (quasi) constant relative risk aversion across generations and rational choice. The next step in the analysis is to solve the model for CRRA preferences, in a subsequent paper. We expect that precautionary savings will be generated under CRRA preferences, due to the contingent PAYG scheme, and therefore additional information about the substitutability between PAYG and funded savings will be added to the model. Further, CRRA preferences provide a better control of the degree of risk aversion, and it may be helpful for conciliating the empirical evidence on attitudes towards risk and on the displacement of household savings by PAYG schemes.

Appendix

Box: The expected PAYG premium

In our stochastic, small open economy, from the perspective of the young alive at time t ; the present value of the expected PAYG premium is the sum of the changes in the current and in the expected discounted consumption, caused by marginal variations in the current and the expected PAYG rates of contribution:

$$\frac{\partial c_{1,t}}{\partial \theta_t} + \frac{\partial c_{1,t}}{\partial \theta} + \frac{1}{1+r} \left[\frac{\partial E_t [c_{2,t+1}]}{\partial \theta_t} + \frac{\partial E_t [c_{2,t+1}]}{\partial \theta} \right] = - \left[1 - \frac{1+\nu}{1+r} \right] w A_t,$$

where, $1 - \frac{1+\nu}{1+r}$ is the expected "golden rule" gap (ν is the rate of growth in the economy and r is the rate of interest); $c_{1,t}$ and $E_t [c_{2,t+1}]$ are derived in equations (11) and (13).

The expected rate of PAYG premium equals the expected "golden rule" gap, which is positive when the economy is strictly dynamic efficient on average ($\nu < r$).

In a central planned economy, intergenerational transfers are not supposed to be costly because we expect the economy to be at the "golden rule" ($r = \nu$), or at least at the "modified golden rule" ($1+r = (1+\nu)(1+\lambda)$), where $1+\lambda \geq 1$ is the rate of time preference (present over future) of the social planner. These are the points associated with maximum consumption and maximum discounted social welfare respectively, in the one-sector neoclassic growth model (see Blanchard and Fischer 1989, p. 45).

In the deterministic closed economy, Diamond (1965) showed that public debt reduces the capital stock in two ways: 1) by lowering the disposable income, and consequently funded savings, because of the debt service (interest payments on internal or external debt); and 2) by substituting debt for capital in the household portfolio (internal debt only). Further, the reduction of capital stock has feedback effects through lower wages that further reduce funded savings and the capital stock. Hence, since public debt reduces the rate of capital accumulation, it may be Pareto improving only when the economy is dynamic inefficient, i.e. when the economy is accumulating capital in excess. The conclusions in Diamond (1965) applies to the PAYG because, under certain conditions, the PAYG social security and *internal* debt are equivalent (see an example in Borgmann 2005, p. 8-11).

In small open economies, the interest rate and the capital stock are exogenously determined. Consequently, the PAYG social security has no consequences for the capital intensity.^a However, from a microeconomic perspective, there is a portfolio effect because the PAYG contribution represents compulsory savings with expected lower return when $\nu < r$, i.e. the social security claim is a dominated asset. In the absence of a risk-sharing motive, there are only corner solutions for the optimal PAYG contribution rates: 0 when $\nu < r$; or 1 otherwise, provided that the young generation can borrow on the account of the future PAYG benefit, according to the lower open-economy interest rate. In an existent PAYG scheme, although the young cannot opt out because of the implicit public debt, a positive PAYG premium can be seen as an opportunity cost that is given exogenously to the small open economy.

^aSee Persson (1985) for the extension of the OLG model in Diamond (1965) to a small open economy, and the current account adjustment following an one-time increase in the public debt. The current paper does not model the external equilibrium.

Box: Social welfare weights and dynamic consistency of the policy (1/2)

Suppose that the government objective function at time t is the ex ante ($_{-\infty}$) sum of the discounted lifetime utilities of generations $\tau = t, \dots, T - 1$:

$$E_{-\infty} [SW_t] = E_{-\infty} \left[\sum_{\tau=t}^{T-1} \lambda_{t,\tau} \{u(c_{1,\tau}^*) + \rho E_{\tau} [u(c_{2,\tau+1}^*)]\} \right]; \quad (a')$$

where, $E_{-\infty}$ denotes ex ante expectations; E_{τ} are the expectations in period τ ; $\lambda_{t,\tau} = \left(\frac{1}{1+\lambda}\right)^{\tau-t}$ is the *social* welfare weight assigned at time t to generation τ ; $\lambda \geq 0$ is the constant discount rate of the government; $\rho < 1$ is the *subjective* weight (future/present); $u(\cdot)$ is the instantaneous utility from consumption; $c_{1,\tau}^*$ and $c_{2,\tau+1}^*$ are the best household responses to the government policy.

The government chooses a sequence of PAYG rates of contribution ($\theta_t; \theta_{t+1} \dots \theta_{T-1}$) that maximizes the objective function in (a'), yielding the following first-order conditions:

$$\frac{\partial E_{-\infty} [SW_t]}{\partial \theta_t} = E_{-\infty} \left[u'(c_{1,t}^*) \frac{\partial c_{1,t}^*}{\partial \theta_t} + \rho E_t \left[u'(c_{2,t+1}^*) \frac{\partial c_{2,t+1}^*}{\partial \theta_t} \right] \right] = 0. \quad (b')$$

$$\frac{\partial E_{-\infty} [SW_t]}{\partial \theta_{\tau}} = E_{-\infty} \left[\begin{array}{l} \frac{\lambda_{t,\tau-1}}{\lambda_{t,\tau}} \rho E_{\tau-1} \left[u'(c_{2,\tau}^*) \frac{\partial c_{2,\tau}^*}{\partial \theta_{\tau}} \right] \\ + u'(c_{1,\tau}^*) \frac{\partial c_{1,\tau}^*}{\partial \theta_{\tau}} + \rho E_{\tau} \left[u'(c_{2,\tau+1}^*) \frac{\partial c_{2,\tau+1}^*}{\partial \theta_{\tau}} \right] \end{array} \right] = 0; \quad (c')$$

$$\tau = t + 1, \dots, T - 1.$$

- The intertemporal condition

Substituting for $u'(c_{1,t}^*)$, from the Euler equation (7) into (b'):

$$\frac{\partial E_{-\infty} [SW_t]}{\partial \theta_t} = \rho E_{-\infty} \left[\frac{T_2}{T_1} (1 + r_{t+1}) \frac{\partial c_{1,t}^*}{\partial \theta_t} E_t [u'(c_{2,t+1}^*)] + E_t \left[u'(c_{2,t+1}^*) \frac{\partial c_{2,t+1}^*}{\partial \theta_t} \right] \right] = 0.$$

Assuming that preferences are such that $\frac{\partial c_{2,t+1}^*}{\partial \theta_t}$ is known at time t although $c_{2,t+1}^* = f(A_{t+1}, \theta_{t+1}; A_t, \theta_t)$ and A_{t+1} and θ_{t+1} are unknown:

$$\frac{\partial E_{-\infty} [SW_t]}{\partial \theta_t} = \rho E_{-\infty} \left[\left(\frac{T_2}{T_1} (1 + r_{t+1}) \frac{\partial c_{1,t}^*}{\partial \theta_t} + \frac{\partial c_{2,t+1}^*}{\partial \theta_t} \right) E_t [u'(c_{2,t+1}^*)] \right] = 0.$$

Assuming that $\frac{\partial c_{1,t}^*}{\partial \theta_t}$ and $\frac{\partial c_{2,t+1}^*}{\partial \theta_t}$ are linear functions of $w_t A_t$ (the efficiency wage), it yields:

$$E_{-\infty} [E_t [u'(c_{2,t+1}^*)] A_t] = 0. \quad (b'')$$

which is the same first-order condition obtained in (15). ■

Box: **Social welfare weights and dynamic consistency of the policy** (2/2)

- The intergenerational conditions

Given, (i) the welfare weights are dynamically consistent: $\lambda_{t,\tau} = \left(\frac{1}{1+\lambda}\right)^{\tau-t}$ (Heijdra and van der Ploeg 2002, p. 629-631); (ii) three stationarity assumptions: "stochastic" steady state, *i.i.d.* and zero-mean shocks, and non-stationary preferences (constant optimal consumption path across generations in spite of positive growth in the "stochastic" steady state). The intertemporal condition in (b') holds for any τ , according to the assumptions in (ii). In addition, given (i) and since $\lambda \geq 0$, the intergenerational conditions in (c') reduce to:

$$E_{-\infty} \left[E_{\tau-1} \left[u' (c_{2,\tau}^*) \frac{\partial c_{2,\tau}^*}{\partial \theta_\tau} \right] \right] = 0 \quad \forall \tau.$$

Using the constant-covariance assumption in (10) and assuming that $\frac{\partial c_{2,\tau}^*}{\partial \theta_\tau}$ is a linear function of wA_τ (the efficiency wage of the next generation), and setting $\tau = t + 1$ yield:

$$E_{-\infty} [E_t [u' (c_{2,t+1}^*) A_{t+1}]] = 0. \quad (c'')$$

which is the same first-order condition obtained in (16). ■

Hence, under stationarity assumptions, the objective function in (a') can be reduced to that in (14) as they yield the same first-order conditions, and thus dynamic consistency of policy implies equal treatment of generations ($\lambda = 0$). This reduction is especially convenient when the government has infinite horizon ($T \rightarrow \infty$), as in the current paper, because when $\lambda = 0$ and $T \rightarrow \infty$ the sum in (a') does not converge.

Notice that $\frac{\partial c_{2,\tau}^*}{\partial \theta_\tau} > 0$ and, without assuming stationarity, the first-order conditions in (c') are for $\tau = t + 1, \dots, T - 1$:

$$E_{-\infty} \left[u' (c_{1,\tau}^*) \frac{\partial c_{1,\tau}^*}{\partial \theta_\tau} + \rho E_\tau \left[u' (c_{2,\tau+1}^*) \frac{\partial c_{2,\tau+1}^*}{\partial \theta_\tau} \right] \right] = -(1 + \lambda) \rho E_{-\infty} \left[E_\tau \left[u' (c_{2,\tau}^*) \frac{\partial c_{2,\tau}^*}{\partial \theta_\tau} \right] \right] < 0;$$

i.e. the ex ante marginal-lifetime-utility loss, due to the PAYG contribution of the young, at time τ , corresponds to the marginal utility gain of the elderly, adjusted by the social and subjective weights (λ and ρ). In contrast, under stationarity assumptions, the ex ante lifetime-marginal-utility loss is zero, according to (b').^a

^a A difference of the same nature, due to social discount, exists between the "golden rule" and the "modified golden rule" growth paths. Diamond (1965, p. 1128-1129) acknowledges that initial conditions and transition costs justify a steady state path with lower capital and consumption levels than the "golden rule". However, in the context of policy benchmark, De La Croix and Michel (2002, p. 91-92) recall that "(...) For Ramsey (1928), the optimal growth problem should not be discounted. (...)", and they show Ramsey's solution for the convergency problem. Blanchard and Fisher (1989, p. 102) refer to the arbitrariness of λ as a parameter that is "unrelated to preferences as captured by the individual utility function".

Diamond (1965, p. 1128) assumed the steady state, before defining the objective function of the central planner as the lifetime utility of a representative generation. The same approach is adopted in the current paper, but for an economy with an infinite-horizon government.

Summary of quadratic preferences

Functional form $u(c_{i,\tau}) = c_{i,\tau} - \frac{a_t}{2} c_{i,\tau}^2$; $c_{i,\tau} \in \{c_{1,t}; c_{2,t+1}\}$

Self insurance $si \equiv mpc - (1+r)mps$

The shares of consumption in the labor income $\frac{c_{1,t}}{wA_t} = mpc \left[1 - \theta_t + \frac{(1+\nu)\bar{\theta} + Cov[\theta_{t+1}, (1+\nu_{t+1})]}{1+r} \right] - (*)_1 \equiv [\cdot]_1$

$\frac{E_t[c_{t+1,2}]}{wA_t} = (1+r)mps \left[1 - \theta_t + \frac{(1+\nu)\bar{\theta} + Cov[\theta_{t+1}, (1+\nu_{t+1})]}{1+r} \right] + (*)_2 \equiv [\cdot]_2$

The parameter a_t

	a	$\frac{1}{wA_t}$	$\frac{1}{wA_t^2}$	$\frac{1}{w}$
$(*)_1 \equiv$	$\frac{si}{(1+r)awA_t}$	$\frac{si}{1+r}$	$\frac{si}{1+r}A_t$	$\frac{si}{(1+r)A_t}$
$(*)_2 \equiv$	$\frac{si}{awA_t}$	si	$si * A_t$	$\frac{si}{A_t}$

Risk aversion coefficients^{1,2}

$-\frac{u''(c_t)}{u'(c_t)} =$	$\frac{a}{1-a[\cdot]wA_t}$	$\frac{1}{\{1-[\cdot]\}wA_t}$	$\frac{1}{\left\{1-\frac{[\cdot]}{A_t}\right\}wA_t^2}$	$\frac{1}{\{1-[\cdot]A_t\}w}$
	\uparrow	\downarrow	\downarrow	\uparrow
$-\frac{u''(c_t)}{u'(c_t)}C_t =$	$\frac{a[\cdot]wA_t}{1-a[\cdot]wA_t}$	$\frac{[\cdot]}{1-[\cdot]}$	$\frac{[\cdot]}{A_t-[\cdot]}$	$\frac{[\cdot]A_t}{1-[\cdot]A_t}$
	\uparrow	$=$	\downarrow	\uparrow

Rational choice (laissez-faire: $\theta_t = 0 \forall t$)

$$u'(c_{t,1}) = 1 - ac_{t,1} \quad 1 - \frac{c_{t,1}}{wA_t} \quad 1 - \frac{c_{t,1}}{wA_t^2} \quad 1 - \frac{c_{t,1}}{w}$$

$$= \frac{mpc}{1+r} > 0$$

$$u'(E_t[c_{t+1,2}]) = 1 - aE_t[c_{t+1,2}] \quad 1 - \frac{E_t[c_{t+1,2}]}{wA_t} \quad 1 - \frac{E_t[c_{t+1,2}]}{wA_t^2} \quad 1 - \frac{E_t[c_{t+1,2}]}{w}$$

$$= mps > 0$$

¹ $[\cdot]$ the shares of consumption in the labor income.

² $\uparrow \downarrow =$ the directions of changes when A_t increases.

Table 1a: The equilibrium with quadratic preferences ($a_t = 1/wA_t$; $\rho = 0.3$; $T_2/T_1 = 15/35$)
Sensitivity analysis of the interest and growth rates

Interest and growth rates per period ($T_1 = 35$ years) annual				Expected PAYG premium	Marginal propensity to save	Ex-ante optimal Cov.	Soc. Sec. contr.	Saving rate PAYG	laissez -faire
$1 + r$	$1 + n$	$1 + g$	$1 + v$	$1 - \frac{1+\nu}{1+r}$	mps	$\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}$	$\bar{\theta}$	$\frac{s_t A_t}{w A_t}$	$\frac{s_t^f A_t}{w A_t}$
baseline									
3,33 1,035	1,1907 1,0050	1,1907 1,0050	1,42 1,010	0,575	0,41	-0,65	0,13	0,25	0,18
3,33 1,035	1,1907 1,0050	1,1500 1,0040	1,37 1,009	0,589	0,41	-0,50	-0,01	0,30	0,18
3,33 1,035	1,1907 1,0050	1,1702 1,0045	1,39 1,010	0,582	0,41	-0,57	0,05	0,28	0,18
3,33 1,035	1,1907 1,0050	1,2116 1,0055	1,44 1,011	0,567	0,41	-0,76	0,23	0,22	0,18
3,33 1,035	1,1907 1,0050	1,2329 1,0060	1,47 1,011	0,560	0,41	-0,90	0,37	0,16	0,18
3,33 1,035	1,1907 1,0050	1,2545 1,0065	1,49 1,012	0,552	0,41	-1,10	0,56	0,09	0,18
3,33 1,035	1,1500 1,0040	1,1907 1,0050	1,37 1,009	0,589	0,41	-0,50	-0,01	0,30	0,18
3,33 1,035	1,1702 1,0045	1,1907 1,0050	1,39 1,010	0,582	0,41	-0,57	0,06	0,28	0,18
3,33 1,035	1,2116 1,0055	1,1907 1,0050	1,44 1,011	0,567	0,41	-0,73	0,20	0,23	0,18
3,33 1,035	1,2329 1,0060	1,1907 1,0050	1,47 1,011	0,560	0,41	-0,82	0,28	0,20	0,18
3,33 1,035	1,2545 1,0065	1,1907 1,0050	1,49 1,012	0,552	0,41	-0,91	0,36	0,17	0,18
3,33 1,035	1,2765 1,0070	1,1907 1,0050	1,52 1,012	0,544	0,41	-1,00	0,45	0,14	0,18
2,37 1,025	1,1907 1,0050	1,1907 1,0050	1,42 1,010	0,403	0,58	-1,28	0,23	0,30	0,18
2,81 1,030	1,1907 1,0050	1,1907 1,0050	1,42 1,010	0,496	0,50	-0,80	0,08	0,32	0,18
3,95 1,040	1,1907 1,0050	1,1907 1,0050	1,42 1,010	0,641	0,33	-0,73	0,33	0,15	0,17
4,67 1,045	1,1907 1,0050	1,1907 1,0050	1,42 1,010	0,696	0,26	-1,88	1,57	-0,19	0,16
2,81 1,030	1,1907 1,0050	1,1500 1,0040	1,37 1,009	0,513	0,50	-0,62	-0,07	0,38	0,18
3,95 1,040	1,1907 1,0050	1,2329 1,0060	1,47 1,011	0,628	0,33	-1,16	0,76	0,02	0,17

Table 1b: The equilibrium with quadratic preferences ($a_t = 1/wA_t$)
Sensitivity analysis of the rate of time preference and the expected retirement length

Interest and growth rates		Rate	Expected	Marginal	Ex-ante optimal	Saving rate					
per period ($T_1 = 35$ years)		time	Retir.	propensity	Cov.	PAYG					
annual	$1 + g$	$1 + v$	length	to save	Soc. Sec.	laissez					
$1 + r$	$1 + n$	$1 + v$	T_1/T_2	mps	contr.	-faire					
		ρ	$1 - \frac{1+r}{1+r}$	mps	$\bar{\theta}$	$\frac{s_t A_t}{w A_t}$					
			$1 - \frac{1+r}{1+r}$	mps	$\frac{Cov[\theta_t, A_t]}{E_{-\infty}[A_t]}$	$\frac{s_t A_t}{w A_t}$					
						$(1 + r) mps$					
baseline											
3,33	1,19	1,42	0,3	0,43	0,57	0,41	-0,65	0,13	0,25	0,18	1,37
1,035	1,005	1,010		(15/35)							
3,33	1,19	1,19	1,42	0,3	1,00	0,57	0,23	-0,80	0,59	0,23	0,77
1,035	1,005	1,005	1,010								
3,95	1,19	1,19	1,42	0,3	1,00	0,64	0,18	-0,58	0,43	0,21	0,70
1,040	1,005	1,005	1,010								
5,52	1,19	1,19	1,42	0,3	1,00	0,74	0,10	-0,33	0,25	0,16	0,54
1,050	1,005	1,005	1,010								
3,33	1,19	1,19	1,42	0,7	0,43	0,57	0,23	-0,80	0,59	0,23	0,77
1,035	1,005	1,005	1,010								
3,33	1,19	1,19	1,42	0,3	0,57	0,57	0,34	-3,27	1,47	0,20	1,15
1,035	1,005	1,005	1,010		(20/35)						
3,33	1,19	1,19	1,42	0,3	0,29	0,57	0,51	-0,29	0,40	0,15	1,71
1,035	1,005	1,005	1,010		(10/35)						

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