A Theory of the Demand for Underwriting*

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Abstract

When a party seeks to pass a risk to another in a market transaction it typically must first provide information about the risk to the potential risk bearer, who determines whether to accept the risk and if so at what price. In the current paper, we examine the demand for underwriting and its effect on equilibrium in an insurance market. Assuming that risk type is known to insurance applicants but not to the insurer, we find sufficient conditions under which low risk buyers are willing to pay for underwriting to obtain greater coverage rather than choose self-selection partial coverage. Thus, underwriting tests serve as a device to increase low risk buyers’ insurance coverage. The results of our analysis indicate that underwriting equilibria would widely exist even under the condition where the separating equilibrium does not hold in the absence of underwriting. Our findings provide an explanation for why empirical studies on adverse selection in some insurance markets have reported evidence of adverse selection, while others have not. We also provide an explanation for why some insurance products are sold with limited or no underwriting.

Keywords: underwriting, insurance, adverse selection, information asymmetry

JEL Codes: D81, D82, G22
1 Introduction

Transfer of risk in a private market inevitably entails underwriting in some form. In practice, the ability to underwrite well is one of the cornerstones of a successful insurance operation. Poor risk selection, in contrast, can result in significant losses and insurer failure. While some forms of risk classification are prohibited by law, underwriting on the basis of race for example; insurers have significant leeway to incorporate information that they feel is necessary in risk selection and pricing.\footnote{Examples of underwriting include automobile insurance companies collecting information about an applicant’s driving history, age, gender, and marital status. In recent years, the utilization of genetic tests as an underwriting tool by life and health insurers has been widely discussed (e.g. Viswanathan et al., 2007).}

The purpose of this paper is to investigate the demand for underwriting and its impact on equilibrium in insurance markets. Particularly, we examine the conditions in which underwriting tests, which we define by the accuracy and the cost of the tests, lead to equilibrium where low-risk consumers obtain greater coverage than self-selection partial coverage. Thus, we argue that many underwriting techniques are financially advantageous to insureds whose costs of coverage are lowered by the exclusion of higher risks from insurance pools, and show that underwriting contributes to reducing adverse selection in insurance markets. To underscore the significance of underwriting techniques in insurance markets, it may be helpful to borrow the concluding remark in Dionne et al. (2001) who study the US auto insurance market:

*We showed ... that risk classification is sufficient, in the sense that there is no residual adverse selection on risk types in the automobile insurance portfolio studied.*

Prior literature has considered the bookends of our analysis. At one extreme, if an insurer’s underwriting has zero probability to reveal an insured’s true risk type, a Rothschild-Stiglitz (RS, hereafter) separating equilibrium may hold (Rothschild and Stiglitz, 1976). At the opposite extreme, perfectly accurate underwriting at zero cost results in a first-best equilibrium, full coverage for each risk type at actuarially fair premiums (e.g. Mossin, 1968). Between these extremes are the more likely actual market cases in which underwriting is neither costless nor perfectly accurate and self-selection does not perfectly reveal a consumer’s risk type. The cur-
rent paper focuses on market outcomes when underwriting is costly and imperfect. Even when
underwriting does not result in perfect classification, we show that it can reduce costs arising
from information asymmetry, thereby resulting in greater risk transfer and less subsidization of
high risks by low risks than would otherwise be the case.

Our model is different from prior studies on information acquisition (see, for instance,
Crocker and Snow, 1992; Doherty and Thistle, 1996; Hoy and Polborn, 2000) in several re-
spects. Most fundamentally, these studies assume that consumers are unaware of their risk
type ex ante and invest in learning their risk type. In contrast, consistent with the literature
on adverse selection, we assumes that individuals know their risk type, but insurers do not.

Second, we consider underwriting as a part of the process of insurance purchasing, such as
information gained through underwriting becomes known to both the potential purchaser and
the insurer. Thus, the test result is not private information held by the insurance purchaser.
This setting is consistent with the ordinary insurance purchasing process. For instance, when
a consumer wants to purchase life insurance, he or she typically needs to provide personal
information to an insurer to obtain a rate. Underwriting tests for life insurance often include
a medical examination conducted by a health professional. The results of the examination are
revealed to both the potential purchaser of insurance and the insurer.

The assumption that underwriting test results are not private information of the insurance
purchaser sets this study apart from earlier work by Doherty and Thistle (1996) and Hoy and
Polborn (2000). Their work assumes that tests, such as genetic tests, are available independent
from the purchase of insurance and that insurers do not have access to the test results.

In contrast to prior signaling models, which entail closing informational disparities through
strategic decision making and which are based on agents facing differentials in signaling costs
linked to their risk, we consider flat underwriting fee across risk types, which are widely applied
in practice.² For instance, the cost of a medical test for life insurance is likely to be identical
for all purchasers.

²Potential subsidization costs paid by low risk individuals due to imperfect underwriting tests vary with
coverage.
Our analysis adds to the body of literature on insurance market equilibrium when information is asymmetric and transactions costs are nonzero. These studies include Allard et al. (1997), which investigates the impact of constant cost sharing on the existence of equilibrium, and Liu and Browne (2007), which considers proportional costs with risk heterogeneity. In these studies, transaction costs are modeled as independent of risk classification. In contrast, we examine the impact of underwriting costs that directly affect information asymmetry.

Although we focus on the example of insurance, our results are generalizable to the market transfer of risk in other contexts. Examples, of which there are many - including home mortgages and the hiring of employees - are characterized by information transfer prior to risk transfer and the cost of information transfer being borne by the party transferring the risk.

The remainder of the article is organized as follows. In Section 2, a brief overview of the literature on adverse selection is presented. Our approach to modeling the insurance market is presented in Section 3. In Section 4 we consider equilibrium when underwriting is perfectly accurate and entails a positive cost. The case of an imperfect underwriting test is introduced in Section 5. A summary of our findings and a discussion of limitations in our work are contained in Section 6.

2 Adverse Selection and Underwriting

The theoretical prediction of the Rothschild and Stiglitz (1976) self-selection model is straightforward; if an insurance policy is fully described by two dimensions, the premium and the quantity of coverage, high risk insurance buyers demand more than low risk buyers do. Presumably the demand differentials between high-risk and low-risk buyers should be observable in the market. Regardless of the strong theoretical prediction, empirical research on the existence of adverse selection in insurance markets is mixed. Studies that do not find evidence that high risks purchase greater amounts of insurance coverage include Cawley and Philipson (1999) for the US life insurance market; Chiappori and Salanié (2000) for the French auto insurance market; Cardon and Hendel (2001) for the U.S. health insurance market; and Saito
(2006) for the Japanese auto insurance market. On the other hand, Finkelstein and Poterba (2002, 2004) find evidence of adverse selection in the UK annuity market.\(^3\) Browne (1992) and Browne and Doerpinghaus (1993) find evidence of adverse selection in the U.S. market for non-group health insurance, and Browne and Doerpinghaus (1994) report empirical results suggesting that informational asymmetries exist in the Medigap insurance market. We suspect that the mixed results of the empirical studies on adverse selection may at least partially be explained by differences in the demand, availability, and cost of underwriting techniques in different markets.

Although the benefit of underwriting in reducing or eliminating adverse selection is evident, the amount of underwriting that occurs in insurance markets varies considerably. One reason for this is that underwriting is costly. National Association of Insurance Commissioners (NAIC) financial statement data on insurance company operations indicate that between 2002 and 2006 the median property and casualty insurer in the U.S. incurred twenty-six cents per premium dollar in underwriting expenses.\(^4\) This suggests that insureds greatly value underwriting as they ultimately pay the costs associated with it. On the other hand, Finkelstein and Poterba (2002), who find adverse selection in the UK annuity market, observe that annuity providers use limited information when underwriting annuities.\(^5\) This contrast suggests that in some markets, but not all, insureds are willing to pay for underwriting.

### 3 Demand and Supply of Insurance Coverage

The insurance market is assumed to be competitive and to consist of three types of players: insurers, buyers (also referred to as applicants) of insurance, and underwriters. The underwriter is set apart from the insurer in our model not to investigate the role of insurance market intermediaries in reducing information asymmetry but to emphasize the role of underwriting

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\(^3\)Puelz and Snow (1994) offer some evidence of adverse selection in the US auto insurance market, but Dionne et al. (2001) show the possibility that their analysis could be misleading.

\(^4\)Median of the ratio of total underwriting expense to premium earned in 2002-2006, NAIC financial statement data. Note that this includes costs that are not directly associated with risk discrimination.

\(^5\)Prices for coverage vary by gender, type of product, market type (compulsory or voluntary), and payment structure.
in insurance transactions. In our model the insurers essentially outsources underwriting to a separate firm. Our results do not differ if the insurer retains the underwriting function. The action space of each player can be characterized as follows:

- **Insurer:** All insurers are homogeneous and behave as if risk-neutral. Each insurer offers only one policy type expressed by two dimensions: coverage quantity, $q$, and the premium, $p$. The offer can be unconditional or depend on the result of an underwriting test. The insurance market is sufficiently large so that an insurer cannot infer the risk type of any applicant from underwriting information that has been obtained from prior applicants.

- **Buyer:** The risk-averse buyer applies for one policy. If it applies for a policy with an unconditional offer, then the buyer receives that amount of coverage. If it applies for a policy that requires taking an underwriting test, the buyer takes the test and pays the associated fee to the underwriter. The result of the test determines the policy which the buyer purchases.

- **Underwriter:** Homogeneous underwriters offer an identical underwriting test characterized by its accuracy, $y$, and its cost, $c$. That is, $U(y, c)$ where $y \in (0, 1]$ and $c \in (0, \infty)$. All market participants are assumed to know the cost of underwriting and its accuracy. An underwriter sets the price at the marginal cost, administers the test, and reports the result of the test to the insurer and buyer.

Assume that individual buyers differs only in their probability of loss. The probability of loss for a high risk (HR), $\pi_H$, and that for a low risk (LR), $\pi_L$, are fixed and $0 < \pi_L < \pi_H < 1$. The proportion of HR buyers and that of LR buyers are denoted by $\lambda$ and $(1 - \lambda)$, respectively. It is assumed that individuals are informed of their own risk types, and the distribution of the risk

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6Although agents and brokers may play an important role in underwriting, our primary interest is underwriting typically conducted by the insurer itself. Underwriting is defined as the process by which an insurer acquires information about an applicant for insurance and uses that information to determine the applicant’s risk type.

7This reporting assumption is different from Doherty and Thistle (1996) and Hoy and Polborn (2000) in that they analyze the cases where insurers cannot observe the test result. We assume that the test results are reported to the insurer once a buyer takes a test.
types in the population is common knowledge. Insurers cannot distinguish between individuals. The expected utility that a HR derives from policy $L = (q_L, p_L)$ is denoted by $V_H(L)$, and the level of utility attained by exchanging premium $p$ for coverage $q$ with an insurer is written as:

$$V_H(L) = \pi_H u(W - p_L + q_L - D) + (1 - \pi_H) u(W - p_L) \quad (1)$$

where $W > 0$ and $D > 0$ denote the buyer’s initial wealth and fixed loss amount, respectively. The utility function, $u$, has the standard risk-averse conditions, $u' > 0$ and $u'' < 0$. The optimal demand for coverage is determined by maximizing the objective function.$^8$

We consider a sequential game in which a new firm decides to enter the existing insurance market by offering policies. When buyers maximize their expected utility, Nash equilibrium identified in this game can be characterized by no policy in the equilibrium making a loss and no policy outside the equilibrium makes a non-negative profit if offered.

Without underwriting, the well-known RS separating equilibrium identified as a set of policies $(H, L)$ may be attained (see Figure 1). If the equilibrium exists, HR buyers acquire full coverage, while LR buyers obtain only partial coverage. However, the separating equilibrium does not exist, if and only if, there is a profitable pooling policy that can attract both HR and LR buyers. This is the case when the population of LRs is sufficiently large relative to that of HRs as pointed out in Rothschild and Stiglitz (1976). A hashed line in Figure 1 illustrates the existence of possible pooling policies.

[Insert Figure 1 here]

Thus, in order for policies with underwriting to be a Nash equilibrium, they must first dominate self-selection separating policies in a market characterized by a separating equilibrium or pooling policies in a market with no equilibrium. Therefore, we utilize those policies as benchmarks for our analysis. One natural concern is the identification of the specific pooling policy we refer to. Since pooling policies cannot establish an equilibrium, there is no unique

$^8$No other risks such as background risk and performance risk are assumed.
pooling policy to represent the possibility of a pooling policy being offered by insurers. However, it is still possible to identify a unique policy with which LR buyers attain the greatest utility among potential pooling policies if it is offered. The policy provides a robust benchmark in that, if a conditional policy that dominates the benchmark pooling policy is offered, any new pooling policy offer makes a loss, thus no pooling policy is offered. The benefit of the benchmarking is to prevent insurers from offering a pooling policy which causes no equilibrium.

An equilibrium with underwriting must satisfy four constraints.

- **HR Participation Constraint**: The expected utility that a HR derives with underwriting must be at least as great as the utility that a HR would derive in the absence of underwriting.

- **LR Participation Constraint**: The expected utility that a LR derives with underwriting must be at least as great as the utility that a LR would derive in the absence of underwriting.

- **Insurer Zero Profit Constraint**: The insurer must earn zero profits due to market competition.

- **No Cream Skimming Constraint**: No new self-selection policy would attract only LR buyer when underwriting policies are offered.

### 4 Perfect Underwriting Test with Positive Fee

We start with the case of a positive underwriting fee for a perfect underwriting test. Underwriters offer a test at a positive cost $c$. The test reveals whether an applicant is a LR or a HR with 100% accuracy. With the underwriting test, fair-odds lines are shifted by the positive intercept $c$ represented by $p_{hc}$ and $p_{lc}$ in Figure 2:

$$p_{ic} = c + \pi_i q_{ic}, i = H, L$$  

(2)
Obviously, HR buyers have no incentive to take the test because it will lead to their being classified accurately as a HR. Thus, the HR participation constraint is non-binding. The underwriting fee could make LR worse off. However, due to the loss of utility a LR incurs by not being able to obtain full coverage without underwriting, it may be possible for LR buyers to be willing to pay additional premium if the benefit of additional coverage exceeds the cost of the fee.

First, we identify conditions where policies with underwriting dominate the self-selection separating policies. The participation constraint for LRs is defined as:

$$V_L(L_c) \geq V_L(L)$$  \hspace{1cm} (3)

where the left hand side is denoted as the LR’s expected utility with full coverage with a positive underwriting cost, $L_c = (q_{Lc}, p_{Lc})$. Explicitly expressed,

$$\pi_L u(W - p_{Lc} + q_{Lc} - D) + (1 - \pi_L)u(W - p_{Lc}) \\
\geq \pi_L u(W - p_L + q_L - D) + (1 - \pi_L)u(W - p_L)$$  \hspace{1cm} (4)

where $p_{Lc} = c + \pi_L q_{Lc}$. Note that the utilities for LRs are still maximized at $q=D$ because a fixed cost does not affect insurance demand.\(^9\) Knowing that LR buyers purchase full coverage, we reduce the LR’s participation constraint to $c \leq \mu_L$ where $\mu_L$ is risk premium defined by $u(W - \pi_L D - \mu_L) = V_L(L)$. The LR’s participation constraint implies that LR buyers seek full coverage $L_c$ by utilizing the underwriting test if the underwriting fee is no more than the LR’s risk premium.

When a pooling policy attracts both risk types, the coverage of our benchmark pooling policy, denoted by $M$, is determined by maximizing LR’s expected utility subject to the actuarially

\(^9\)The optimization problem is defined as $\max_q V_L(L_c)$ subject to $q_c \geq 0$ and $q_c \leq D$
fair premium for the pooling policies:

\[ V_L(M) = \pi_L u(W - p_M + q_M - D) + (1 - \pi_L) u(W - p_M) \]  

(5)

where \( p_M = [\lambda \pi_H + (1 - \lambda) \pi_L] q_M \). The corresponding risk premium for LR, denoted as \( \mu_{ML} \), is defined by \( u(W - \pi_L D - \mu_{ML}) = V_L(M) \).

Thus, as long as a perfect classification is possible, the LR buyer’s participation constraint can be reduced to the association between risk premiums and the underwriting fee. The LR will pay the fee for underwriting to reveal their risk type if and only if the underwriting fee is no more than the risk premium. If there is a pooling policy that LRs prefer to the separating policy, LR applicants are willing to pay up to the risk premium identified by the benchmark pooling policy for the underwriting test. Otherwise, they will choose the pooling policy, and equilibrium will fail to exist. Thus, the upper bound on the underwriting fee depends on the underlying market. Regardless of whether a separating equilibrium or a no equilibrium result holds in the market, LR will pursue underwriting if its cost is less than the benefit they can derive through revealing their risk type. Therefore, we can unify the conditions to be satisfied as follows:

\[ c \leq \min \mu^* \]  

(6)

where \( \mu^* = \{\mu_L, \mu_{ML}\} \). This is a robust condition for LR buyers’ demand for underwriting in terms of the underlying market.

In the case when an underwriting test is perfectly accurate, the underwriting policy chosen by a LR always satisfies the no cream skimming constraint. Note that HR buyers do not choose the conditional policy because of the perfect revelation of their risk type and rather choose their self-selection policy, \( H \). Therefore, any new unconditional policy which attracts LR buyers would also attract HR buyers, and thereby results in an immediate loss.

As a result, the separating equilibrium is \((H, L_c)\) where LR buyers opt for the underwriting test if the cost of the underwriting test is less than the risk premium corresponding to the underlying market. Otherwise, the underwriting test does not have any impact on the RS...
separating equilibrium and no Nash equilibrium for pooling policies. Thus, this simple setting shows that underwriting tests in the market allow LR buyers to obtain more complete coverage in return for the fee paid for underwriting.

5 Imperfect Underwriting Test

Underwriting techniques employed in practice are typically imperfect in that a HR may be classified as a LR and a LR may be classified as a HR. To investigate the effects of this imperfection in a simple manner, the underwriting test is now assumed to reveal a HR applicant’s true risk type with less than 100% accuracy, while a LR buyer is always revealed as a LR.\footnote{Relaxing this assumption to allow the test to identify wrongly a LR as a HR does not change the nature of our analysis. Details are available from the author.} Note that accuracy is defined as probability $y$. The test correctly identifies a HR buyer as a HR with probability $y$; however, with probability $(1-y)$ the test identifies a HR buyer as a LR.

We now consider the case of a positive underwriting fee for an imperfect underwriting test.\footnote{It is assumed that all insurance applicants who took the underwriting test purchase the conditional policy. Thus, all applicants pay the fee for the underwriting test. We also examined the case in which some insurance applicants who took the underwriting test decide not to purchase the conditional policy after observing the test result. Although the discussion is not reported in this paper, the result is available upon request.} While both HR buyers and LR buyers always utilize imperfect underwriting when it is costless, they may or may not purchase the underwriting test when a positive cost is associated with it. Therefore, there are four possibilities to be considered; 1) Both HR and LR buyers demand underwriting; 2) Only LR buyers demand underwriting; 3) Only HR buyers demand underwriting; and 4) Neither HR nor LR buyers demand underwriting. The cases in which LRs do not demand underwriting have no underwriting equilibrium due to a lack of fund for subsidy. The remaining two cases are discussed below.

First, the case where only LR buyers demand underwriting test is investigated because it is analogous to a perfect underwriting in that perfect classification is attainable. Therefore, both HR buyers and LR buyers demand a full coverage. In Section 5.2, we examine the case where both HR buyers and LR buyers demand underwriting test, which results in a partial coverage for a pooling policy.
5.1 Only LRs Demand Underwriting

First we consider the HR buyer’s constraint, $EV_{Hc} < V_H$, with which HR buyers are willing to take its unconditional separating policy, $H$. It is explicitly expressed as follows:

$$y[\pi_H u(W - p_{Hc} + q_{Hc} - D) + (1 - \pi_H) u(W - p_{Hc})]$$

$$+ (1 - y)[\pi_H u(W - p_{Ls} + q_{Ls} - D) + (1 - \pi_H) u(W - p_{Ls})]$$

$$< \pi_H u(W - p_H + q_H - D) + (1 - \pi_H) u(W - p_H)$$

(7)

where $p_{Hc} = c + \pi_H q_{Hc}$ and $p_{Ls} = c + \pi_{Ls} q_{Ls}$. The subsidy for a HR buyer identified as a LR (HL hereafter) is indicated by the subscript $s$.\(^{12}\) The left hand side of the inequality is the HR’s expected utility when the HR demands the underwriting test, and the right hand side is the expected utility when the HR chooses the self-selection policy. Knowing $q_H = q_{Hc} = D$, we restate the inequality by:

$$\mu_{Hc}(c, y) > 0$$

(8)

where $\mu_{Hc}$ is the HR buyer’s risk premium defined by $u(W - \pi_H D - \mu_{Hc}) = EV_{Hc}$.

This inequality states that HR buyers do not demand the underwriting if it is an unfair bet. Although this HR’s constraint is identified by the unconditional separating policy, $H$, it is also sufficient for the cases in which pooling policies attract LR buyers without underwriting. Since any new unconditional pooling policy results in a loss when conditional policies satisfying the LR’s participation constraint are offered, HR buyers do not deviate from their separating policy. Thus, when LR buyers demand policies with underwriting test, HR buyers do not demand the test if it is an unfair bet.

Under the condition where HR applicants do not utilize the underwriting test due to the large cost relative to the accuracy, LR buyers do not need to subsidize HL buyers. Hence, the no cream skimming constraint is satisfied as before. If any new unconditional policy that attracts LR buyers from an existing conditional policy is offered, it also attracts HR buyers and causes an immediate loss for an insurer.

\(^{12}\)The detail regarding the subsidy is discussed in Lemma 5.2.1.
Proposition 5.1.1. \((H, L_c)\) is a full coverage separating equilibrium where only LR applicants demand underwriting test, \(U=(y, c)\), if both (6) and (8) are satisfied.

In spite of the simplicity of this result, the implication of underwriting demanded by only LR applicants is not trivial. For example, when underwriters can offer a highly accurate underwriting test such as a genetic test, this type of market equilibrium is likely to exist. Because of the high accuracy, HR buyers do not take the underwriting test. The existence of such tests makes it possible to attain perfect risk classification.

In practice, the coexistence of an unconditional policy for HRs and a conditional policy for LRs can be observed in life insurance markets. While ordinary personal life insurance tends to require a medical test, which allows low risk individuals to purchase a policy at an appropriate rate given their risk, there are insurers who sell policies requiring very little or no underwriting. Examples include credit life policies and some policies marketed to seniors.\(^{13}\)

In this subsection, we investigated the conditions under which only the HR applicants demand the underwriting test. Generally, such an equilibrium is likely to exist when the underwriting test is considerably accurate. In contrast, when the underwriting test is not too accurate and the fee is also not too expensive, the next case where both the HR and LR buyers demand underwriting test is likely to occur.

5.2 Both HR and LR Demand Underwriting

In this section, first we identify the HR buyer’s participation constraint. Then Lemma 5.2.1 establishes necessary and sufficient conditions for LRs to be willing to pool with misclassified HRs in a contract that earns zero profit, and Lemma 5.2.2 addresses the condition needed for the pool to be resistant to cream skimming.

The HR’s participation constraint is defined as \(EV_{Hc} \geq V_H\), and is reduced to:

\[
\mu_{Hc}(c, y) \leq 0 \tag{9}
\]

\(^{13}\)See, for instance, http://law.freeadvice.com/insurance_law/life_insurance_law/life_insurance_medical_questions.htm and http://www.onedollarglobeinsurance.com/seniorlifeinsurance
This inequality states that HR buyers require the underwriting to be at least a fair bet. Otherwise, HR buyers prefer an unconditional policy.

**Lemma 5.2.1.** When underwriters offer a test, $U=(y, c)$, LR buyers are willing to subsidize HL buyers by accepting an additional subsidization rate, $s$, sufficient for the insurer to earn a zero profit if and only if there exists a conditional policy $A = (q_A, p_A)$ such that

$$V_L(A) \geq \max V^*$$

where $V^* = \{V_L(L), V_L(M)\}$ and

$$p_A = c + q_A[\pi_L + \lambda(\pi_H - \pi_L)\frac{1-y}{1-\lambda y}]$$

**Proof.** Inequality (10) is the LR participation constraint that requires LR buyers not be worse off due to subsidization. To justify (11), the insurer zero profit constraint, we consider the condition that the total premium must be equal to the expected claim payment:

$$(1 - \lambda)\pi_Ls q_A + \lambda (1 - y)\pi_L s q_A$$

$$= (1 - \lambda)\pi_L q_A + \lambda (1 - y)\pi_H q_A$$

where $\pi_L s = \pi_L + s$. The subsidy rate is proportional to coverage because the subsidy paid to HLs by LRs depends on the optimal coverage for the pooling policy. Then, it can be rewritten as:

$$s = \lambda(\pi_H - \pi_L)\frac{1-y}{1-\lambda y}.$$  

The optimal coverage for the conditional pooling policy is determined where the LR’s expected utility with premium (11) is maximized.

Let $q_{Lsc}$ represent the optimal coverage underwriting policy for LRs and HLs that includes the subsidization cost and the underwriting fee. In order for the set of policies $(H_c, L_{sc})$ to
be a semi-separating equilibrium where both HR and LR applicants demand the imperfect underwriting test, the policies must satisfy the following no cream skimming constraint to avoid nonexistence of the equilibrium.

Let $EV_{Hc}$ represent the HR’s expected utility and $V_L(L_{sc})$ be LR’s utility when both HR and LR buyers demand underwriting. Figure 3 illustrates the cases where a new unconditional policy $K$ attracts only LR buyers and achieves a non-negative profit given that the set of policies $(H_c, L_{sc})$ is offered.

[Insert Figure 3 here]

Graphically, the constraint is satisfied if the HR’s indifference curve is located above the LR’s indifference curve between $p_L$ and $p_{Lsc}$ (See Figure 4). This constraint requires a certain level of accuracy for the underwriting test to restrict the subsidy burden on LR buyers. Intuitively, if the HRs derive too much benefit from imperfect underwriting, a new unconditional policy $K$ is likely to exist and an equilibrium with underwriting is not possible. This no cream skimming constraint is formalized as follows:

[Insert Figure 4 here]

Lemma 5.2.2. Given the set of policies $(H_c, L_{sc})$ is offered, there is no new unconditional policy that achieves a non-negative profit if and only if:

$$q_{L^*} \leq q_{L^{**}}$$

where $q_{L^*}$ is defined by $EV_{Hc} = V_H(L^*)$ and $L^* = (q_{L^*}, \pi_{L^*} L^*)$, and $q_{L^{**}}$ is defined by $V_L(L_{sc}) = V_L(L^{**})$ and $L^{**} = (q_{L^{**}}, \pi_{L^*} L^{**})$.

Proof. Suppose that $q_{L^{**}} < q_{L^*}$. Then, there exists a policy $K$ such that $q_{L^{**}} < q_K < q_{L^*}$, to which only LR buyers are willing to deviate from $L_{sc}$. Thus, a new policy $K$ attains a non-negative profit. For the necessary condition to hold, policies providing a greater level of utility than $L_{sc}$ to LR buyers attract HR buyers, implying $q_{L^{**}} < q_{L^*}$. □
Proposition 5.2.3. \( (H_c, L_sc) \) is a semi-separating equilibrium where both HR and LR applicants demand an underwriting test, \( U=(y, c) \), if conditions (9), (10), and (14) are satisfied.

Proof. This is obvious from the lemmas.

Implication for an increase of the LR buyers’ demand for coverage can be easily observed. As shown in (11), the premium of the conditional policy includes both a fixed underwriting fee, \( c \), and a subsidy rate, \( s \), proportional to the coverage. With regard to the buyer’s optimal choice of coverage given a constant underwriting fee, the additional subsidy rate inversely affects the demand for coverage. Note that the slope of the fair-odds line for the unconditional pooling policy can be rewritten as \( \pi_M = \pi_L + \lambda(\pi_H - \pi_L) \) and the additional slope, \( \lambda(\pi_H - \pi_L) \), represents the subsidy rate paid by LR buyers. Analogously, for the conditional policy, corresponding subsidy rate is represented by \( \lambda(\pi_H - \pi_L) \frac{1}{1-\lambda y} \). Thus, the fraction component captures the association with the unconditional pooling policy coverage.

Since we assume a positive proportion of HR, \( \lambda > 0 \), and the imperfect underwriting test, \( y \in (0, 1) \), then \( \frac{1-y}{1-\lambda y} \in (0, 1) \). And it is obvious that the accuracy of the test, \( y \), is inversely associated with the ratio. For instance, as \( y \rightarrow 1 \), the fraction converges to zero. That is, a perfectly accurate test reduces the subsidy rate to zero, implying a full coverage. In contrast, as \( y \rightarrow 0 \), the fraction converges to one, corresponding to our benchmark unconditional pooling coverage. Figure 5 illustrates the case that underwriting test, if it is demanded, allows LR buyers to demand more than the unconditional pooling policy coverage.

Thus, LR’s demand could significantly increase by an introduction of underwriting test, especially when the underlying market is characterized by separating policies without underwriting test, (see Appendix A for the numerical example). Another aspect of the effect on coverage is that HR buyers identified as a LR reduce their coverage due to the pooling policy. This also contribute to a reduction of coverage demand differential between LR and HR buyers.

[Insert Figure 5 here]

Furthermore, we argue that underwriting equilibrium concept offers a potential resolution for nonexistence of the Nash equilibrium (Rothschild and Stiglitz, 1976). As discussed in Lemma
5.2.2, the conditional policy, $L_{sc}$, satisfying the no cream skimming constraint can avoid any new unconditional policy offer which attracts only LR buyers. The constraint can be generally satisfied with simply more accurate underwriting tests than the LR’s participation constraint requires.

6 Conclusion

Underwriting is a process that allows insurers to classify risks and price them accordingly. Underwriting results in higher insurance prices for those classified as high risks and lower prices for low risks. Rothschild and Stiglitz (1976) have found that in the absence of underwriting, asymmetric information results in LR obtaining less insurance coverage than they would in a full information market. This paper explicitly examines when underwriting is demanded and how it effects the demand for insurance. We identify sufficient conditions with which the coverage demand differential between HR and LR is significantly reduced when an underwriting test is offered. Our analysis can be categorized into the following distinct cases.

Consider the cases that only LR buyers demand underwriting. This case tend to arise when an underwriter offers a highly accurate underwriting test, and a notable finding is that an imperfect underwriting test taken only by LRs can result in an equilibrium with perfect classification. This results if the underwriting fee induces HR to reveal their risk type by not taking the test. For instance, we observe that life insurance markets for senior applicants show a congruence with this type of equilibrium. While there are insurers that sell senior individuals conditional policies, those who believe they are HR are attracted by unconditional policies that can be typically purchased through direct marketing. Thus, insurers selling unconditional policies to HRs and those marketing conditional policies to LRs coexist in the market.

We also examined the cases where both HR and LR demand underwriting. When the proportion of HR buyers is relatively large, our results implies that LR buyer’s demand for coverage could be significantly increased in equilibrium when a relatively accurate underwriting test is available. While a large proportion of HR buyers in a market allows for a relatively
large underwriting fee for LR buyers, it also increases potential subsidization costs for LR buyers due to misclassification resulting from an imperfect underwriting test. Thus, in order for an underwriting equilibrium to exist, an accurate underwriting test, which reduces the subsidization costs for LRs, must be offered. In contrast, when the proportion of HR buyers is relatively small, the maximum fee for an underwriting test is relatively restricted due to the potential pooling policy in the absence of underwriting. Established underwriting techniques may be used to classify risk types at a reasonable fee.

In this paper, we show that the existence of underwriting tests serves as a devise to increase insurance coverage for LR buyers, and it may partially explain why empirical findings on adverse selection are mixed. One explanation is that the availability and the desirability of underwriting tests are quite different from one type of insurance market to another. As discussed earlier, the UK annuity market may be one of the extreme examples of a market where underwriting is not desired (e.g. Finkelstein and Poterba, 2002, 2004). The health insurance market for elderly with a relatively large proportion of HR buyers seems to be another instance of a no underwriting equilibrium (e.g. Browne and Doerpinghaus, 1994). The large population of HR buyers suggests the possibility of a separating equilibrium. The underwriting costs would be significant if many HRs took the underwriting test hoping to be misclassified. Our model predicts that underwriting is not demanded in such a market unless a highly accurate underwriting test is offered.

In summary, the demand for underwriting is affected by factors including the cost and accuracy of the test as well as the ratio of HR buyers to LR buyers in the market. Our model predicts that the demand for insurance increases as a consequence of underwriting when inexpensive and accurate underwriting tests are available in the market.

We have provided anecdotal support for our theoretical findings. Empirical tests of our results are warranted.

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14 Other factors such as the degree of risk-aversion, initial wealth and loss amount also affects the cost of partial coverage and the demand for underwriting consequently.
Appendix

A. Numerical Illustration

Suppose that the probability of loss for LR buyers is 5% and that for HR buyers is 10%. Further, assume that 30% of buyers are HR. All applicants have $200,000 of initial wealth, a potential loss of $100,000, and an identical exponential utility function, \( u(w) = -\alpha e^{-\alpha w} \) where the parameter \( \alpha \) is 0.00001. Figure 6 shows that potential coverage that LR and HL buyers obtain when both LR and HR buyers take underwriting test and the equilibrium exists. The minimum coverage, $72,172, at underwriting accuracy \( y=0 \) corresponds to the coverage of our benchmark pooling policy, and the coverage increases as the accuracy is improved. Without underwriting test, a separating policy only offers $39,165 and dominates potential pooling policies. Thus, underwriting test could significantly increase coverage for LR buyers.

[Insert Figure 6 here]

We next consider the maximum underwriting fee that makes an equilibrium attainable.\(^{15}\) LR buyers will not take a test with a positive fee if the accuracy of the underwriting test is less than 27% (see LR participation in Figure 7) due to the subsidization costs they would incur. Given that the accuracy is at least 27%, the maximum underwriting fee that LR buyers are willing to pay increases as the accuracy of the test increases. In contrast, the HR participation constraint (see HR participation in Figure 7) has a downward slope, and it crosses with the LR’s constraint at 83%. Thus, beyond 83% accuracy for both HR buyers and LR buyers to demand underwriting, the maximum payable underwriting fee must decrease as accuracy increases. This is because greater accuracy reduces the benefit to HRs of gambling in underwriting.

[Insert Figure 7 here]

\(^{15}\)Only when constant absolute risk aversion is assumed, optimal coverage and the maximum underwriting fee can be separately identified.
To be part of an equilibrium, a policy must satisfy no cream skimming constraint as well. In Figure 7, an underwriting test must lie below the constraint curve, which imposes at least 62% accuracy. Thus, semi-separating policies \( (H_c, L_{sc}) \) where both LR and HR demand the underwriting test can be an equilibrium corresponding to Proposition 5.2.3, if underwriters offer a test for a positive fee that falls below both the no cream skimming constraint and the HR participation constraint in Figure 7. For instance, if underwriting test with 80% accuracy \( (y=0.8) \) is available in the market, the conditional policy, \( L_{sc} \), attains the coverage, \( q_{sc} = $91,985 \), which is increased by $52,820 (135\%) from the coverage of LR buyer’s unconditional separating policy.
References


Figure 1: Existence of the RS Separating Equilibrium

Figure 2: Perfect Underwriting Test with Underwriting Fee
Figure 3: Existence of Unconditional Policies that Attract only LRs

Figure 4: Non-Existence of Unconditional Policies that Attract only LRs

Figure 5: Partial Coverage Attained with Imperfect Underwriting Test
Figure 6: Potential Coverage Obtained by LR buyers

Figure 7: Accuracy of Underwriting Test and the Acceptable Fee ($\lambda=0.3$)