Fraudulent Claims and Nitpicky Insurers

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Abstract

Insurance fraud is a major source of inefficiency in insurance markets. A self-justification of fraudulent behavior is that insurers are bad payers who start nitpicking if an opportunity arises, even in circumstances where the good-faith of the policyholders is not in dispute. We relate this nitpicking activity to the inability of insurers to commit to an auditing policy. Reducing the indemnity payments acts as an incentive device for the insurer since auditing is profitable even if the claim is not fraudulent. We show that while nitpicking is not optimal when insurers are able to commit to their auditing strategy, it is optimal when they are unable to do so, which explains the prevalence of nitpicking in insurance markets. As optimal cut levels are bounded above, the insurer must sometimes refrain his possibility to reduce the indemnity payment below what is legally possible. We also investigate the reputation problem assuming policyholders incur a morale cost defrauding the insurer that decreases with the intensity of the insurer’s nitpicking activity. We show that despite this induced morale effect, nitpicking remains optimal.

1 Introduction

Insurance fraud is widely considered to be a major source of inefficiency in insurance markets. Insurance fraud may be hard, for instance when someone

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fakes an accident or a theft or commits arson to collect money illegally from insurance companies. It may be soft when opportunistic people tell “little white lies” to their insurer because “everybody does it.” It takes the form of claims fraud when policyholders invent fictitious claims or exaggerate claims, but insurance fraud also includes underwriting fraud when insurance seekers misrepresent their risk exposure or do not disclose relevant information about their risk. Although it is a recognized fact that insurance fraud costs insurance companies and tax payers billions of dollars every year, it is indeed striking to observe how insurance defrauders (at least those involved in soft fraud) often do not perceive insurance claim padding as an unethical behavior and even tend to practice some kind of self-justification. A common view among consumers holds that insurance fraud would just be the rational response to the unfair behavior of insurance companies. Tennyson (1997, 2002) emphasizes that the psychological attitude toward insurance fraud is related to the perception of the fairness of insurance firms by policyholders. She shows that negative perceptions of insurance institutions are related to attitudes toward filing exaggerated claims. For instance, Tennyson (2002) shows that consumers who are not confident of the financial stability of their insurer and those who find auto insurance premiums to be burdensomely high are more likely than others to find fraud acceptable. Thus, consumers would tend to rationalize and justify their fraudulent claims through their negative perceptions of insurance companies. They would neutralize the psychological costs of their inappropriate behavior by considering it as the counterpart of the firms’ unfair behavior: “It’s their fault; if they had been fair with me, I would not have done it” (Strutton et al., 1994, 1997). An eye for an eye would thus be the rule of the insurance fraud game.

Insurance fraud belongs to the set of aberrant consumer behaviors (ACB), defined by Fullerton & Punj (1993) as “behavior in exchange settings which violates the generally accepted norms of conduct in such situations and which is therefore held in disrepute by marketers and by most consumers.” While it has been evidenced that people are more accepting of fraudulent behavior when directed at large organizations like insurance companies (Gneezy, 2005), the perception of unfairness reveals to be the principal justification of insurance fraud. Fukukawa et al. (2007) use a questionnaire to examine the factors that influence the decision-making of ACB across five scenarios.

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1 According to the Coalition Against Insurance Fraud, Insurance fraud steals at least $80 billion every year in the US. See CAIF (2011)
including exaggerating an insurance claim, but also changing a price tag, returning a stained suit, copying software from a friend and taking a quality towel from an hotel. Four factors emerged from a Principle Component Analysis, with among them the perception of unfairness relating to business practice.\(^2\) Fukukawa et al. (2007) show that the perceived unfairness factor is dominant in characterizing the occurrence of the scenario where individuals exaggerate claims and that its effect on insurance fraud is significantly larger than on the other aberrant behavior scenarios.

The perception of unfairness behavior of insurance companies is often associated with the popular view according to which insurers would be bad payers that would start nitpicking if an opportunity arises. Dionne & Gagné (2001) have shown with data from Québec that the amount of the deductible in automobile insurance is a significant determinant of the reported loss, at least when no other vehicle is involved in the accident, and thus when the presence of witnesses is less likely. This suggests that the higher the deductible, the higher the propensity of drivers to file fraudulent claims. Although a deductible is a clause of the insurance contract that cannot be interpreted as a bad faith attitude of the insurer, the result of Dionne and Gagné sustains the idea that the larger the part of an accident cost born by a policyholder, the larger the incentives he or she feels to defraud. In the same vein, the results of an experimental study by Miyazaki (2009) show that higher deductibles result in weaker perception that claim padding is an unethical behavior. As summarized by Miyazaki (2009), the results indicate “some degree of perceived corporate unfairness, wherein consumers feel that the imbalance in favor of the firm has to be balanced by awarding the claimant a higher dollar amount.”

It is a fact that consumer surveys and specialized websites are full of stories in which policyholders complain about the unfairness of their insurance company. The insurance industry affects resources to reduce misunderstandings and to arbitrate disputes, for instance through an Insurance Ombudsman Service or an Insurance Claim Complaints Bureau. Apart from the fact

\(^2\)The Perceived Unfairness factor is comprised of items related to the perception of unfair business practice, for instance because the insurer is overcharging, or because ACB is nothing but retaliation against some inadequate practice or because of weak business performance. Other factors are labeled Evaluation (loading variables relating to the easiness to engage in ACB or to the general attitude toward ACB), Social Participation (with variables representing the social external encouragement to ACB) and Consequence (measuring the extent to which the outcomes of ACB are seen as beneficial or harmful).
that disputes are sometimes induced by the deliberate bad faith of one of the two parties, more often than not consumers’ complaints are motivated by the complexity of insurance contracts and by the difficulty to figure out all possible contingencies to which the contract may apply.\textsuperscript{3} Imagining the innumerable concrete scenarios in which the insurance claims may take place is out of reach for most individuals. It is indeed often said that policyholders should watch out for insurance small print, but as expressed by Daniel Kahnemann: “What psychology and behavioral economics have shown is that people don’t think very carefully. They’re influenced by all sorts of superficial things in their decision-making, and they procrastinate and don’t read the small print.”\textsuperscript{4} This feature of human psychology applies to insurance contracts. n fact, insurance policies are usually very precise. They specify the various contingencies in which claims can be filed by policyholders, with exclusions and limits on payments. These clauses are often designed to lead policyholders to exert the appropriate effort. However, they also frequently allow the insurer a certain amount of leeway once an accident occurred. Even in circumstances where the good-faith of the policyholders is not in dispute,\textsuperscript{5} the insurer may use these clauses to reduce the indemnity payment. This leeway may be at the origin of the feeling that the insurer legally profits from a situation where the policyholder is undoubtedly in good faith but the small print of the contract allows the insurer to deny the claim or to reduce the indemnity payment.\textsuperscript{6}

\textsuperscript{3}An example, among thousands, drawn from The Telegraph (7 December 2011) illustrates how complexity may jeopardize the efficiency of risk sharing through insurance contracts: “A reader from Hampshire tells how she was caught out by the small print in her travel insurance policy when she took a taxi to Gatwick to catch a flight to Luxor. She was delayed by traffic and bad weather conditions and ended up missing her flight, and her holiday. Her insurer refused to pay out for the lost holiday (nearly £3,000), because the policy only specified cover travel by \textit{scheduled public transport}. Insurance small print is one of the most common stumbling blocks for travelers and potentially the most expensive.”

\textsuperscript{4}Drawn from: "Psychologist and Nobel-winning economist Daniel Kahnemann on why people don’t make rational choices" in \textit{Time}, December 5, 2011.

\textsuperscript{5}E.g., in cases where the clauses of the insurance contract are related neither with the origin of the accident nor with its severity and where the policyholder was not aware that his behavior would trigger the cancelation or a reduction in the indemnity payment.

\textsuperscript{6}Consider the following examples drawn from www.thisismoney.co.uk (25th May 2011), a financial website. In all cases, customers have their claims rejected because they have not declared a fact that appears completely irrelevant to the claim, even when they weren’t asked about it: “A woman who contracted leukaemia denied a payout under a critical
What is the logic of such behaviors? Why do insurers sometimes start nitpicking about claims, although the honesty of their customers is not in dispute? This is a true puzzle because nitpicking induces some degree of uncertainty in the way the insurance contract is enforced, and for that reason it reduces the efficiency of the insurance coverage. Consequently, even if nitpicking is reflected in lower insurance premiums, the competition between insurers should lead them to offer the most efficient coverage and to refrain from such an apparently inefficient behavior. If nitpicking is so widespread in the insurance industry, its raison d’être must be related to insurance market mechanisms and not to the deviant behavior of some unscrupulous opportunistic insurers. The objective of this paper is to analyze this issue.

Our starting point is the behavior of insurers that are confronted with claims fraud. Insurers spend resources to monitor claims through a spectrum of verification procedures that go from the settling of the apparently honest claims in a routine way to the referral of most dubious claims to a Special Investigative Unit (SIU). Red flags and sometimes advanced scoring techniques may be used to channel claims in the most efficient way. The principles that guide these audit mechanisms have been analyzed in costly verification models where insureds have private information about their losses and insurers can verify claims by incurring an audit cost.\footnote{See Picard (2001) for a survey and Dionne et al. (2009) for an analysis of the link between claims auditing and the use of red flags, including scoring techniques.}

Among various issues, this literature has shown how the ability of insurers to commit to an auditing policy may affect the efficiency of auditing mechanisms. For example, given the cost of scrutinizing a claim, it may not be profitable to transmit suspicious claims to the SIU if the fraud rate is low. In the most simple audit models, it is optimal to fully deter fraud through claims verification if insurers can commit to audit claims with a
given probability whatever the hit rate. On the contrary, the fraud rate remains positive if insurers are unable to commit (see Picard, 1996). In more sophisticated models with heterogeneous policyholders, the impossibility to commit induces a larger fraud rate than what would be optimal otherwise (see Dionne & Gagné, 2001). The fact that there is some positive fraud rate provides incentives to monitor claims, but this residual fraud will be reflected in higher insurance premiums paid by policyholders and ultimately by a less efficient risk sharing through insurance contracting.

Although the intensity of the commitment issue can be weakened in a dynamic setting where insurers can acquire the reputation of being tough auditors (Krawczyk, 2009) or when the monitoring of claims can be delegated to an independent agent in charge of investigating claims (Mehmud & Mookherjee, 1989) or to a common agency (Picard, 1996), this commitment problem remains an issue. In what follows, we reconsider the commitment problem in a setting where insurers can exploit their superior information about the legal enforcement of insurance contracts.

When insurers verify a claim, they can detect whether it is fraudulent or not. For the sake of simplicity, we assume that this detection mechanism works perfectly well: if an audit is triggered, it allows the insurer to detect with certainty whether the claim is fraudulent or not. However, auditing claims also provides information to the insurer that may sometimes allow him to partially or fully deny obligation to indemnify the policyholder, although the good faith of the latter is not in question. Thus, the insurance policy is viewed as an incomplete contract in the sense that there exist occurrences of the loss (or circumstances of the accident) under which the insurer can deny or reduce the indemnity payment also it would have been better not to do so. Indeed, conditioning the indemnity payment on such circumstances reduces the efficiency of the insurance coverage without exerting any incentive effect on policyholders. Thus, behaving in such a way would be a suboptimal strategy if the insurer could commit on the probability with which an audit is triggered. However, if the insurer cannot commit, then the possibility of exploiting such information acts as an incentive device for the insurer. To put it differently, if the insurer is aware of the fact that auditing may be profitable even if the claim is not fraudulent, then he will be prompted to verify claim for a lower rate of fraud than if the only motive for auditing is to detect cheaters.

We first investigate this issue in section 2 by considering a simple two-state setting in which the occurrence of an accident corresponds to a unique
loss level and insurers cannot commit to their audit strategy. In such a framework, we show that it is optimal to adopt a nitpicking strategy that consists in reducing on average a positive proportion of the contractual insurance indemnity on audited claims. We establish that nitpicking would not be optimal if insurers were able to commit to their auditing strategy whatever the percentage of fraudulent claims. Thus the prevalence of nitpicking in insurance market reveals the existence of the commitment problem. We then investigate more closely the insurer’s nitpicking strategy by characterizing the optimal indemnity cut given the information on each specific case audited. We show that the indemnity cuts are bounded above, meaning that the insurer refrains from reducing the indemnity payment in cases where he would be legally entitled to do so. We then investigate the reputation problem. Here we assume policyholders incur a morale cost defrauding the insurer that decreases with the intensity of the insurer’s nitpicking activity. We show that despite this induced morale effect, nitpicking remains optimal. Finally, in section 3, we extend our results to a more general setting in which losses may be more or less important and we show that nitpicking remains optimal. Section 4 concludes.

2 The model

2.1 Nitpicking and optimal insurance contracts

Consider an insurance company providing coverage to individuals (households or businesses) against an accident that occurs with probability $\pi$ and results in a loss $L$. Denote by $P$ the insurance premium and $I$ the contractual indemnity. Assume that the insurer knows (statistically) that a proportion $\alpha$ of the claims are fraudulent: they are filed by individuals who have not experienced an accident. Accordingly, the insurance company follows an audit rule which consists in verifying the genuineness of the accident for a proportion $\beta$ of the total claims. Such an audit is costly, and we denote by $c$ the verification cost, with $c < L$. Audit reveals without ambiguity a fraudulent claim, i.e. that no accident did occur, in which case the insured receives no indemnity and incurs a litigation cost $B$.

If an accident did occur, this auditing activity results in an assessment of the indemnity given to the insured which depends of the evidences that the insured is able to provide regarding the extent of his loss, that he did.
effectively follow the requirements specified in the insurance contract to be entitled an indemnity, that the accident effectively corresponds to the insurance policy, and so on. The way the insurance contract is written, the many loopholes and ambiguities that its wording contains, allow the insurer to be very creative and efficient in such a nitpicking activity, resulting to an effective indemnity equal to \((1 - \tilde{z})I\) where \(\tilde{z}\) correspond to the fraction of the contractual reimbursement that the insurer is able to save. These savings vary case by case and thus \(\tilde{z}\) is a random variable. However, since the insurance policy is written by the insurance company and final decisions depend on the instructions received by claims handlers, the insurer somehow controls the intensity of his nitpicking activity, which is characterized by \(q \in [0,1]\). In a contract with nitpicking intensity \(q\) the insurance company save on average a fraction \(q\) of the contractual indemnity \(I\). More precisely, the nitpicking technology is such that \(\tilde{z}\) is distributed according to the c.d.f. \(F(z,q) = \Pr\{\tilde{z} \leq z|q\}\) over \([0,1]\), where \(q = E[\tilde{z}|q]\) and \(\partial F/\partial q \leq 0\). Hence \(q = 0\) corresponds to the case where the contractual indemnity \(I\) is paid without any restriction to all claimants when fraud has not been detected. When \(q > 0\), a fraction \(\tilde{z}\) of the contractual indemnity \(I\) is retained and an increase in \(q\) shifts \(\tilde{z}\) in the sense of first-order stochastic dominance. Here the nitpicking technology is taken as given and characterized by this family of probability distributions \(F(\cdot,q)\) indexed by \(q\). We will later show how an optimal nitpicking technology may be derived. In any case, we assume that nitpicking does not induce any additional cost and thus the insurer can commit to this behavior. Consequently, the nitpicking intensity \(q\) may be considered as the implicit part of the insurance contract, \(P\) and \(I\) being the explicit part.

Thus, the expected cost of claims (indemnity + audit cost) per policyholder is given by

\[
C = [\pi(1 - \beta q) + (1 - \pi)(1 - \beta)\alpha]I + [\pi + (1 - \pi)\alpha]\beta c
\]  

The first term in (1)corresponds to the per-individual expected indemnity payment, where honest and fraudulent claims amount to outlays equal to \(\pi(1 - \beta q)I\) and \((1 - \pi)(1 - \beta)\alpha I\) respectively. Note in particular that \(\beta q\) corresponds to the fraction of contractual indemnities not paid on average to honest policyholders. The second term corresponds to the average audit cost per policyholder: a contract results in an honest claim with probability \(\pi\) and a fraudulent one with probability \((1 - \pi)\alpha\), and claims are audited with probability \(\beta\) entailing a cost \(c\).
The expected utility of an insured is given by

\[
Eu = \pi \{ (1 - \beta)u(w - P - L + I) + \beta E[u(w - P - L + (1 - \tilde{z})I)|q] \} \\
+ (1 - \pi) \{ (1 - \alpha)u(w - P) + \alpha [(1 - \beta)u(w - P + I) + \beta u(w - P - B)] \}
\]

(2)

The first term corresponds to the expected utility in case an accident occurs: with probability \(1 - \beta\) the insurance company decides to pay the indemnity without auditing, while with probability \(\beta\) the claim is audited which results to a partial repayment of the contractual indemnity which depends on the insurer’s nitpicking process. The second term corresponds to the no-accident situation, the insured being honest with probability \(1 - \alpha\) or filing a fraudulent claim with probability \(\alpha\), in which case he receives the indemnity if no audit occurs, but he faces litigation costs \(B\) if he is spotted.\(^8\)

When insurers cannot commit to audit claims with a given probability, the fraudulent strategy of the insured (probability \(\alpha\)) and the audit strategy of the insurer (\(\beta\)) must be mutually best responses. As usual in audit models without commitment, insurers and policyholders play mixed strategies such that they are indifferent between the alternatives they may choose from (auditing or not, and defrauding or being honest respectively). Using (1) and (2), one can easily verify that this is the case with an insurance contract \((I, P, q)\) if \(c > qI\), resulting in mixed strategies given by

\[
\alpha^*(I, q) = \frac{\pi(c - qI)}{(1 - \pi)(I - c)} 
\]

(3)

and

\[
\beta^*(I, P) = \frac{u(w - P + I) - u(w - P)}{u(w - P + I) - u(w - P - B)}. 
\]

(4)

In particular, we may observe that the equilibrium fraud rate \(\alpha^*\) is decreasing with respect to \(I\) and \(q\), which shows that fraud may be decreased either by increasing the contractual indemnity or by nitpicking more intensely. If \(c \leq qI\), then fraud fully vanishes at equilibrium, i.e., \(\alpha^*(I, q) = 0\), because the return on nitpicking is larger than the audit cost, so that insurers will monitor claims even if there is no fraud. At equilibrium, either policyholders are indifferent between defrauding and being honest (when \(c > qI\)) or there

\(^8\)We assume in this section that individuals do not feel social or moral pressure behaving dishonestly. We consider moral restraints in the following.
is no fraud (when \( c \leq qI \)), which allows us to rewrite the insureds’ expected utility \( E u^* \) as a function of \( I, P \) and \( q \):

\[
E u^*(I, P, q) \equiv (1 - \pi)u(w - P) + \pi u(w - P - L + I) - \pi \beta^*(I, P)[u(w - P - L + I) - E[u(w - P - L + (1 - \bar{z})I)|q]]
\]

and the insurer per-individual cost to

\[
C^*(I, q) \equiv [\pi + (1 - \pi)\alpha^*(I, q)]I = \pi(1 - q)I^2/(I - c).
\]

Assuming a competitive insurance market, the optimal insurance contract maximizes the insured’s expected utility. Thus, it solves

\[
\max_{I, P, q}\{E u^*(I, P, q) : P \geq C^*(I, q)\}. \tag{5}
\]

**Lemma 1** If insurers are not allowed to cut the contractual reimbursement (i.e. if nitpicking is impossible: \( q = 0 \)), then the optimal contract involves overinsurance: \( I > L \).

Lemma 1 reminds us that the impossibility of insurers to commit to their auditing strategy leads them to offer insurance contracts with indemnity larger than loss (see Boyer, 2004 and Picard, 2009 for models with claims fraud and underwriting fraud respectively). The intuition is straightforward and is illustrated figure 1. Insurers are incited to audit claims if the expected gain of auditing is larger than the cost, which is the case if the fraud rate is larger than \( \pi c/(1 - \pi)(I - c) \). If fraud were not at stake, then insurers would offer full coverage contract with \( I = L \). Graphically, the optimal insurance contract correspond of the tangency point between the indifference curve of the policyholder \( E u^*_c \) and the fair insurance line \( P = \pi I \). However, if the insured can defraud and insurers cannot commit to their auditing strategy, the insurance premium is no longer given by \( P = \pi I \) but by \( P = C^*(I, 0) \). The tangency point between the zero profit line and the expected utility curve \( E u^* \) is above and on the right of the commitment case at \( I = I^* \): Slightly increasing \( I \) over \( L \) maintains the insurer’s incentives at the right level for a lower fraud rate, and thus with a lower premium, and ultimately that will be favorable to the insured.

Of course, the question is to determine if it is optimal that insurance companies nitpick claims. Proposition 1 shows that this is actually the case.
Figure 1: Optimal insurance contract without commitment.

**Proposition 1** The optimal insurance contract entails nitpicking: $q > 0$.

**Corollary 1** Similar computations under the constraint $I \leq L$ give $q > 0$.

Proposition 1 and its Corollary show that insurers will always practice some degree of nitpicking. The intuition follows the same line as the one of Lemma 1. Indeed (3) shows that increasing $q$ leads to a first-order decrease in the equilibrium fraud rate, because the gains drawn from nitpicking provide an additional incentives to monitor claims. Nitpicking also induces some degree of uncertainty in the insurance coverage, which reduces the attractiveness of the contract for policyholder. However, at the first order, the incentive effect dominates the risk sharing effect since there is overinsurance when $q = 0$. Consequently practicing some degree of nitpicking is favorable to the insureds themselves. This conclusion still holds if overinsurance were prohibited, say because of a strong risk of moral hazard: the status quo...
situation without nitpicking would then be at $I = L$, with an unchanged conclusion.

### 2.2 Optimal nitpicking strategy

We have assumed sofar that the nitpicking technology was given to the insurer, and that his only choice was the expected level of indemnity cut: i.e., the insurer had to choose within a family of probability distributions $F(z, q)$ indexed by the nitpicking intensity $q$. In practice, nitpicky insurers may have some leeway depending on the information they obtain on the characteristics of the claim through auditing. The insurer then has to decide whether or not and to what extent he will decide to use this information to reduce the indemnity payment under the contractual indemnity although auditing didn’t reveal any fraudulent behavior. Thus, we may consider in a more realistic way that the insurer has to decide indemnity cut based on the information which is available to him after auditing. This information is summarized by a random variable $\tilde{x}$ distributed over $[0, 1]$ with c.d.f. $G(x)$ and density $g(x) = G'(x)$. We interpret $\tilde{x}$ as the maximum cut in the indemnity payment the insurer is legally entitled to apply, given all relevant available information on the claim. The insurer may chose to cut up to what he is legally possible, i.e. his indemnity cut strategy $z$ (i.e. the fraction of the contractual indemnity he will not pay) is a function of $x$ that must satisfy $z(x) \leq x$ for all $x$ in $[0, 1]$. It is straightforward to verify that the non-negative profit constraint is obtained by substituting $Ez(\tilde{x}) \equiv \int_0^1 z(x)dG(x)$ to $q$ in function $C^\ast$. Now the policyholder’s expected utility $Eu^\ast$ depends on $I, P$ and on function $z(\cdot)$ and it may be written as

$$Eu^\ast = (1 - \pi)u(w - P) + \pi[1 - \beta^\ast(I, P)]u(w - P - L + I) + \pi\beta^\ast(I, P) \int_0^1 u(w - P - L + (1 - \tilde{z}(x))I)dG(x)$$

Thus, still assuming a competitive insurance market, the optimal insurance contract and nitpicking strategy solve

$$\max_{I, P, z(\cdot)} \{Eu^\ast(I, P, z(\cdot)) : P \geq C^\ast(I, Ez(\tilde{x})), 0 \leq z(\tilde{x}) \leq \tilde{x}\}.$$

The resulting nitpicking strategy is characterized is the following proposition.
**Proposition 2** The optimal nitpicking strategy \( z(\cdot) \) is characterized by a ceiling \( \hat{x} > 0 \) such that:

\[
z(x) = \begin{cases} 
  x & \forall x < \hat{x} \\
  \hat{x} & \forall x \geq \hat{x}
\end{cases}
\]

Proposition 2 states that the optimal strategy of the insurer is to reduce as much as possible the indemnity to what is legally possible up to ceiling \( \hat{x} \). The intuition of this result is as follows. The optimal nitpicking strategy results from a trade-off between on one hand the incentive advantage derived by the insurer and on the other hand the negative effect of nitpicking on the risk coverage provided to policyholders. The incentives advantage depends on the average reduction in indemnity payment \( Ez(\hat{x}) \). For a given average reduction \( Ez(\hat{x}) \), the most efficient strategy in terms of risk sharing would involve a uniform percentage of reduction in the indemnity payment when a claim is audited. However, this is not a feasible strategy because auditing may not provide any relevant information to the insurer that would allow him to decrease the payment. The optimal strategy consists in reducing the payment as much as possible (i.e. with \( z^*(x) = x \)), but without creating too much disturbances in the risk coverage, hence the ceiling \( \hat{x} \). In other words, if auditing has not revealed any fraudulent behavior, then the insurer should not exploit its information to decrease the indemnity over \( \hat{x} \) even if it could do so.

If insurers could commit to their auditing strategy, the optimal contract \( \{I, P, z(\cdot)\} \) would maximize the expected utility of honest individuals \( Eu^* \) under the zero profit constraint, which becomes

\[
P \geq \pi [I + \beta^*(I, P)(c - Ez(\hat{x}))].
\]

**Proposition 3** If the insurer can commit to its auditing strategy then nitpicking is suboptimal: the insurer chooses \( z(x) = 0 \) for all \( x \).

Proposition 3 shows that the insurer should choose \( z(x) \equiv 0 \) if it were able to commit to audit claims whatever the fraud rate. Indeed, nitpicking would be suboptimal in such a case because it would artificially create an additional risk for the policyholder. Note that this risk is detrimental to the honest policyholder whose claim is audited, without reducing the indemnity payment for claims that are not audited. In fact, under the commitment assumption, payment should optimally be distorted in the other direction, by increasing the indemnity paid to the honest claims that are audited and reducing the
indemnity when the claim is not audited. Nitpicking goes exactly in the opposite direction.

2.3 Nitpicking intensity and incentives

The nitpicking strategies analyzed above correspond to the optimal behavior of an insurer who is legally constrained in the way he determines the cut on contractual reimbursement. If there were no legal restraint on such cuts,\(^9\) the insurer would impose a unique cut, say \(q\), on the contractual reimbursement of all audited claims. As the indemnity cut is a commitment device that gives incentives to the insurer to effectively audit claims, this unique cut rate must be related to the proportion of fraudulent claims. This relationship appears clearly in the definition of \(\alpha^*(I, P)\) given by (3) which reveals that fraudulent claims are filled as long as \(q < c/I\). An interesting question is to determine the optimal proportion of fraudulent claims if insurers were not legally constrained in their rebate. This optimal proportion must be solution of

\[
\max_{I, P, q} \{Eu^*(I, P, q) : P \geq C^*(I, q), q \leq c/I\}.
\]

We show in the appendix that

**Proposition 4** If there is no legal restraint on the level of rebate that insurers can apply to the contractual indemnity of audited claims, then the optimal contract is such that \(q = c/I\) and no fraudulent claims are filed at equilibrium. Moreover, in that case the optimal contract would involve underinsurance (i.e., \(I < L\)) if litigations costs \(B\) are large enough.

Proposition 4 highlights the strength of nitpicking: if no legal limit were imposed on indemnity cuts, it would be more efficient to provide audit incentives through such an activity -which apparently contraries the policyholders’ interest- than by tolerating a positive rate of fraud in the market.

2.4 Insurers’ reputation and morale cost

So far we have considered that all individuals are inclined to defraud the insurers. They behave without morale restraint, having no value judgment neither on the mere fact of defrauding on their part, nor on the insurers’

\(^9\)That would correspond to the case where \(G(x) = 0\) for all \(0 \leq x < 1\) and \(G(1) = 1\).
reputation of nitpicking. In that case, we have shown that nitpicking was part of the optimal strategy of the competitive insurers because it is a commitment device guarantying that claims are audited. However, applying unilaterally rebates on contractual reimbursement for legitimate claims is certainly a cause of disputes and surely it induces resentment on the insureds’ part. This nitpicking could tarnish the insurers’ reputation, which in turn may have a repercussion on the insureds’ behavior. Hence nitpicking may have the counterproductive result of inducing more individuals to file fraudulent claims.

To investigate this problem, consider that individuals incur morale costs when they defraud the insurers, and these costs depend on the insurers’ reputation. The more insurers are known for nitpicking, the less the individuals’ cost of defrauding them. Hence, when individuals behave honestly, they incur no morale cost, while their net revenue is reduced by \( \gamma(q) > 0 \) when they file a fraudulent claim, with \( \gamma'(q) < 0 \), where \( q = E_\tilde{x}(\tilde{x}) \). Compared to the previous sections, it is easily verified that only the audit probability (4) is affected by these costs. It now depends on \( q \) and becomes

\[
\beta^*(I, P, q) = \frac{u(w - P - \gamma(q) + I) - u(w - P)}{u(w - P - \gamma(q) + I) - u(w - P - \gamma(q) - B)}.
\]

with \( \partial \beta^*/\partial q > 0 \): the larger the nitpicking intensity, the larger the audit probability that dissuades policyholders from defrauding. Hence the optimal contract is a solution to program (5) where the audit probability that affects the insured’s expected utility \( E_u^* \) is given by (4) with \( \beta^*(I, P, q) \) instead of \( \beta^*(I, P) \). The fraud rate \( \alpha^*(I, q) \) that is necessary to induce auditing is independent from the link between the intensity of nitpicking and defrauders’ morale costs. Thus, the logics of the reasoning that explained why nitpicking is optimal is not affected by the existence of such costs.

**Proposition 5** The optimal insurance contract still entails nitpicking, i.e. \( q > 0 \), when defrauders incur morale costs that decrease with \( q \). However, the morale cost induced by fraud decreases the insureds’ welfare.

We have observed that nitpicking would be a suboptimal behavior if insurers could commit to their audit strategy. This conclusion is still valid, and even reinforced if nitpicking induces lower morale costs. Indeed, when \( q \) is increasing from 0 to a positive value, the audit rate \( \beta^*(I, P, q) \) that
discourages fraud increases. This clear-cut difference between the commitment and no-commitment cases reinforces the idea that if nitpicking is a widespread phenomenon in the insurance industry despite its adverse effects on the perception of corporate fairness and incentives to defraud, that may be because such an opportunistic behavior provides additional incentives to monitor claims.

3 Continuum of losses

Let us now consider the case where nitpicking occurs in a setting where accidents generate insurable losses that may differ from one claim to another. More explicitly, we assume that conditionally to the occurrence of the accident, the loss \( \ell \) is a random variable distributed over an interval \((0, L]\) according to the c.d.f. \( F(\ell) \) with density \( f(\ell) = F'(\ell) \). In that case, there are two reasons for the insurer to audit claims: firstly to verify the occurrence of the accident and secondly to assess the extent of the loss compared to the insured’s claim. So, in addition to detect claims that do not correspond to a true accident, the audit of claims also intends to spot the individuals who exaggerate their losses.

The further developments are guided by the following intuition. Because losses differ between claims, individuals’ incentives to overstate losses also differ: an individual having experienced a large loss expects to receive a large indemnity from the insurer if his claim is honest. Relative to this indemnity, an overstatement of a given size of his loss would only lead to a relatively small increase in the indemnity if the claim is not audited, or to a reimbursement corresponding to the insurer’s assessment of the loss and litigation costs in case of an audit. Moreover, the policyholder may expect a larger probability to have his claim audited if he declares a larger loss. When designing the optimal insurance contract and audit strategy, the insurer has to account for these incentives to overstate losses. For simplicity, we here consider the case where individuals do not have morale costs of defrauding.

Let \( EU(\ell, \hat{\ell}) \) be the expected utility of an individual with loss \( \ell \) who overstates his loss by announcing \( \hat{\ell} \) larger than \( \ell \) and let \( EU(\ell) \) be the expected utility in the case of an honest claim \( \hat{\ell} = \ell \). Let \( \beta(\hat{\ell}) \) be the audit probability for a claim \( \hat{\ell} \). The indemnity actually paid is now a function \( I(\cdot) \) of the claim \( \hat{\ell} \) if there is no audit, and of the true loss, possibly reduced by the nitpicking activity of the insurer, if the claim is audited. In this second case, the
indemnity payment is \( I([1-z(\bar{x})]\ell) \), where \( \bar{x} \) is defined as before and \( z(\bar{x}) \) is the reduction rate in the assessment of the claim, with \( z(x) \leq x \) for all \( x \) in \([0, 1]\). We simplify matters by restricting attention to insurance contracts without overinsurance, possibly because overinsurance would induce adverse consequences in terms of moral hazard. We thus assume \( 0 \leq I(\ell) \leq \ell \) for all \( \ell \). The policyholder still incurs the litigation cost \( B \) if auditing reveals fraud. Thus, we may write

\[
EU(\ell, \hat{\ell}) \equiv \beta(\hat{\ell})Eu(w-P-\ell+I([1-z(\bar{x})]\ell)-B)+[1-\beta(\hat{\ell})]u(w-P-\ell+I(\hat{\ell}))
\]

if \( \hat{\ell} > \ell \) and

\[
EU(\ell) \equiv \beta(\ell)Eu(w-P-\ell+I([1-z(\bar{x})]\ell))+[1-\beta(\ell)]u(w-P-\ell+I(\ell)).
\]

An individual with loss \( \ell > 0 \) tells the truth (i.e. announces \( \hat{\ell} = \ell \)) if the incentive constraints \( EU(\ell) \geq EU(\ell, \hat{\ell}) \) are satisfied for all \( \hat{\ell} \) in \((\ell, L]\). For individuals who did not experience an accident, the incentives constraints are written as

\[
u(w-P) \geq \beta(\hat{\ell})u(w-P-B)+[1-\beta(\hat{\ell})]u(w-P+I(\hat{\ell})) \tag{7}
\]

for all for all \( \hat{\ell} \) in \((0, L]\). Hence, for these individuals, defrauding is equivalent to choosing one of the lotteries \( Z(\hat{\ell}) \equiv \{-B, \beta(\hat{\ell}); I(\hat{\ell}), 1-\beta(\hat{\ell})\} \), \( \hat{\ell} \in (0, L]\), and their incentive constraints are satisfied if these insured, with wealth \( w-P \), do not benefit from choosing one of these lotteries \( Z(\hat{\ell}) \).

**Lemma 2** If individuals display NIARA preferences\(^{10} \) and if the insurance contract \( \{P, I(\cdot), \beta(\cdot), z(\cdot)\} \) is such that \( I(\ell) \) and \( \beta(\ell) \) are non-decreasing in \( \ell \), then all policyholders with losses \( \ell > 0 \) are deterred to file a fraudulent claim if the incentive constrains of the individuals with no loss are satisfied.

The intuition of Lemma 2 is simple. For a type \( \ell \) individual (with \( \ell = 0 \) if he has not experienced any accident), filing a fraudulent claim \( \hat{\ell} \) larger than \( \ell \) is a risky choice: he gains \( I(\hat{\ell}) - I(\ell) \) if his claim is not audited, and he has to pay \( B \) in case of an audit. Note that when \( \ell > 0 \), the earnings in case of truthful revelation of the loss may also be uncertain in case of nitpicking. However, under the assumptions made in the Lemma, the wealth of an individual who does not defraud is always larger when \( \ell = 0 \) than when

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\(^{10}\)Non Increasing Absolute Risk Aversion.
\( \ell > 0 \). If the individuals’ absolute risk aversion is not increasing with wealth and if policyholders are deterred to file fraudulent claims (which is a risky venture) when \( \ell = 0 \), then \textit{a fortiori} they will choose not to defraud when \( \ell > 0 \).

We assume in the following that the optimal insurance contract satisfies the condition of Lemma 2. Of course this will have to be checked. Then incentive constraints are satisfied for all \( \ell \) if they are satisfied for \( \ell = 0 \). In that case, the optimal audit strategy is such that (7) is binding for all \( \ell > 0 \), leading no-loss individuals to be indifferent between being honest or announcing any claim \( \ell \in (0, L] \). The audit probability written as a function \( \beta(\ell) \) of the claim size thus satisfies

\[
\beta(\ell) = \frac{u(w - P + I(\ell)) - u(w - P)}{u(w - P + I(\ell)) - u(w - P - B)}.
\]

In practice, the cost of auditing claims is increasing with the size of the claim. For simplicity, we will assume that this cost is proportional to the announced loss, i.e. it is given by \( c \ell \) for claim \( \ell \), with \( 0 < c < 1 \). Under the assumptions made in Lemma 2, we may consider the case where only no-loss individuals file fraudulent claims.\textsuperscript{11} The expected cost \( C(\ell) \) of the claims of size \( \ell \) per policyholder then satisfies

\[
C(\ell) h(\ell) = \pi \{ \beta(\ell) [c \ell + EI([1 - z(\bar{x})])\ell] + [1 - \beta(\ell)] I(\ell) \} f(\ell) + (1 - \pi) \sigma p(\ell) \{ \beta(\ell) c \ell + [1 - \beta(\ell)] I(\ell) \}
\]

where \( h(\ell) \) is the density function of the size of the claims, \( \sigma \) is the probability that a no-loss individual decides to defraud the insurer, and \( p(\ell) \) is the density of the size of fraudulent claims over \((0, L]\). In other terms, a defrauder (necessarily a no-loss individual) chooses to announce \( \hat{\ell} \in [\ell, \ell + d\ell] \) with probability \( p(\ell)d\ell \) and claims will be in the same interval with probability \( h(\ell)d\ell \). The insurer is indifferent between auditing those claims or not auditing if

\[
p(\ell) = \frac{\pi \{ EI([1 - z(\bar{x})])\ell + c \ell - I(\ell) \} f(\ell)}{\sigma(1 - \pi)[I(\ell) - c \ell]}
\]

We must have \( \int_0^L p(\ell)d\ell = 1 \) for \( p(\cdot) \) to be a density function, which yields

\textsuperscript{11}Under the assumptions made in Lemma 2, honesty is an optimal behavior when \( \ell > 0 \) if it is optimal when \( \ell = 0 \).
the fraud probability of no-loss individuals

\[
\sigma = \frac{\pi}{1 - \pi} \left\{ \int_0^L \frac{EI([1 - z(\tilde{x})]\ell)}{I(\ell) - c\ell} f(\ell)d\ell - 1 \right\}.
\]

The claims probability distribution then satisfies

\[
h(\ell) = \pi f(\ell) + (1 - \pi)\sigma p(\ell) = \pi f(\ell) \frac{EI([1 - z(\tilde{x})]\ell)}{I(\ell) - c\ell}
\]

At equilibrium, \( C(\ell) = \pi I(\ell) \) and the expected cost of a contract for the insurer simplifies to

\[
C = \int_0^L \pi I(\ell) h(\ell)d\ell = \pi E \left[ \frac{I(\ell)I([1 - z(\tilde{x})]\tilde{\ell})}{I(\tilde{\ell}) - c\ell} \right].
\]

The expected utility of the insured is given by

\[
Eu^*(I(\cdot), P, z(\cdot)) = (1 - \pi)u(w - P) + \pi Eu(w - P - \tilde{\ell} + I(\tilde{\ell})) - \pi E[\beta(\tilde{\ell})\{u(w - P - \tilde{\ell} + I(\tilde{\ell})) - u(w - P - \tilde{\ell} + I([1 - z(\tilde{x})]\tilde{\ell})]\}]
\]

Assuming a competitive insurance market, the optimal contract satisfies

\[
\max_{I(\cdot), z(\cdot), P} \left\{ Eu^*(I(\cdot), P, z(\cdot)) : P \geq \pi E \left[ \frac{I(\ell)I([1 - z(\tilde{x})]\tilde{\ell})}{I(\tilde{\ell}) - c\ell} \right], 0 \leq I(\tilde{\ell}) \leq \tilde{\ell} \right\}
\]

where \( \beta(\cdot) \) is given by 8 and where expectation is taken with respect to \( \tilde{x} \) and \( \tilde{\ell} \). Program 9 is a generalization of the problem derived in Section 1.2 to the case where losses may have different sizes, with a similar conclusion:

**Proposition 6** When individuals incur losses of different size, the optimal insurance contract entails some degree of nitpicking by insurers: \( Ez(\tilde{x}) > 0 \).

## 4 Conclusion

To be completed
References


Appendix

A  Proof of Lemma 1

Let $L(I, P, q, \lambda) \equiv Eu^*(I, P, q) + \lambda[P - C^*(I, q)]$ be the Lagrangian of program (5), and let $I^*(q), P^*(q), \lambda^*(q)$ be the optimal indemnity, premium and Lagrange multiplier for a given nitpicking level $q$. We get

$$
(\partial L/\partial P)|_{q=0} = -[(1-\pi)u'(w-P^*(0))+\pi u'(w-P^*(0)-L+I^*(0))]+\lambda^*(0) = 0
$$

$$
(\partial L/\partial I)|_{q=0} = \pi u'(w-P^*(0)-L+I^*(0)) - \lambda^*(0)\pi(I^*(0)-2c)I^*(0)/(I^*(0)-c)^2 = 0
$$

and, as $(I-c)^2 > I^2 - 2cI,$

$$
\lambda^*(0) = (1-\pi)u'(w-P^*(0))+\pi u'(w-P^*(0)-L+I^*(0)) > u'(w-P^*(0)-L+I^*(0))
$$

which implies

$$
u'(w-P^*(0)) > u'(w-P^*(0) - L + I^*(0))
$$

hence $I^*(0) > L$.

B  Proof of Proposition 1

Denoting $\psi(q) \equiv L(I^*(q), P^*(q), q, \lambda^*(q))$, the Envelope Theorem gives

$$
\psi'(q) = \pi \beta^*(I, P) \frac{\partial E}{\partial q} [u(w - P^*(q) - L + (1 - \tilde{z})I)|q]_{I=I^*(q)} + \frac{\lambda^*(q)\pi I^*(q)^2}{I^*(q) - c}.
$$

Integrating by part, allows us to write

$$
E[u(w-P-L+(1-\tilde{z})I)|q] = u(w-P-L)+I \int_0^1 u'(w-P-L+(1-z)I)F(z, q)dz,
$$

and thus

$$
\frac{\partial E}{\partial q} [u(w-P^*(q) - L + (1-\tilde{z})I)|q]_{I=I^*(q)} = I^*(q) \int_0^1 u'(w-P^*(q) - L + (1-z)I^*(q)) \frac{\partial F(z, q)}{\partial q} dz.
$$
Hence, from the mean-value theorem there exists $\hat{z}(q) \in (0, 1)$ such that
\[
\frac{\partial E}{\partial q} [u(w - P^*(q) - L + (1 - \hat{z})I)|q]\bigg|_{I = I^*(q)} \\
= I^*(q)u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)) \int_0^1 \frac{\partial F(z, q)}{\partial q} dz \\
= -I^*(q)u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)),
\]
where the last equality is obtained by differentiating the identity
\[
q = E[\tilde{z}|q] = 1 - \int_0^1 F(z, q)dz,
\]
which gives $\int_0^1 \frac{\partial F(z, q)}{\partial q} dz = -1$. As $\beta^* < 1$, we have
\[
\psi'(q) > \pi [-I^*(q)u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)) + \lambda^*(q)I^*(q)^2/(I^*(q) - c)],
\]
and finally, using $\hat{z}(0) = 0$ and $u'(w - P^*(0) - L + I^*(0)) < \lambda^*(0)$, we get
\[
\psi'(0) > \pi \lambda^*(0)I^*(0) [-1 + I^*(0)/(I^*(0) - c)] = \pi c \lambda^*(0)I^*(0)/(I^*(0) - c) > 0,
\]
which implies $q^* > 0$. □

C Proof of proposition 2

Pointwise maximization with respect to $z(x)$ yields
\[
[-\beta^* I u'(w - P - L + (1 - z(x))I) + \lambda I^2/(I - c)]g(x) \geq 0, \quad (10)
\]
for all $x \in [0, 1]$ with an equality if $z(x) < x$. Thus, we have $\beta^* u'(w - P - L + (1 - z(x))I) = \lambda I/(I - c)$ when $z(x) < x$, and $\beta^* u'(w - P - L + (1 - z(x))I) < \lambda I/(I - c)$ when $z = x$. As $u(\cdot)$ is concave, $u'(w - P - L + (1 - z)I)$ is strictly increasing in $z$, implying that the optimal solution is characterized as in Proposition 2, with threshold $\hat{x}$ defined by
\[
\beta^* u'(w - P - L + (1 - \hat{x})I) = \lambda I/(I - c),
\]
for optimal values of $P, I$ and $\lambda$. □

D Proof of proposition 3

Let $\{I, P, z(\cdot)\}$ be an optimal contract when the insurer can commit. Suppose it is such that $E[z(\bar{x})] > 0$. Consider the alternative contract $\{\hat{I}, \hat{P}, \tilde{z}(\cdot)\}$ with
\[ \hat{P} = P, \hat{z}(x) \equiv 0 \] and \[ \hat{I} = I - \beta^*(I, P) E[z(\bar{x})]. \]

We have
\[
\hat{P} = P \geq \pi[I + \beta^*(I, P)(e - E[z(\bar{x})]I)] \\
= \pi[\hat{I} + c\beta^*(I, \hat{P})] \\
> \pi[\hat{I} + c\beta^*(\hat{I}, \hat{P})],
\]

where the last inequality results from \( \hat{I} < I \) and \( \partial \beta^*/\partial I > 0 \). Hence \( \{\hat{I}, \hat{P}, \hat{z}(\cdot)\} \) is feasible when the insurer can commit. Let \( E_u \) and \( E_{\hat{u}} \) denote the policyholder’s expected utility for \( \{I, P, z(\cdot)\} \) and \( \{I, P, \hat{z}(\cdot)\} \) respectively.

Using the concavity of \( u(\cdot) \) and \( \hat{P} = P \) yields
\[
E_u = \pi\{(1 - \beta^*(I, P, q))u(w - P - L + I) + \beta^*(I, P, q)E[u(w - P - L + (1 - \hat{z}(\bar{x}))I)]\} \\
+ (1 - \pi)u(w - P) \\
> \pi\{u(w - P - L + I - \beta^*(I, P)E[z(\bar{x})]I) + (1 - \pi)u(w - P)\} \\
= \pi u(w - \hat{P} - L + \hat{I}) + (1 - \pi)u(w - \hat{P}) = E_{\hat{u}},
\]

which contradicts the optimality of \( \{I, P, z(\cdot)\} \). \( \blacksquare \)

**E Proof of proposition 4**

Denote by \( \mathcal{L}(I, P, q, \lambda) \equiv E u^*(I, P, q) + \lambda[P - C^*(I, q)] \) the Lagrangian of the insurer’s program and assume that \( q^* < c/I^* \) at the optimum. The first-order optimality conditions with respect to \( I \) and \( q \) are given by
\[
\frac{\partial \mathcal{L}}{\partial I} = \pi[(1 - \beta^*)u'(w - P - L + I) + (1 - q)\beta^*u'(w - P - L + (1 - q)I)] \\
- (\partial \beta^*/\partial I)[u(w - P - L + I) - u(w - P - L + (1 - q)I)] \\
+ \lambda(1 - q)(I - 2c)/(I - c)^2 = 0.
\]

and
\[
\frac{\partial \mathcal{L}}{\partial q} = \pi I[-\beta^*u'(w - P - L + (1 - q)I) + \lambda I/(I - c)] = 0. \quad (11)
\]

Rearranging terms of (11) and using (12) yield
\[
(1 - \beta^*)u'(w - P - L + I) = \frac{\partial \beta^*}{\partial I}[u(w - P - L + I) - u(w - P - L + (1 - q)I)] \\
- \lambda \frac{(1 - q)Ic}{(I - c)^2}
\]

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where
\[
\frac{\partial \beta^*}{\partial I} = \frac{u'(w - P + I)[u(w - P) - u(w - P - B)]}{[u(w - P + I) - u(w - P - B)]^2}
\]

Using the concavity of \( u(\cdot) \) we have
\[
u(w - P + I) - u(w - P) > u'(w - P + I)I
\]
and, using (12),
\[
u(w - P - L + I) - u(w - P - L + (1-q)I) < qIu'(w - P - L + (1-q)I) = q\lambda I^2/[(I-c)\pi \beta^*].
\]

Consequently
\[
\frac{\partial \beta^*}{\partial I}[u(w - P - L + I) - u(w - P - L + (1-q)I)] < \frac{q\lambda I^2}{\pi(I-c)} \frac{u'(w - P + I)[u(w - P) - u(w - P - B)]}{[u(w - P + I) - u(w - P - B)][u(w - P + I) - u(w - P)]] < \frac{q\lambda I}{\pi(I-c)} u(w - P + I) - u(w - P - B)
\]

We thus have
\[
(1 - \beta^*)u'(w - P - L + I) < \frac{q\lambda I}{I-c} \frac{\lambda(1-q)Ic}{(I-c)^2} = \lambda I \frac{qI - c}{(I-c)^2} < 0
\]
which is impossible since \( \beta^* \leq 1 \). We thus have \( q = c/I \) implying \( \alpha^* = 0 \).

As \( q = c/I \) at the optimum, the insurer cost (1) simplifies to \( C = \pi I^* \) which is also the premium at the optimum, i.e. \( P^* = \pi I^* \). Hence, the program of the insurer may be written as max \( I \) \( \phi(I) - \pi K(I) \) where
\[
\phi(I) \equiv \pi u(w - L + (1-\pi)I) + (1-\pi)u(w - \pi I)
\]
with \( \phi'(L) = 0, \phi''(I) < 0 \), and
\[
K(I) \equiv \beta^*(I, \pi I)[u(w - L + (1-\pi)I) - u(w - L + (1-\pi)I - c)].
\]

Consequently, we have \( I < L \) for the optimal contract iff \( K'(L) > 0 \) which may be written as
\[
\frac{u'(w - \pi L - c) - u'(w - \pi L)}{u(w - \pi L) - u(w - \pi L - c)} < \frac{\left(\frac{d\beta^*(I, \pi I)}{dI}\right)|_{I=L}}{1 - \beta^*(L, \pi L)} = (12)
\]
Differentiating yields
\[
\frac{d\beta^*(I, \pi I)}{dI} = \frac{M(I)}{u(w + (1 - \pi)I) - u(w - \pi I - B)},
\]
where
\[
M(I) \equiv \pi u'(w - \pi I) - (1 - \pi)u'(w + (1 - \pi)I) - \beta^*(I, \pi I)[\pi u'(w - \pi I - B) - (1 - \pi)u'(w + (1 - \pi)I)].
\]

As the LHS of (13) is positive, we first have to determine the conditions for \(d\beta^*(I, \pi I)/dI|_{I=L}\) to be positive. This is the case if \(M(L) > 0\), hence if either
\[
\frac{u'(w + (1 - \pi)L)}{u'(w - \pi L - B)} > \frac{\pi}{1 - \pi} \quad \text{and} \quad \beta^* > \frac{(1 - \pi)u'(w + (1 - \pi)L) - \pi u'(w - \pi L)}{(1 - \pi)u'(w + (1 - \pi)L) - \pi u'(w - \pi L - B)}
\]
or
\[
\frac{u'(w + (1 - \pi)L)}{u'(w - \pi L)} < \frac{\pi}{1 - \pi} \quad \text{and} \quad \beta^* < \frac{\pi u'(w - \pi L) - (1 - \pi)u'(w + (1 - \pi)L)}{\pi u'(w - \pi L - B) - (1 - \pi)u'(w + (1 - \pi)L)}.
\]

As
\[
u(w + (1 - \pi)I) - u(w - \pi I) < u'(w - \pi I)I
\]
and
\[
u(w + (1 - \pi)I) - u(w - \pi I - B) > (I + B)u'(w + (1 - \pi)I)
\]
from the concavity of \(u(\cdot)\), we have
\[
\beta^* < \frac{I}{I + B} \frac{u'(w - \pi I)}{u'(w + (1 - \pi)I)}.
\]

Hence, in the first case, we must have
\[
\frac{L}{L + B} \frac{u'(w - \pi L)}{u'(w + (1 - \pi)L)} > \frac{(1 - \pi)u'(w + (1 - \pi)L) - \pi u'(w - \pi L)}{(1 - \pi)u'(w + (1 - \pi)L) - \pi u'(w - \pi L - B)}
\]
which is not satisfied for \(B\) small while the condition on \(\pi\) requires a small \(B\) to be satisfied. In the second case, the condition on \(\pi\) does not depend on
B (and it is verified whatever \( \pi > 0 \) if \( L \) is large enough) while a sufficient condition for the second requirement is given by
\[
\frac{L}{L + B} u'(w - \pi L) < \frac{\pi u'(w - \pi L) - (1 - \pi) u'(w + (1 - \pi) L)}{\pi u'(w - \pi L - B) - (1 - \pi) u'(w + (1 - \pi) L)}
\]
which is not satisfied when \( B = 0 \), but as the LHS decreases with \( B \) while the RHS increases, it does for \( B \) large enough.

In that case, as
\[
\frac{\left. \frac{d\beta^*(I, \pi I)}{dI} \right|_{I = L}}{(1 - \pi)\beta^*(L, \pi L)} = \frac{(1 - \beta^*)(1 - \pi) u'(w + (1 - \pi) L) + \pi u'(w - \pi L) - \beta^* \pi u'(w - \pi L - B)}{(1 - \pi) [u(w + (1 - \pi) L) - u(w - \pi L)]}
\]
\[
> \frac{(1 - \beta^*) [\pi u'(w - \pi L) - (1 - \pi) u'(w + (1 - \pi) L)]}{(1 - \pi) [u(w + (1 - \pi) L) - u(w - \pi L)]}
\]
using the concavity of \( u(\cdot) \), a sufficient condition for \( I^* < L \) is given by
\[
\frac{u'(w - \pi L - c) - u'(w - \pi L) \pi u'(w - \pi L) - (1 - \pi) u'(w + (1 - \pi) L)}{u(w - \pi L) - u(w - \pi L - c)} \cdot \frac{(1 - \pi) [u(w + (1 - \pi) L) - u(w - \pi L)]}{1 - \beta^*(L, \pi L)} < 1
\]
As \( \beta^* \) decreases with \( B \), this condition is more likely to be satisfied when \( B \) is large. Finally, using L’Hospital rule, we have for small \( c \)
\[
\frac{u'(w - \pi L - c) - u'(w - \pi L)}{u(w - \pi L) - u(w - \pi L - c)} \approx \frac{u''(w - \pi L)}{u'(w - \pi L)} \equiv A(w - \pi L)
\]
and the condition becomes
\[
\frac{\pi u'(w - \pi L) - (1 - \pi) u'(w + (1 - \pi) L)}{(1 - \pi) [u(w + (1 - \pi) L) - u(w - \pi L)]} < \frac{1 - \beta^*(L, \pi L)}{A(w - \pi L)}.
\]

\section{Proof of proposition 5}

The arguments are similar to the ones given in the proof of proposition 5. Denoting by \( \psi(q) \equiv L(I^*(q), P^*(q), q, \lambda^*(q)) \), the envelop theorem gives
\[
\psi'(q) = \pi \beta^*(I, P, q) \frac{\partial E}{\partial q} [u(w - P^*(q) - L + (1 - \tilde{\pi}) I^*(q))|q] + \frac{\lambda^*(q) \pi I^*(q)^2}{I^*(q) - c}
\]
\[
- \pi \frac{\partial \beta^*(I, P, q)}{\partial q} \{[u(w - P - L) - I] - E[u(w - P - L + (1 - \tilde{\pi}) I)|q]\},
\]

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where the last bracketed term tends to 0 when \( q \to 0 \). As a result, we have \( \psi'(0) > 0 \) which implies \( q^* > 0 \). 

\[ \]

**G Proof of lemma 2**

We may write

\[ EU(\ell, \hat{\ell}) \leq \beta(\hat{\ell})Eu(w-P-\ell+I([1-z(\hat{x})]\ell)-B)+[1-\beta(\hat{\ell})]u(w-P-\ell+I(\hat{\ell})+I(\ell)) \]

since \( I(\ell) \geq 0 \) for all \( \ell \). Furthermore, using \( \beta(\hat{\ell}) \geq \beta(\ell) \) for all \( \hat{\ell} > \ell \) implies

\[ EU(\ell) \geq \beta(\hat{\ell})Eu(w-P-\ell+I([1-z(\hat{x})]\ell)+[1-\beta(\hat{\ell})]u(w-P-\ell+I(\ell)). \]

Hence, a sufficient condition for \( \ell \)-loss individuals to tell the truth is given by

\[ \beta(\hat{\ell})\{Eu(w-P-\ell+I([1-z(\hat{x})]\ell)-Eu(w-P-\ell+I([1-z(\hat{x})]\ell-B) \]

\[ \geq [1-\beta(\hat{\ell})\{u(w-P-\ell+I(\hat{\ell})+I(\ell))-u(w-P-\ell+I(\ell)) \]

for all \( \hat{\ell} > \ell \). As

\[ Eu(w-P-\ell+I([1-z(\hat{x})]\ell))-Eu(w-P-\ell+I([1-z(\hat{x})]\ell-B) \]

\[ = \int_0^B \int_0^1 u'(w-P-\ell+I([1-z(\hat{x})]\ell)-b)dG(x)db \]

\[ \geq \int_0^B u'(w-P-\ell+I(\ell)-b)db \]

\[ = u(w-P-\ell+I(\ell))-u(w-P-\ell+I(\ell)-B) \]

Since \( I(\cdot) \) is non-decreasing, we have \( u'(w-P-\ell+I(\ell)-b) \leq u'(w-P-\ell+I([1-z(\hat{x})]\ell)-b) \). Consequently, a sufficient condition for \( \ell \)-loss individuals to tell the truth is given by

\[ u(w-P-\ell+I(\ell)) \geq \beta(\hat{\ell})\{u(w-P-\ell+I(\ell)-B)+[1-\beta(\hat{\ell})]u(w-P-\ell+I(\hat{\ell})+I(\ell)). \]

If (7) is satisfied for all \( \hat{\ell} \), then individuals with wealth \( w-P \) do not benefit from choosing one of the lotteries \( Z(\hat{\ell}) \equiv (-B, \beta(\hat{\ell}); I(\hat{\ell}), 1-\beta(\hat{\ell}), \hat{\ell} \in (0, L) \). Hence, if preferences are NIARA and using \( I(\ell) \leq \ell \), it is also true for individuals with wealth \( w-P-\ell+I(\ell) \leq w-P \). 

\[ \]

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H Proof of proposition 6

Denote by
\[ \mathcal{L}(I(\cdot), P, z(\cdot), \lambda) = Eu^*(I(\tilde{\ell}), P, z(\tilde{x}))+\lambda\{P-\pi E\left[ I(\tilde{\ell})\left[1 - z(\tilde{x})\right]\ell/(I(\tilde{\ell}) - c\ell) \right] \} \]
the Lagrangian of program (9) neglecting the constraint on \( I(\tilde{\ell}) \). Suppose that \( z(x) = 0 \) for all \( x \in [0,1] \) at an optimal solution. The first-order optimality condition for \( P \) simplifies to
\[ \partial \mathcal{L}/\partial P = -(1 - \pi)u'(w - P) - \pi Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) + \lambda = 0. \tag{13} \]

We deduce from the concavity of \( u(\cdot) \) and \( I(\ell) \leq \ell \) for all \( \ell \) that \( Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) \geq u'(w - P) \) and (14) implies \( Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) \geq \lambda \) with a strict inequality if \( I(\ell) < \ell \) on a measurable subset of \((0,L)\). For the indemnity, a point-wise maximization gives
\[ \partial \mathcal{L}/\partial I(\ell) = \pi f(\ell)\{u'(w - P - \ell + I(\ell)) - \lambda[I(\ell) - 2c\ell]I(\ell)/[I(\ell) - c\ell]^2 \} \geq 0 \tag{14} \]
with an equality if \( I(\ell) < \ell \). As \( [I(\ell) - 2c\ell]I(\ell) < [I(\ell) - c\ell]^2 \), we would have
\[ u'(w - P - \ell + I(\ell)) < \lambda \leq Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) \]
for all \( I(\ell) < \ell \) which is impossible, hence a contradiction. We thus have
\( I(\ell) = \ell \) for all \( \ell \in (0,L) \). Regarding the nitpicking strategy, as we have
\[ \frac{\partial \mathcal{L}}{\partial z(x)} = -\pi g(x)E\left[ \tilde{\ell}I'(\left[1 - z(x)\right]\ell)\beta^*(\tilde{\ell})u'(w - P - \tilde{\ell} + I([1 - z(x)]\tilde{\ell})) + \frac{\lambda I(\tilde{\ell})}{I(\tilde{\ell}) - c\ell} \right], \]
if \( z^*(x) = 0 \) for some \( x \in [0,1] \), it must be the case that
\[ E[\tilde{\ell}I'(\tilde{\ell})\beta^*(\tilde{\ell})u'(w - P - \tilde{\ell} + I(\tilde{\ell})) + \lambda I(\tilde{\ell})/[I(\tilde{\ell}) - c\ell]] < 0 \]
which does not depend on \( x \) and implies that effectively, \( z^*(x) = 0 \) for all \( x \in [0,1] \). As \( I(\tilde{\ell}) = \ell \) when \( z^*(x) = 0 \) for all \( x \in [0,1] \), this leads to the condition
\[ u'(w - P)E[\tilde{\ell}\beta^*(\tilde{\ell})] + \lambda/(1 - c) < 0 \]
which is impossible. \( \blacksquare \)