Perfect Competition in Markets with Adverse Selection

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Abstract

Adverse selection is an important problem in many markets. Governments typically respond to it with complex regulations, which include mandates, community rating, subsidies, risk adjustment, and the regulation of contract characteristics. This paper proposes a perfectly competitive model of a market with adverse selection. Prices are determined by firms making zero profits, and the set of traded contracts is determined by entry into new contracts being unprofitable. Crucially for applications, contract characteristics are endogenously determined, consumers may have multiple dimensions of private information, and an equilibrium always exists. The equilibrium corresponds to the limit of a differentiated products Bertrand game.

We apply the model to show that mandates can increase efficiency, but have unintended consequences. An insurance mandate that forces consumers to purchase a minimum level of coverage can increase adverse selection on the intensive margin and lead some consumers to purchase less coverage. Optimal regulation addresses adverse selection on both the extensive and the intensive margins, can be described by a sufficient statistics formula, and includes elements that are commonly used in practice.
1 Introduction

Policy makers and market participants consider adverse selection a first-order concern in many markets. These markets are often heavily regulated, if not subject to outright government provision, as in social programs like unemployment insurance and Medicare. Government interventions are typically complex, involving the regulation of contract characteristics, personalized subsidies, community rating, risk-adjustment, and mandates. However, most models of competition take contract characteristics as given, considerably limiting the scope of normative and even positive analyses of these policies.

Standard models face three basic limitations. The first limitation arises in the Akerlof (1970) model, which, following Einav et al. (2010a), is used by most of the recent applied work. The Akerlof lemons model considers a market for a single contract with exogenous characteristics, making it impossible to consider the effect of policies that affect contract terms. In contrast, the Spence (1973) and Rothschild and Stiglitz (1976) models do allow for endogenous contract characteristics. However, they restrict consumers to be heterogeneous along a single dimension, despite evidence on the importance of multiple dimensions.

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1Van de Ven and Ellis (1999) survey health insurance markets across eleven countries, focusing on risk adjustment, that is, cross-subsidies from insurers who enroll cheaper consumers to those who enroll more expensive consumers. The survey gives a glimpse on regulations that are typically used. Seventeen out of eighteen markets use risk adjustment. Eleven use community rating, which forbids price discrimination based on some observable characteristics, such as age or preexisting conditions. These measures are also used by private sponsors. For example, large corporations in the United States typically limit the insurance options made available to their employees. Benefits consulting firms often advise companies to risk-adjust contributions due to concerns about adverse selection. This is in contrast to setting uniform employer contributions, what is known as a “fixed-dollar” or “flat rate” model (Pauly et al., 2007; Cutler and Reber, 1998).

2Recent papers using this framework include Handel et al. (2013); Hackmann et al. (2014); Mahoney and Weyl (2014); Scheuer and Smetters (2014).

3Many authors have highlighted the importance of taking the determination of contract characteristics into account and the lack of a theoretical framework to deal with this. Einav and Finkelstein (2011) say that “abstracting from this potential consequence of selection may miss a substantial component of its welfare implications [...]. Allowing the contract space to be determined endogenously in a selection market raises challenges on both the theoretical and empirical front. On the theoretical front, we currently lack clear characterizations of the equilibrium in a market in which firms compete over contract dimensions as well as price, and in which consumers may have multiple dimensions of private information.” According to Einav et al. (2009), “analyzing price competition over a fixed set of coverage offerings [...] appears to be a relatively manageable problem, characterizing equilibria for a general model of competition in which consumers have multiple dimensions of private information is another matter. Here it is likely that empirical work would be aided by more theoretical progress.”

4Chiappori et al. (2006) highlight this shortcoming: “Theoretical models of asymmetric information typically use oversimplified frameworks, which can hardly be directly transposed to real-life situations. Rothschild and Stiglitz’s model assumes that accident probabilities are exogenous (which rules out moral hazard), that only one level of loss is possible, and more strikingly that agents have identical preferences which are moreover perfectly known to the insurer. The theoretical justification of these restrictions is straightforward: analyzing a model of “pure,” one-dimensional adverse selection is an indispensable first step. But their empirical relevance is dubious, to say the least.”
of private information.\textsuperscript{5} Moreover, the Spence model suffers from rampant multiplicity of equilibria, while the Rothschild and Stiglitz model often has no equilibrium.\textsuperscript{6}

This paper develops a competitive model of adverse selection. The model incorporates three key features, motivated by the central role of contract characteristics in policy and by recent empirical findings. First, the set of traded contracts is endogenous, allowing us to study policies that affect contract characteristics. Second, consumers may have several dimensions of private information, engage in moral hazard, and exhibit deviations from rational behavior such as inertia and overconfidence.\textsuperscript{7} Third, equilibria always exist and yield sharp predictions. Equilibria are inefficient, and even simple interventions can raise welfare. Nevertheless, standard regulations have important unintended consequences once firm responses are accounted for.

The key idea in our model is to consistently apply the price-taking logic of the standard Akerlof (1970) and Einav et al. (2010a) models to the case of endogenous contract characteristics. The prices of traded contracts are set so that every contract makes zero profits. Moreover, whether a contract is offered depends on whether the market for that contract unravels, exactly as in the Akerlof (1970) model with a single product. For example, consider a candidate equilibrium in an insurance market and a policy that is not traded at a price of $1,100. Suppose that consumers would start purchasing the policy were its price to fall below $1,000. Consider what happens as the price of the policy falls from $1,100 to $900 and buyers flock in. One case is that buyers are bad risks, with an average cost of, say, $1,500. In this case, it is reasonable for the policy not to be traded because there is an adverse selection death spiral in the market for the policy. Another case is that buyers are good risks, with an expected cost of, say, $500. In that case, the fact that the policy is not traded is inconsistent with free entry, since any firm who entered the market for this policy would make positive profits.

We formalize this idea as follows. The model takes as given a set of potential contracts and a distribution of consumer preferences and costs. A contract specifies all relevant characteristics, except for a premium or price. Equilibria determine both prices and the contracts that are traded. A weak equilibrium is a set of prices and an allocation such that all consumers optimize and prices equal the expected cost of supplying each contract. However, there is a large number of weak equilibria because this notion imposes little discipline on which contracts are traded. For example, there are always weak equilibria where no con-

\textsuperscript{5}See Finkelstein and McGarry (2006); Cohen and Einav (2007); Fang et al. (2008).

\textsuperscript{6}According to Chiappori et al. (2006), “As is well known, the mere definition of a competitive equilibrium under asymmetric information is a difficult task, on which it is fair to say that no general agreement has been reached.” See also Myerson (1995).

tracts are bought because prices are high, and prices are high because the expected cost of a non-traded contract is arbitrary.

We make an additional equilibrium requirement that formalizes the idea that entry into non-traded contracts is unprofitable. We require equilibria to be robust to a small perturbation of fundamentals. Namely, equilibria must survive in economies with a set of contracts that is similar to the original, but finite, and with a small mass of agents who demand all contracts and have low costs. The definition avoids pathologies related to conditional expectation over measure zero sets because all contracts are traded in a perturbation, much like the notion of a proper equilibrium in game theory (Myerson, 1978). The second part of our refinement is similar to that used by Dubey and Geanakoplos (2002) in a model of competitive pools.

Competitive equilibria always exist and make sharp predictions in a wide range of applied models that are particular cases of our framework. The equilibrium matches standard predictions in the models of Akerlof (1970), Einav et al. (2010a), and Rothschild and Stiglitz (1976) (when their equilibrium exists). Besides the price-taking motivation, we give strategic foundations for the equilibrium, showing that it is the limit of a game-theoretic model of firm competition. We discuss in detail the relationship between our equilibrium and standard solution concepts below.

To understand the importance of contract characteristics and different dimensions of heterogeneity, we apply our model to the effects of different policy interventions. To ensure that the effects are quantitatively plausible, we calibrate a parametric health insurance model based on Einav et al. (2013). Consumers have four dimensions of private information, giving a glimpse of equilibrium behavior beyond standard one-dimensional models. There is moral hazard, so that welfare-maximizing regulation is more nuanced than simply mandating full insurance. We calculate the competitive equilibrium with firms offering contracts covering from 0% to 100% of expenditures. There is considerable adverse selection in equilibrium, creating scope for regulation.

We calculate the equilibrium under a mandate that requires purchase of insurance with actuarial value of at least 60%. Figure 1 depicts the mandate’s impact on coverage choices. A non-equilibrium model would simply predict that many consumers migrate from lower coverage to 60%. In equilibrium, however, the influx of cheaper consumers into the 60% policy reduces its price. This leads some of the consumers who were purchasing more comprehensive plans to reduce their coverage levels. Taking equilibrium effects into account, the mandate has important unintended consequences. The mandate forces some consumers to increase their purchases to the minimum quality standard, but also increases adverse selection on the intensive margin. Despite these unintended consequences, mandates may increase social welfare. Under our benchmark parameters, the increase in Kaldor-Hicks welfare from the
mandate equals $127 per consumer. We compute the welfare-maximizing regulation, and find that it involves subsidies to address adverse selection on the intensive margin, and increases welfare by $279 relative to the unregulated market.

These results are consistent with the view that there is scope for government intervention in markets with adverse selection. Moreover, interventions that do not take into account their effect on contract characteristics may miss much of the potential welfare gains. To understand this point more generally, besides varying calibration parameters, we theoretically analyze the sources of inefficiency. We focus on a bare-bones case, ignoring redistribution, the ability to price discriminate on some observables, and assuming simple substitution patterns. Markets with adverse selection are inefficient because the incremental price of coverage is the same for all consumers and, therefore, differs from the incremental cost of coverage. Welfare-maximizing regulation is summarized by a sufficient statistics formula. Moreover, the optimal regulation formula is closely related to risk-adjustment policies that are commonly employed in insurance markets. While this suggests that interventions in the intensive margin are important, we caution that finer points of the optimal regulation formula have to be modified in richer settings, and leave a comprehensive analysis to future work (Azevedo and Gottlieb (in preparation)).
2 Model

2.1 The Model

We consider competitive markets with a large number of consumers and free entry of identical firms operating at an efficient scale that is small relative to the market. To model the gamut of behavior relevant to policy discussions in a simple way, we take as given a set of potential contracts, preferences, and costs of supplying contracts.\(^8\) We restrict attention to a group of consumers who are indistinguishable with respect to characteristics over which firms can price discriminate.

Formally, firms offer contracts (or products) \(x\) in \(X\). Consumer types are denoted \(\theta\) in \(\Theta\). Consumer type \(\theta\) derives utility \(U(x, p, \theta)\) from buying contract \(x\) at a price \(p\), and it costs a firm \(c(x, \theta) \geq 0\) measured in units of a numeraire to supply it. Utility is strictly decreasing in price. There is a positive mass of consumers, and the distribution of types is a measure \(\mu\).\(^9\) An economy is defined as \(E = [\Theta, X, \mu]\).

2.2 Clarifying Examples

The following examples clarify the definitions, limitations of the model, and the goal of deriving robust predictions in a wide range of selection markets. Parametric assumptions in the examples are of little consequence to the general analysis, so that some readers may prefer to skim over details. We begin with the classic Akerlof (1970) model, which is the dominant framework in applied work. It is simple enough that the literature mostly agrees on equilibrium predictions.

Example 1. (Akerlof) Consumers choose whether to buy a single insurance product, so that \(X = \{0, 1\}\). Utility is quasilinear,

\[
U(x, p, \theta) = u(x, \theta) - p,
\]

and the contract \(x = 0\) generates no cost or utility, \(u(0, \theta) \equiv c(0, \theta) \equiv 0\). Thus, in equilibrium it has a price of 0. All that matters is the joint distribution of willingness to pay \(u(1, \theta)\) and costs \(c(1, \theta)\), which is given by the measure \(\mu\).

A competitive equilibrium in the Akerlof model has a compelling definition and is amenable to an insightful graphical analysis. Following Einav et al. (2010a), let the demand curve \(D(p)\) be the mass of agents with willingness to pay higher than \(p\), and let \(AC(q)\) be the average cost

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\(^8\)This approach is similar to Veiga and Weyl (2014a,b) and to the value and cost functions of Einav et al. (2009, 2010a).

\(^9\)The relevant \(\sigma\)-algebra and detailed assumptions are described below.
Figure 2: Weak equilibria in the (a) Akerlof and (b) Rothschild and Stiglitz models.

Notes: Panel (a) depicts demand $D(p)$ and average cost $AC(q)$ curves in the Akerlof model, with quantity in the horizontal axis, and prices in the vertical axis. The equilibrium price of contract $x = 1$ is denoted by $p^*$. Panel (b) depicts two weak equilibria of the Rothschild and Stiglitz model. The horizontal axis depicts contracts, and the vertical axis prices. $IC^L$ and $IC^H$ are indifference curves of type $L$ and $H$ consumers. $L$ and $H$ denote the contract-price pairs chosen by each type in these weak equilibria, which are the same as in Rothschild and Stiglitz (1976) when Bertrand equilibrium exists. The relative size of the circles $L$ and $H$ represents the mass of agents of each type. The bold curves $p(x)$ (black) and $\tilde{p}(x)$ (gray) depict two weak equilibrium price schedules. $p(x)$ is also a (refined) equilibrium price, but $\tilde{p}(x)$ is not.

of the $q$ consumers with highest willingness to pay. An equilibrium in the Akerlof (1970) and Einav et al. (2010a) sense is given by the intersection between the demand and average cost curves, depicted in Figure 2a. At this price and quantity, consumers behave optimally and the price of insurance equals the expected cost of providing coverage. If the average cost curve is always above demand, then the market unravels and equilibrium involves no transactions.

This model is restrictive in two important ways. First, contract terms are exogenous. This is important because market participants and regulators often see distortions in contract terms as crucial. In fact, many of the interventions in markets with adverse selection regulate contract dimensions directly, aim to affect them indirectly, or try to shift demand from some type of contract to another. It is impossible to consider the effect of these policies in the Akerlof model. Second, there is a single non-null contract. This is also considerably restrictive. For example, Handel et al. (2013) approximate American health insurance exchanges by assuming that they offer only two types of plans (corresponding to $x = 0$ and $1$).

\footnote{Formally, under appropriate assumptions, the definitions are
\begin{align*}
D(p) &= \mu(\{\theta : u(1, \theta) \geq p\}) \\
AC(q) &= \mathbb{E}[c(1, \theta)|\mu, u(1, \theta) \geq D^{-1}(q)].
\end{align*}
\(x = 1\), and that consumers are forced to choose one of them.\(^{11}\) Likewise, Hackmann et al. (2014) and Scheuer and Smetters (2014) lump the choice of buying any health insurance as \(x = 1\).

The next example, the Rothschild and Stiglitz (1976) model, endogenously determines contract characteristics. However, consumer preferences are highly stylized. Still, this model already exhibits problems with existence of equilibrium, and there is no consensus about equilibrium predictions.

**Example 2.** (Rothschild and Stiglitz) Each consumer may buy an insurance contract in \(X = [0, 1]\), which insures her for a fraction \(x\) of a possible loss of \(l\). Consumers differ only in the probability \(\theta\) of a loss. Their utility is

\[
U(x, p, \theta) = \theta \cdot v(W - p - (1 - x)l) + (1 - \theta) \cdot v(W - p),
\]

where \(v(\cdot)\) is a Bernoulli utility function and \(W\) is wealth, both of which are constant in the population. The cost of insuring individual \(\theta\) with policy \(x\) is \(c(x, \theta) = \theta \cdot x \cdot l\). The set of types is \(\Theta = \{L, H\}\), with \(0 < L < H < 1\). The definition of an equilibrium in this model is a matter of considerable debate, which we address in the next section.

We now illustrate more realistic multidimensional heterogeneity with an empirical model of preferences for health insurance used by Einav et al. (2013).

**Example 3.** (Einav et al.) Consumers are subject to a stochastic health shock \(l\) and, after the shock, decide the amount \(e\) they wish to spend on health services. Consumers are heterogeneous in their distribution of health shocks \(F_\theta\), risk aversion parameter \(A_\theta\), and moral hazard parameter \(H_\theta\).

For simplicity, we assume that insurance contracts specify the fraction \(x \in X = [0, 1]\) of health expenditures that are reimbursed. Utility after the shock equals

\[
CE(e, l; x, p, \theta) = [(e - l) - \frac{1}{2H_\theta}(e - l)^2] + [W - p - (1 - x)e],
\]

where \(W\) is the consumer’s initial wealth. The privately optimal health expenditure is \(e = l + H_\theta \cdot x\), so that in equilibrium

\[
CE^*(l; x, p, \theta) = W - p - l + l \cdot x + \frac{H_\theta}{2} \cdot x^2.
\]

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\(^{11}\)In practice, Affordable Care Act exchanges offer bronze, gold, silver and platinum plans, with approximate actuarial values ranging from 60% to 90%. Within categories plans vary in important dimensions such as the quality of their hospital networks. Silver is the most popular option, and over 10% of adults were uninsured in 2014.
Einav et al. (2013) assume constant absolute risk aversion (CARA) preferences before the health shock, so that ex-ante utility equals

\[ U(x, p, \theta) = \mathbb{E}[\exp\{-A_\theta \cdot CE^*(l; x, p, \theta)\} | l \sim F_\theta]. \]

For our numerical examples below, we assume that losses are normally distributed with mean \( M_\theta \) and variance \( S_\theta^2 \), which leaves us with four dimensions of heterogeneity.\(^{12}\) Calculations show that the model can be described with quasilinear preferences as in equation (1), with willingness to pay and cost functions

\[ u(x, \theta) = x \cdot M_\theta + \frac{x^2}{2} \cdot H_\theta + \frac{1}{2} x(2 - x) \cdot S_\theta^2 A_\theta, \text{ and} \]

\[ c(x, \theta) = x \cdot M_\theta + x^2 \cdot H_\theta. \]

The formula decomposes willingness to pay into three terms. The first term \( xM_\theta \) equals average covered expenses, which is also part of firm costs. The second term \( x^2H_\theta/2 \) represents the utility from overconsumption of health services, which is caused by moral hazard. Note that the moral hazard component costs twice as much to firms as consumers are willing to pay for it. Finally, the last term represents the insurance value of policy \( x \), which is greater for agents who are more risk averse and who have higher variance in health status, and costs nothing to risk-neutral insurers.

The previous example illustrates that the framework can fit multidimensional heterogeneity in a realistic empirical model. Moreover, it can incorporate ex-post moral hazard through the definitions of the utility and cost functions. The model can fit other types of consumer behavior, such as ex-ante moral hazard, non-expected utility preferences, overconfidence, or inertia to abandon a default choice. It can also incorporate administrative or other per-unit costs on the supply side. Moreover, it is straightforward to consider more complex contract features, including deductibles, copays, stop-losses, franchises, network quality, managed restrictions on expenses, and so on.

In this last example and in other models with complex contract spaces and rich heterogeneity, there is no agreement on a reasonable equilibrium prediction. Unlike the Rothschild and Stiglitz model, where there is controversy about what the correct prediction is, in this case the literature offers almost no possibilities.

\(^{12}\)Because of the normality assumption, losses and expenses may be negative in the numerical example. We report this parametrization because the closed form solutions for utility and cost functions make the model more transparent, and because qualitative features of the equilibria are similar to more realistic parametric assumptions.
2.3 Assumptions

The assumptions we make are mild enough to include all the examples above, so applied readers may wish to skip this section. On a first read, it is useful to keep in mind the particular case where $X$ and $\Theta$ are compact subsets of Euclidean space, utility is quasilinear as in equation (1), and $u$ and $c$ are continuously differentiable. These assumptions are considerably stronger than what is needed, but they are weak enough to incorporate most models in the literature. We begin with technical assumptions.

Assumption 1. (Technical Assumptions) $X$ and $\Theta$ are compact and separable metric spaces. Whenever referring to measurability we will consider the Borel $\sigma$-algebra over $X$ and $\Theta$, and the product $\sigma$-algebra over the product space. In particular, we take $\mu$ to be measurable with respect to the Borel $\sigma$-algebra.

Note that $X$ and $\Theta$ can be infinite dimensional, and the distribution of types can admit a density with infinite support, may be a sum of point masses, or a combination of the two. We now consider a more substantive assumption.

Assumption 2. (Bounded Marginal Rates of Substitution) There exists a constant $L$ with the following property. Take any $p \leq p'$ in the image of $c$, any $x, x'$ in $X$, and any $\theta \in \Theta$. Assume that

$$U(x, p, \theta) \leq U(x', p', \theta),$$

that is, that a consumer prefers to pay more to purchase contract $x'$ instead of $x$. Then, the price difference is bounded by

$$p' - p \leq L \cdot d(x, x').$$

That is, the willingness to pay for an additional unit of any dimension of insurance is bounded. The assumption is simpler to understand when utility is quasilinear and differentiable. In this case it is equivalent to the absolute value of the derivative of $u$ being uniformly bounded.

Assumption 3. (Continuity) The functions $U$ and $c$ are continuous.

Continuity of the utility function is not very restrictive because of Berge’s Maximum Theorem. Even with moral hazard, utility is continuous under standard assumptions. Continuity of the cost function is more restrictive. It implies that we can only consider models with moral hazard where the payoffs to the firm vary continuously with agent types and the contract. This typically fails in models where agents change their actions discontinuously with small changes in a contract. Nevertheless, it is possible to include some models with moral hazard in our framework. See Kadan et al. (2014) Section 9 for a discussion of how to define a metric over a contract space, starting from a description of actions and states.
3 Competitive Equilibrium

3.1 Definition of a Weak Equilibrium

We now define a minimalistic equilibrium notion, a weak equilibrium, requiring only that firms make no profits and consumers optimize. A vector of \textbf{prices} is a measurable function $p : X \rightarrow \mathbb{R}$, with $p(x)$ denoting the price of contract $x$. An \textbf{allocation} is a measure $\alpha$ over $\Theta \times X$ such that the marginal distribution satisfies $\alpha|\Theta = \mu$. That is, $\alpha(\{\theta, x\})$ is the measure of $\theta$ types purchasing contract $x$.\footnote{This formalization is slightly different than the traditional way of denoting an allocation as a map from types to contracts. We take this approach because different agents of the same type may buy different contracts in equilibrium, as in Chiappori et al. (2010).} We are often interested in the expected cost of supplying a contract $x$, and use the following shorthand notation for conditional moments:

$$
\mathbb{E}_x[c|\alpha] = \mathbb{E}[c(\tilde{x}, \tilde{\theta})|\alpha, \tilde{x} = x].
$$

That is, $\mathbb{E}_x[c|\alpha]$ is the expectation of $c(\tilde{x}, \tilde{\theta})$ according to the measure $\alpha$ and conditional on $\tilde{x} = x$. Note that such expectations depend on the allocation $\alpha$. When there is no risk of confusion we omit $\alpha$, writing simply $\mathbb{E}_x[c]$. Similar notation is used for other moments.

\textbf{Definition 1.} The pair $(p^*, \alpha^*)$ is a \textbf{weak equilibrium} if

1. For each contract $x$ firms make no profits. Formally,

$$p^*(x) = \mathbb{E}_x[c|\alpha^*]$$

almost everywhere according to $\alpha^*$.

2. Consumers select contracts optimally. Formally, for almost every $(\theta, x)$ with respect to $\alpha^*$, we have

$$U(x, p^*(x), \theta) = \sup_{x' \in X} U(x', p^*(x'), \theta).$$

Note that this is a price-taking definition, not a game-theoretic one. Consumers optimize taking prices as given, as do firms, who also take the average costs of buyers as given. Note also that there is no individual rationality requirement. This can be modeled by assuming that there exists a null contract that costs nothing and provides zero utility.

A final observation is that we only ask that prices equal expected costs almost everywhere according to $\alpha$.\footnote{The reason is that conditional expectation is only defined almost everywhere. Although it is possible to understand all of our substantive results without recourse to measure theory, we refer interested readers to Billingsley (2008) for a formal definition of conditional expectation.} In particular, weak equilibria place no restrictions on the prices of contracts.
that are not purchased. As demonstrated in the examples below, this is a serious problem with this definition, and the reason why a stronger equilibrium notion is necessary.

3.2 Equilibrium Multiplicity and Free Entry

We now illustrate that weak equilibria are compatible with a wide variety of outcomes, most of which are unreasonable in a competitive marketplace.

Example 2′. (Rothschild and Stiglitz - Multiplicity of Weak Equilibria) We first revisit Rothschild and Stiglitz’s (1976) original equilibrium. They set up a Bertrand game with identical firms and showed that, when a Nash equilibrium exists, it has allocations given by the points $L$ and $H$ in Figure 2b. High-risk consumers buy full insurance $x_H = 1$ at actuarially fair rates $p_H = H \cdot l$. Low-risk types purchase partial insurance, with actuarially fair prices reflecting their lower risk. The level of coverage $x_L$ is just low enough so that high-risk agents do not wish to purchase contract $x_L$. That is, $L$ and $H$ are on the same indifference curve $IC^H$ of high types.

Note that we can find weak equilibria with the same allocation. One example of weak equilibrium prices is the curve $p(x)$ is Figure 2b. The zero profits condition is satisfied because the prices of the two contracts that are traded, $x_L$ and $x_H$, equal the average cost of providing them. The optimization condition is also satisfied because the price schedule $p(x)$ is above the willingness to pay given by the curves $IC^L$ and $IC^H$. Therefore, no consumer wishes to purchase a different bundle.

However, many other weak equilibria exist. One example is the same allocation with the $\tilde{p}(x)$ curve in Figure 2b. Again, firms make no profits because the prices of $x_H$ and $x_L$ are actuarially fair, and consumers are optimizing because the price of other contracts is higher than their indifference curves.

There are also weak equilibria with completely different allocations. For example, it is a weak equilibrium for all agents to purchase full insurance, and for all other contracts to be priced so high that no one wishes to buy them. This does not violate the zero profits condition because the expected cost of contracts that are not traded is arbitrary. This weak equilibrium has full insurance, which is the first-best outcome in this model. It is also a weak equilibrium for no insurance to be sold, and for prices of all contracts with positive coverage to be prohibitively high. Therefore, weak equilibria provide very coarse predictions, with the Bertrand solution, full insurance, complete unraveling, and many other outcomes all being possible.

In a market with free entry, however, some weak equilibria are more reasonable than others. Take the weak equilibrium with complete unravelling. Suppose firms enter the market a policy with positive coverage, driving down its price. Initially no consumers would
purchase the policy, and firms would continue to break even. As prices decrease enough to reach the indifference curve of type $H$, they would start buying. At this point, firms make money because risk averse consumers are willing to pay a premium for insurance, so that the average cost is higher than the price. Therefore, this weak equilibrium conflicts with the idea of free entry. A similar tâtonnement eliminates the full insurance weak equilibrium. If firms enter the market for partial insurance policies, driving down its price, they would initially not attract any consumers. However, as the price decreases enough to reach the indifference curve of type $L$, they would make money because they would attract only the safest consumers.

This argument also eliminates the weak equilibrium associated with $\tilde{p}(x)$. Let $x_0 < x_L$ be a non-traded contract with $\tilde{p}(x_0) > p(x)$. Suppose firms enter the market for $x_0$, driving down its price. Initially no consumers would purchase $x_0$, and firms would continue to break even. As prices decrease enough to reach $p(x_0)$, the $L$ types become indifferent between purchasing $x_0$ or not. If they decrease any further, all the $L$ types would purchase contract $x_0$. At this point, firms would lose money because the average cost would be higher than the price.\footnote{To see why, note that in this weak equilibrium the $L$ types buy $x_L$ at an actuarially fair price. Therefore, the only reason why they would purchase less insurance is if firms sell it at a loss.} The price of $x_0$ would be driven down to $p(x_0)$, at which point it is no longer advantageous for firms to enter. We are then left with $p(x)$ and the allocation in Figure 2b as the unique equilibrium prediction.

### 3.3 Definition and Existence of an Equilibrium

We now define an equilibrium concept that formalizes the free entry argument. Equilibria are required to be robust to small perturbations of a given economy. A perturbation has a large but finite set of contracts approximating $X$. The perturbation adds a small measure of behavioral agents, who always purchase each of the existing contracts and impose no costs on firms. The point of considering the perturbed economy is that all contracts are traded, eliminating the paradoxes associated with defining the average cost of non-traded contracts.

We introduce, for each contract $x$, a behavioral consumer type who always demands contract $x$. We write $x$ for such a behavioral agent, and extend the utility and cost functions so that $U(x, p, x) = \infty$, $U(x', p, x) = 0$ if $x' \neq x$, and $c(x, x) = 0$. For clarity, we refer to non-behavioral types as standard types.

**Definition 2.** Consider an economy $E = [\Theta, X, \mu]$. A perturbation of the economy $E$ is an economy where the set of contracts is a finite set $\bar{X} \subseteq X$, and there is a small mass of behavioral agents demanding each contract in $\bar{X}$. Formally, a **perturbation** $(E, \bar{X}, \eta)$ is an economy $[\Theta \cup \bar{X}, \bar{X}, \mu + \eta]$, where $\bar{X} \subseteq X$ is a finite set, and $\eta$ is a strictly positive measure.
The next definition says that a sequence of perturbations converges to the original economy if the set of contracts fills in the original set of contracts, and if the total mass of behavioral agents converges to 0.

**Definition 3.** A sequence of perturbations \((E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}\) converges to \(E\) if

1. Every point in \(X\) is the limit of a sequence \((x^n)_{n \in \mathbb{N}}\) with each \(x^n \in \bar{X}^n\).
2. The mass of behavioral types \(\eta^n(\bar{X}^n)\) converges to 0.

We now define what it means for a sequence of equilibria of perturbations to converge to the original economy.

**Definition 4.** Take an economy \(E\) and a sequence of perturbations \((E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}\) converging to \(E\), with weak equilibria \((p^n, \alpha^n)\). The **sequence of perturbed equilibria** \((p^n, \alpha^n)_{n \in \mathbb{N}}\) **converges to a price allocation pair** \((p^*, \alpha^*)\) of \(E\) if

1. The allocations \(\alpha^n\) converge weakly to \(\alpha^*\).
2. For every sequence \((x^n)_{n \in \mathbb{N}}\), with each \(x^n \in \bar{X}^n\) and limit \(x \in X\), \(p^n(x^n)\) converges to \(p^*(x)\). \(\text{\cite{footnote:17}}\)

We are now ready to define an equilibrium.

**Definition 5.** The pair \((p^*, \alpha^*)\) is an **equilibrium** of \(E\) if there exists a sequence of perturbations that converges to \(E\) and an associated sequence of weak equilibria that converges to \((p^*, \alpha^*)\).

The most transparent way to understand how equilibrium formalizes the free entry idea is to return to the Rothschild and Stiglitz model from Example 2. Recall that there is a weak equilibrium where no one purchases insurance and prices are high. But this is not an equilibrium. In any perturbation, if standard types do not purchase insurance, prices are driven to 0 by behavioral types, contradicting the high prices. Likewise, the weak equilibrium corresponding to \(\tilde{p}\) in Figure 2b is not an equilibrium. Consider a contract \(x_0 < x_L\) with \(\tilde{p}(x_0) > p(x_0)\). In any perturbation, if prices are close to \(\tilde{p}\) then none of the standard types

\footnote{Note that both an economy and its perturbations have a set of types contained in \(\Theta \cup X\), and contracts contained in \(X\). To save on notation, we extend distributions of types to be defined over \(\Theta \cup X\) and allocations to be defined over \((\Theta \cup X) \times X\). With this notation, measures pertaining to different perturbations are defined in the same space.}

\footnote{Note that, in a perturbation, prices are only defined for a finite subset \(\bar{X}^n\) of contracts. The definition of convergence is strict in the sense that, for a given contract \(x\), prices must converge to the price of \(x\) for any sequence of contracts \(x^n\) converging to \(x\).}
would want to purchase $x_0$. But this would make the price of $x_0$ equal to 0 because the only way to sustain positive prices in a perturbation is by attracting standard types. In fact, equilibria of perturbations sufficiently close to $E$ involve most $L$ types purchasing contracts similar to $x_L$, and most $H$ types purchasing contracts similar to $x_H$. The price of any contract $x_0 < x_L$ must be such that $L$ types are indifferent between $x_0$ and $x_L$. There is a small mass of $L$ types purchasing $x_0$ to maintain the indifference. If prices were lower, then the $L$ types would flood the market for $x_0$ and make firms lose money. If prices were higher, no $L$ types would purchase $x_0$. The only equilibrium is that corresponding to $p(x)$ in Figure 2b (this is proven in Appendix B).

This example clarifies that the mechanics of equilibrium are similar to the standard analysis of the Akerlof model from Example 1. In the example depicted in Figure 2a, the only equilibrium is that associated with the intersection of demand and average cost.\footnote{There are other weak equilibria in the example in Figure 2a, but the only equilibrium is the intersection between demand and average cost. For example, it is a weak equilibrium for no one to purchase insurance, and for prices to be very high. But this is not an equilibrium. The reason is that, in a perturbation, behavioral agents make the average cost curve well-defined for all quantities, including 0. The perturbed average cost curve is continuous, equal to 0 at a quantity of 0, and slightly lower than the original. As the mass of behavioral agents shrinks, the perturbed average cost curve approaches its value in the original economy. Consequently, the only equilibrium is the standard solution, where demand and average cost intersect.} This is similar to the way that prices for $x_L$ and $x_H$ are determined in example 2. If the average cost curve were always above the demand curve, the only equilibrium would be complete unraveling. This is analogous to the way that the market for contracts other than $x_L$ and $x_H$ unravels in Example 2.

There are two ways to think about the equilibrium requirement. One is that it consistently applies the logic of the Akerlof (1970) and Einav et al. (2010a) models to the case where there is more than one potential contract. This is similar to the intuitive free entry argument discussed in Section 3.2. Another interpretation is that the definition demands a minimal degree of robustness with respect to perturbations, while paradoxes associated with conditional expectation do not occur in perturbations. This rationale is similar to proper equilibria (Myerson, 1978).

We now show that equilibria always exist.

\textbf{Theorem 1.} \textit{Every economy has an equilibrium.}

The proof is based on two observations. First, equilibria of perturbations exist by a standard fixed-point argument. Second, equilibrium price schedules in any perturbation are uniformly Lipschitz. This is a consequence of the bounded marginal rate of substitution (Assumption 2). The intuition is that, if prices increased too fast with $x$, no standard types would be willing to purchase more expensive contracts. This is impossible, however, because a contract cannot have a high equilibrium price if it is only purchased by the low-cost behavioral types. We then apply the Arzelà–Ascoli Theorem to demonstrate existence of equilibria.
Figure 3: Equilibrium (a) and optimal (b) price schedules in the multidimensional health insurance model, Example 3.

Notes: Panel (a) illustrates equilibrium prices and quantities in the multidimensional health insurance model, Example 3. The solid curve denotes prices. The size of the circles represent the mass of consumers purchasing each contract, and its height represents the average loss parameter of such consumers, that is $E_x[M]$. Panel (b) illustrates the equilibrium demand profile. Each of the 5,000 points represents a randomly drawn type from the population. The horizontal axis represents expected health shock $M_\theta$, and the vertical axis represents the absolute risk aversion coefficient $A_\theta$. The colors represent the level of coverage purchased in equilibrium.

Existence only depends on the assumptions of Section 2.3. Therefore, equilibria are well-defined in a broad range of theoretical and empirical models. Equilibria exist not only in stylized models, but also in rich multidimensional settings. Figure 3 plots an equilibrium of the Einav et al. model (Example 3). Equilibrium makes sharp predictions, displays adverse selection, with costlier consumers purchasing higher coverage, and consumers sort across the four dimensions of private information. We return to this example below, when we turn to applications.

4 Discussion

This section establishes consequences of competitive equilibrium, and discusses the relationship to existing solution concepts.

4.1 Equilibrium Properties

We begin by describing some properties of equilibria.

Proposition 1. Let $(p^*, \alpha^*)$ be an equilibrium of economy $E$. Then:

1. The pair $(p^*, \alpha^*)$ is a weak equilibrium of $E$. 


2. For every contract \( x' \in X \) with strictly positive price, there exists \((\theta, x)\) in the support of \( \alpha^* \) such that

\[
U(x, p^*(x), \theta) = U(x', p^*(x'), \theta) \quad \text{and} \quad c(x', \theta) \geq p(x').
\]

That is, every contract that is not traded in equilibrium has a low enough price for some consumer to be indifferent between buying it or not, and the cost of this consumer is at least as high as the price.

3. The price function is \( L \)-Lipschitz, and, in particular, continuous.

4. If \( X \) is a subset of Euclidean space, then \( p^* \) is Lebesgue almost everywhere differentiable.

The proposition shows that equilibria have several regularity properties. By part 1, they are weak equilibria. Moreover, equilibrium prices are always continuous and differentiable almost everywhere, as in the examples. Finally, part 2 shows that the price of any out-of-equilibrium contract is either equal to 0, or low enough so that some type is indifferent between buying it or not. Moreover, the cost of selling to this indifferent type is at least as high as the price. Intuitively, these are the consumer types who make the market for that contract unravel.\(^{19}\)

The following corollary notes that the proposition offers a simple way to find equilibria. For instance, it gives a simple way to prove that the unique equilibrium in Example 2 is that associated with \( p(x) \) in Figure 2b.

**Corollary 1.** If \((p^*, \alpha^*)\) is the unique weak equilibrium satisfying any subset of the conditions 2-4 in Proposition 1, then it is the unique equilibrium.

### 4.2 Strategic Foundations

Our equilibrium concept can be justified as the limit of a strategic model. This relates our work to the large literature on game-theoretic competitive screening models. Moreover, the assumptions on the strategic game clarify the limitations of our model and the situations where we expect competitive equilibrium to be a reasonable prediction.

We consider such a strategic setting in Appendix C. We start from a perturbation \((E, \bar{X}, \eta)\). Each contract has \( n \) differentiated varieties, and each variety is sold by a different

\(^{19}\)These conditions are necessary but not sufficient for an equilibrium. The reason is that the existence of a type satisfying the conditions in Part 2 of the proposition does not imply that the market for a contract \( x \) would unravel in a perturbation. This may happen because, with multiple dimensions of private information, there can be other types who are indifferent between purchasing \( x \) or not, and some of them may have lower costs. A simple example where this happens are models with preferences similar to Chang (2010); Guerrieri and Shimer (2013), where contracts specify the probability of trading and prices are set competitively.
firm. Consumers have logit demand shocks for the varieties, with a common semi-elasticity parameter $\sigma$. We assume that firms have a small efficient scale. To capture this in a simple way, we assume that each firm can only serve up to a fraction $\bar{q}$ of consumers. Firms cannot turn away consumers, as would be the case with community rating regulations. Thus, the three key parameters are the number of varieties of each contract $n$, the semi-elasticity of demand $\sigma$, and the maximum scale of each firm $\bar{q}$.

We consider symmetric Bertrand-Nash equilibria, where firms independently set prices. Proposition C1 shows that Bertrand-Nash equilibria exist as long as $1/n < \bar{q}$ and $\bar{q}$ is bounded above by a constant. Therefore, equilibria exist as long as there are enough firms, and the efficient scale is sufficiently small. Moreover, this is true regardless of the semi-elasticity parameter. Even if firms are close to the limit of no differentiation, equilibria exist for a fixed scale and number of firms. At a first blush, this result seems to contradict the finding that the Rothschild and Stiglitz model often has no Nash equilibrium (Riley, 1979). The reason why our definition works is that the profitable deviations in the Rothschild and Stiglitz model rely on firms setting very low prices and attracting a sizable portion of the market. However, this is not possible if firms have small scale and cannot turn consumers away. Besides establishing existence of an equilibrium, Proposition C1 shows that profits per contract are bounded above by $2/\sigma$. Therefore, as firms become undifferentiated, profits converge to 0.

Proposition C2 then shows that, for a fixed number of varieties $n$ and scale $\bar{q}$ satisfying the conditions for existence, Bertrand-Nash equilibria converge to a competitive equilibrium as the semi-elasticity parameters converge to infinity. Thus, our competitive equilibrium corresponds to the limit of this game-theoretic model.

These results have three consequences. First, convergence to competitive equilibrium is a relatively brittle result because it depends on the Bertrand assumption, on the number of varieties and maximum scale satisfying a pair of inequalities, and on semi-elasticities growing at a fast enough rate relative to those parameters. This is to be expected because existing strategic models lead to very different conclusions with small changes in assumptions. Second, although convergence depends on special assumptions, it is not a knife-edge case. There exists a non-trivial set of parameters for which equilibria are justified by a strategic model. Finally, the sufficient conditions give some insight into situations in which competitive equilibrium is a reasonable solution concept. Namely, when there are many firms, efficient scales are small relative to the market, and firms are very undifferentiated. The results do not imply that markets with adverse selection are always close to perfect competition. Indeed, market power is often an issue in such markets (see Dafny, 2010; Dafny et al., 2012; Starc, 2014). Nevertheless, our sufficient conditions are similar to those in markets without adverse selection: the presence of many, undifferentiated firms, with scales that are small relative to the market.
4.3 Relationship with the Literature

Our price-taking approach is reminiscent of the early work by Akerlof (1970) and Spence (1973). Multiplicity of weak equilibria is well-known since Spence’s (1973) analysis of labor market signaling. Much like the discussion in Section 3, Spence (1973) noted that a large variety of outcomes are possible under weak equilibria.

The literature addressed equilibrium multiplicity in three ways. One strand of the literature employed game-theoretic equilibrium notions and restrictions on consumer heterogeneity, typically in the form of ordered one-dimensional sets of types. This is the case in the competitive screening literature, initiated with Rothschild and Stiglitz’s (1976) Bertrand game, which led to the issue of non-existence of equilibria. Subsequently, Riley (1979) showed that Bertrand equilibria do not exist for a broad (within the one-dimensional setting) class of preferences, including the standard Rothschild and Stiglitz model with a continuum of types. Wilson (1977), Miyazaki (1977), Riley (1979), and Netzer and Scheuer (2014), among others, proposed modifications of Bertrand equilibrium so that an equilibrium always exists. It has long been known that the original Rothschild and Stiglitz game has mixed strategy equilibria, but only recently Luz (2013) has characterized them.

The literature on refinements in signaling games shares the features of game-theoretic equilibrium notions and restrictive type spaces. In order to deal with the multiplicity of price-taking equilibria described by Spence, this literature modeled signaling as a dynamic game. However, since signaling games typically have too many sequential equilibria, Banks and Sobel (1987), Cho and Kreps (1987), and several subsequent papers proposed equilibrium refinements that eliminate multiplicity.

Another strand of the literature considers price-taking equilibrium notions, like our work, but imposes additional structure on preferences, such as Bisin and Gottardi (1999, 2006), following work by Prescott and Townsend (1984). Notably, Dubey and Geanakoplos’ (2002) model of competitive pools introduced a refinement of weak equilibria using behavioral agents. This is similar to our construction in the case of a finite set of contracts. The main difference is that they consider the additional pool structure, a finite set of pools, and specification of states of the world and endowments for each household. Our framework imposes no such structure, and can thus accommodate standard applied models such as Example 3. Gale (1992), like us, considers general equilibrium in a setting with less structure than the

\[\text{As Crocker and Snow (1985a) show, the Miyazaki-Wilson equilibrium is constrained Pareto efficient in that its allocations maximize the low-risk type’s utility subject to incentive and zero profits constraints. The Rothschild-Stiglitz equilibrium is only constrained efficient when it coincides with the Miyazaki-Wilson equilibrium. Even when constrained efficient, these allocations have undesirable distributional properties as they attribute full weight to low risks. In fact, information-constrained regulators can implement more desirable allocations with taxes that redistribute from low- to high-risk consumers (Crocker and Snow (1985b)).}\]

\[\text{There has also been work on this type of game with nonexclusive competition. Attar et al. (2011) show that nonexclusive competition leads to outcomes similar to the Akerlof model.}\]
insurance pools. However, he refines his equilibrium with a stability notion based on Kohlberg and Mertens (1986). More recent contributions have considered general equilibrium models where firms can sell the right to choose from menus of contracts (Citanna and Siconolfi, 2014).

Our results are related to this previous work as follows. In standard one-dimensional models with single crossing, our unique equilibrium corresponds to what is usually called the “least-costly separating equilibrium” in the signaling literature. Thus, our equilibrium prediction is the same as in models without cross-subsidies, such as Riley (1979) and Bisin and Gottardi (2006), and the same as Rothschild and Stiglitz (1976) when their equilibrium exists. It also coincides with Banks and Sobel (1987) and Cho and Kreps (1987) in the settings they consider. It differs from equilibria that involve cross-subsidization, such as Wilson (1977), Miyazaki (1977), Hellwig (1987), and Netzer and Scheuer (2014). Our equilibrium differs from mixed strategy equilibria of the Rothschild and Stiglitz (1976) model, even as the number of firms increases. This follows from the Luz (2013) characterization. In the case of a pool structure and finite set of contracts, our equilibria are the same as in Dubey and Geanakoplos (2002).

Another strand of the literature considers preferences with less structure. Chiappori et al. (2006) consider a very general model of preferences within an insurance setting. This paper differs from our work in that they consider general testable predictions without specifying an equilibrium concept, while we derive sharp predictions within an equilibrium framework. Rochet and Stole (2002) consider a competitive screening model with firms differentiated as in Hotelling (1929), where there is no adverse selection. Their Bertrand equilibrium converges to competitive pricing as differentiation vanishes, which is the outcome of our model. However, Riley’s (1979) results imply that no Bertrand equilibrium would exist if one generalizes their model to include adverse selection.

Like our paper, Veiga and Weyl (2014b) propose a model with endogenous contract characteristics in a multidimensional framework. They consider an oligopoly model of competitive screening in the spirit of Rochet and Stole (2002), but where each firm can offer a single contract. Contract characteristics are determined by a simple first-order condition, as in the Spence (1975) model. Moreover, their model can incorporate imperfect competition, which is beyond the scope of the present work. Our numerical results suggest that the two models agree on many qualitative predictions. For example, insurance markets provide inefficiently low coverage, and increasing heterogeneity in risk aversion seems to attenuate adverse selection. In this sense their findings are complementary to ours.

The key difference is that their model has a single traded contract, while our model endogenously determines the set of traded contracts. In their model, when perfectly competitive
equilibria exist, all firms offer the same contract. In contrast, a rich set of contracts is offered in our equilibrium. For example, in the case of no adverse selection (when costs are independent of types), our equilibrium is for firms to offer all products priced at cost, which corresponds to the standard notion of perfect competition. A colorful illustration is tomato sauce. The Veiga and Weyl (2014b) model predicts that a single type of tomato sauce is offered cheaply, with characteristics determined by the preferences of average consumers. In contrast, our prediction is that many different types of tomato sauce are sold at cost. Italian style, basil, garlic lover, chunky, mushroom, and so on. In a less gastronomically titillating example, insurers offer a myriad types of life insurance: term life, universal life, whole life, combinations of these categories, and many different parameters within each category. Our results on the convergence of Bertrand equilibria suggest that the two models are appropriate in different situations. Their model of perfect competition seems more relevant when there are few firms, which are not very differentiated, the fixed cost of creating a new contract is high, and it is a good strategy for firms to offer products of similar quality as their competitors. That is, when firms herd on a particular type of contract, so that the symmetric equilibrium is plausible.

5 Application: Equilibrium Effects of Mandates

5.1 Calibration

We calibrated the multidimensional health insurance model in Example 3 to illustrate the equilibrium concept and understand equilibrium effects of policy interventions. To understand what effects are quantitatively plausible, we calibrated the model based on Einav et al. (2013)’s preference estimates from employees in a large US corporation.

We considered linear contracts and normal losses. The advantage of this parametric restriction is that willingness to pay and costs are transparently represented by Equation (2). Consumers differ along four dimensions: expected health loss, standard deviation of health losses, moral hazard, and risk aversion. All parameters are readily interpretable from Equation (2). We assumed that the distribution of parameters in the population is lognormal.
Table 1: Calibrated distribution of consumer types

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>H</th>
<th>M</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.0E-5</td>
<td>1,330</td>
<td>4,340</td>
<td>24,474</td>
</tr>
</tbody>
</table>

Log covariance

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.25</td>
<td>-0.01</td>
<td>-0.12</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>$\sigma_{\log H}^2$</td>
<td>-0.03</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>0.20</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: In all simulations of the health insurance example consumer type parameters are distributed log normally, with the moments in the table.

of Einav et al. (2013), as in Table 1. There are two exceptions. We reduced average risk aversion because linear contracts involve losses in a much wider range than the contracts in their data. The lower value of risk aversion better matched the substitution patterns in the data because constant absolute risk aversion models do not work well across different ranges of losses (Rabin, 2000 and Handel and Kolstad, 2013). The other exception is the log variance of moral hazard, which we vary in our simulations. In our baseline we set $\sigma_{\log H}^2$ to 0.28. See Appendix D for details on the calibration.

To calculate an equilibrium, we used a perturbation with 26 evenly spaced contracts, and added a mass equal to 1% of the population as behavioral agents. We then used a standard fixed point algorithm. In each iteration, consumers choose optimal contracts taking prices as given. Prices are adjusted up for unprofitable contracts and down for profitable contracts. Prices consistently converge to the same equilibrium for different initial values.

The equilibrium is depicted in Figure 3a. It features adverse selection with respect to the average loss, in the sense that, on average, consumers who purchase more coverage have higher losses. Moreover, agents sort across contracts in accordance to preferences. This is illustrated in Figure 3b, which displays the contracts purchased by agents with different expected loss and risk aversion parameters. The figure corroborates the existence of adverse selection on average loss, as agents with higher expected losses tend to choose more generous contracts. However, even for the same levels of risk aversion and expected loss, different consumers choose different contracts due to other dimensions of heterogeneity.

Although there is adverse selection, equilibria do not feature a complete “death spiral,” where no contracts are sold. This is not true in general, as in some examples the market completely unravels.\(^{26}\) It is also possible that the support of traded contracts is a strict large compact set does not meaningfully impact the numerical results.

\(^{26}\)Recall that the Akerlof model, in which a death spiral sometimes occurs, is a particular case of our framework. See Hendren (2013) for sufficient conditions under which markets unravel in the case of a fixed loss, no moral hazard, and consumers who only differ in the loss probability.
subset of all contracts. Whenever this is the case, buyers with the highest willingness to pay for each contract that is not traded value it below their own average cost (Proposition 1). That is, the markets for non-traded contracts are shut down by an Akerlof-type death spiral.

5.2 Policy Interventions

This section investigates the effects of policy interventions. We focus on a mandate requiring consumers to purchase at least 60% coverage. Equilibrium is depicted in Figures 1 and 4a. With the mandate, about 85% of consumers get the minimum coverage. Moreover, some consumers who originally chose policies with greater coverage switch to the minimum amount after the mandate. In fact, the mandate increases the fraction of consumers who buy 60% coverage or less, as only 80% of consumers did so before the mandate.

Some consumers reduce their coverage because the mandate exacerbates adverse selection on the intensive margin. With the mandate, many cheap consumers purchase the minimum coverage. This reduces the price of the 60% policy, attracting consumers who were originally purchasing more generous policies. In equilibrium consumers sort across policies so that prices are continuous (as must be the case by Proposition 1). This leads to a lower but steeper price schedule, so that some consumers choose less coverage.

Despite this unintended consequence, the mandate increases Kaldor-Hicks welfare in the baseline example by $127 per consumer.\(^\text{27}\) This illustrates that competitive equilibria are inefficient, and that even coarse policy interventions can have large benefits.

We calculated the price schedule that maximizes Kaldor-Hicks welfare (Figure 4b). This

\(^{27}\text{Note that some mandates can decrease welfare, as shown by Einav et al. (2010b).}\)
Table 2: Welfare and coverage under different scenarios

<table>
<thead>
<tr>
<th>X</th>
<th>Equilibrium</th>
<th>( \sigma_H^2 = 0.28 )</th>
<th>( \sigma_H^2 = 0.98 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Welfare</td>
<td>Equilibrium</td>
<td>Efficient</td>
</tr>
<tr>
<td>[0, 1]</td>
<td>0 0.46</td>
<td>279 0.8</td>
<td>0 0.43</td>
</tr>
<tr>
<td>[0.60, 1]</td>
<td>127 0.62</td>
<td>267 0.8</td>
<td>201 0.61</td>
</tr>
<tr>
<td>0, 0.90</td>
<td>97 0.66</td>
<td>250 0.9</td>
<td>137 0.63</td>
</tr>
<tr>
<td>0.60, 0.90</td>
<td>123 0.62</td>
<td>258 0.83</td>
<td>180 0.61</td>
</tr>
<tr>
<td>0, 0.60, 0.90</td>
<td>58 0.53</td>
<td>258 0.83</td>
<td>91 0.51</td>
</tr>
</tbody>
</table>

Notes: The table reports Kaldor-Hicks welfare as defined in Section 6, and the welfare of the unregulated market with \( X = [0, 1] \) is normalized to 0. Equilibrium calculations were performed with 1% behavioral agents. When the set of contracts includes an interval we added a contract every 0.04 coverage. Note that welfare is optimized with a tolerance of 1% gain in each iteration. Due to this tolerance our estimated welfare under efficient pricing is slightly higher with \( X = [0.60, 1] \) than with \( X = [0, 1] \), but we know theoretically that these are at most equal.

is the schedule that would be implemented by a regulator who maximizes welfare, can use cross subsidies, but does not posses more information than the firms do. The optimal price schedule is much flatter than the unregulated market or the mandate. That is, optimal regulation involves subsidies across contracts, aimed at reducing adverse selection on the intensive margin. Optimal prices increase welfare by $279 from the unregulated benchmark. Hence, addressing distortions related to contract characteristics can considerably increase welfare.

We considered variations of the model to understand whether the results are representative. Expected coverage and welfare are reported in Table 2, for different sets of contracts and log variance of moral hazard. Equilibrium behavior is robust to both changes. Moreover, in all cases, optimal regulation considerably increases welfare with respect to the 60% mandate. This is true if we move from contracts 0%, 60%, and 90% to 60% and 90%. Moreover, we replicate the result in Handel et al. (2013) and Veiga and Weyl (2014a) that the markets with only 60% and 90% contracts almost completely unravel. This suggests that our equilibrium results are not driven by idiosyncrasies in our parametric model.

Finally, the variance in moral hazard does not have a large qualitative impact on equilibrium, but considerably changes optimal regulation. For example, when \( X = [0, 1] \), the optimal allocation in the high moral hazard scenario gives about 84% coverage to all consumers, which is quite different from the rich menu in Figure 4b. This is because consumers with higher moral hazard tend to buy more insurance, but it is socially optimal to give them less insurance. Therefore, a regulator may give up using prices to screen consumers (we discuss this in detail below). From a broader perspective, this numerical result shows that the relative importance of different sources of heterogeneity can have a large impact on optimal
policy. Therefore, taking multiple dimensions of heterogeneity into account is important for government intervention.

6 Market Failure

This section gives an overview of the sources of market failure in competitive equilibria and explains the mechanics behind the numerical results. More specifically, we show that competitive equilibria are inefficient, that even simple regulations like mandates can increase welfare, and that optimal interventions involve subsidies on the intensive margin.

A general formal treatment of optimal regulation would require solving a multidimensional screening problem, which would make the exposition more involved. Moreover, it is possible to understand the key issues by focusing on the simple case below. Therefore, for clarity, we proceed informally, and refer readers to Azevedo and Gottlieb (in preparation) for a mathematically rigorous treatment.

We use the insurance terminology, although the analysis does not rely on parametric assumptions. The set of contracts is \( X = [0, 1] \). Preferences are quasilinear as in equation (1), ignoring income effects. We abstract away from redistributive issues, reclassification risk, the possibility to discriminate across subgroups of consumers, and assume that consumers have restrictive substitution patterns across contracts. We treat conditional expectations as functions, even though conditional expectations are only formally defined almost everywhere.

6.1 Inefficiency

We now show that the competitive equilibrium is inefficient when consumers have private information about costs. In this sense, our model echoes the standard view of economists, regulators, and market participants that adverse and advantageous selection causes market outcomes to be inefficient.

Define the (Kaldor-Hicks) welfare generated by an equilibrium \((p, \alpha)\) as

\[
\hat{u} - c d\alpha.
\]

(3)

Assume that cost, utility, and the endogenously determined price functions are continuously differentiable.

To understand the wedge between privately and socially optimal choices, fix a consumer \( \theta \). Denote marginal utility and marginal cost functions as \( mu(x, \theta) = \partial_x u(x, \theta) \) and \( mc(x, \theta) = \partial_x c(x, \theta) \). The consumer trades off the price against her willingness to pay for coverage, as in Figure 5. In an interior choice such as \( x_{eq} \), she equates marginal utility and the price \( p'(x) \) of an additional unit of coverage. However, the socially optimal choice \( x_{eff} \) equates
marginal utility and marginal cost. Therefore, whenever costs vary among consumers, there
is no reason to expect market outcomes to be efficient.

Figure 5: Inefficiency of private choices.

Notes: The figure illustrates the decision of an individual consumer \( \theta \). The privately optimal decision \( x^{eq} \) is to equate marginal utility \( mu \) with the price of an additional unit of coverage \( p' \). The socially optimal choice \( x^{eff} \) equates marginal utility with marginal cost \( mc \). The figure also depicts the average marginal cost curve of consumers purchasing each contract \( x \), in bold and denoted \( E_x[mc] \), and the marginal cost curves of other consumers purchasing \( x^{eq} \) as faded curves.

To understand the sources of inefficiency, define the intensive margin selection coefficient as

\[ S_I(x) = \partial_x E_x[c] - E_x[mc]. \]

\( S_I(x) \) is the cost increase per additional unit of insurance minus the average marginal cost of a unit of insurance. In other words, \( S_I(x) \) is how much costs increase with an increase in coverage due to selection. This coefficient is positive if, locally around a contract \( x \), agents who purchase more coverage are more costly, and it is negative if agents who purchase more coverage have lower costs. Thus, \( S_I \) is closely related to the positive correlation test of Chiappori and Salanié (2000). Moreover, it is natural to say that there is adverse selection around \( x \) if \( S_I(x) \) is positive, and advantageous selection if \( S_I(x) \) is negative. This definition of adverse selection is a local property. It is possible that there is adverse selection in one region of the contract space, and advantageous selection in another region.

The wedge can be divided in two components. First, in a multidimensional model many different types purchase contract \( x \). This is the case, for example, in our calibrated example, as depicted in Figure 3b. Therefore, there is no reason for the marginal cost of any single buyer of \( x \) to equal \( p'(x) \). Second, even the average marginal cost \( E_x[mc] \) of agents buying
where $\epsilon(x, \theta)$ is defined as

$$
\epsilon(x, \theta) = \frac{1}{x} \cdot \frac{\mu(x, \theta)}{\partial_{xx} u(x, \theta) - p''(x)}.
$$
although the perturbation extracts revenues from agents purchasing contracts higher than $x_0$, this is moot because we are ignoring redistribution and cost of public funds. Therefore, the total welfare effect equals

$$\mathbb{E}_{x_0}[\epsilon \cdot (mu - mc)] \cdot \frac{f(x_0) \cdot x_0}{p'(x_0)} \cdot dxdp'$$

plus higher-order terms, which become insignificant as $dx$, $dp'$, and $dp'/dx$ converge to 0. Consequently, optimal subsidies satisfy, for all $x$,

$$\mathbb{E}_x[\epsilon \cdot (p' - mc)] = 0.$$  

This expectation can be decomposed as

$$0 = \mathbb{E}_x[\epsilon] \cdot \mathbb{E}_x[p' - mc] - \text{Cov}_x[\epsilon, mc]$$

Rearranging this equation we find that optimal subsidies are given by the simple formula

$$t'(x) = S_I(x) - \frac{\text{Cov}_x[\epsilon, mc]}{\mathbb{E}_x[\epsilon]}.$$  

The intuition for the optimal regulation formula (4) is as follows. The term $S_I(x)$ compensates firms for the incremental costs that are due to selection. This term is closely related to risk-adjustment, a common policy intervention in insurance markets. Actual risk adjustment policies cross-subsidize insurance plans based on observable enrollee characteristics that are predictive of costs. Formulae differ in what characteristics are used, how expected costs are measured, and whether prospective or retrospective data is used. Both private and public sponsors use risk adjustment. For example, in employer-provided health insurance, plans are often priced according to expected costs of the average employee. Despite these differences, and practical shortcomings of actual risk adjustment policies, policymakers and market participants often believe that it is important for cross subsidies to lean against the wind of advantageous or adverse selection, consistent with the $S_I(x)$ term. Moreover, the differences

\footnote{This equals the percentage change in the choice of $x$ for a one percent of $p'(x)$ parallel shift in the $p'(\cdot)$ curve.}

\footnote{We caution readers that this formula is not mathematically rigorous because conditional expectations are only defined almost everywhere according to $\alpha$. Interested readers are again referred to Azevedo and Gottlieb (in preparation) for a formal mathematical analysis.}

\footnote{Practical hurdles include measurement biases (Wennberg et al., 2013), gaming by firms (Brown et al., 2014; Geruso and Layton, 2014), and failure to capture all relevant dimensions of heterogeneity (Shepard, 2014; Einav et al., 2015).}
between the optimal subsidy and the usual risk adjustment formulae clarifies that cross subsidies should fulfill economic objectives, as opposed to simply being equal to selection in a statistical sense. This is consistent with the shortcomings of purely statistical risk adjustment pointed out by Glazer and McGuire (2000) and Einav and Levin (2015).

To understand the intuition for the covariance term, recall the two sources of inefficiency identified in Section 6.1: marginal costs of any given agent are different than the price of additional coverage, and marginal costs and prices differ even on average. The risk-adjustment term $S_I(x)$ addresses this second source of inefficiency by eliminating adverse or advantageous selection from the point of view of the firms. The covariance term deals with the first source of inefficiency, which is due to the variation in marginal costs among different buyers of contract $x$. Hence, this term sorts consumers more efficiently. If buyers of $x$ with higher marginal costs also have more elastic demands, then the formula calls for a higher slope of prices $p'(x)$ than under pure risk adjustment, which induces the high cost consumers to purchase lower levels of coverage. If this correlation is negative, then incremental prices should be lower, inducing consumers with lower costs to purchase more insurance.

This formula is consistent with the numerical findings in Section 5. Namely, we found that optimal prices involved cross subsidies on the intensive margin to reduce adverse selection not only in whether consumers participate in the market or not, but also in what kinds of products they purchase.

Finally, our intuitive discussion considered the case where optimal regulation has an interval of contracts being offered. However, we saw in the numerical simulations that, with higher heterogeneity in moral hazard, the optimal allocation may involve all consumers purchasing the same contract. The intuition of why this can happen is simple. In the parametric example, the first-best contract for consumer $\theta$ can be calculated by equating marginal utility and marginal cost, which gives

$$x_{fb}^\theta = \frac{A_\theta S_\theta^2}{A_\theta S_\theta^2 + H_\theta}.$$

Note that this expression is decreasing in the moral hazard parameter. All things equal, the social planner prefers to provide less coverage to consumer who are more likely to engage in moral hazard. However, consumers with higher moral hazard parameters always wish to purchase more insurance. Hence, if all heterogeneity is in moral hazard, the planner prefers not to screen consumers and, instead, assigns the same bundle to everyone. This phenomenon has been described by Guesnerie and Laffont (1984) in one-dimensional screening models, who call it non-responsiveness.
7 Conclusion

This paper builds a competitive model of adverse selection. The model has endogenously determined contract characteristics and permits rich consumer heterogeneity, unlike standard models like Akerlof (1970) and Rothschild and Stiglitz (1976). Our equilibrium concept extends the logic of the Akerlof (1970) model beyond the case of a single contract and robustly makes sharp predictions. More specifically, Theorem 1 guarantees that an equilibrium exists, and the examples illustrate that equilibria often make sharp predictions.

Equilibria are inefficient and even simple interventions can increase welfare. Our numerical examples suggest that standard policies can have important unintended consequences. Moreover, policy interventions should take into account not only whether consumers participate in the market, but also effects on contract characteristics. This is in concert with the view of regulators, who often implement policies aimed at affecting these characteristics, and with Einav et al. (2009), who have suggested that contract characteristics may be important.

References


