Decision-making on cyber risk management: Interaction between market insurance and risk control measures under loss aversion

Abstract

This paper studies decision-making on cyber risk management in the presence of interdependent risk by considering market insurance and risk control measures: self-protection and self-insurance. We construct an economic framework with a descriptive decision model under loss aversion to reflect a market behavior, which has not been examined in the literature. The model demonstrates the effect of loss aversion on potential decision-making with interaction between cyber insurance and risk controls in different scenarios. We find that an agent with the reference point of self-protection as an essential effort against cyber risk is more likely to give up additional risk management tools (market insurance and self-insurance), supporting the presence of a fatalistic behavior against cyber loss in the current cyber-insurance market. We further compare our findings with empirical evidence on the frequency rate of cyber loss and find that it is more likely to implement additional measures particularly for small-sized companies as time goes on. The focus of our paper is on cyber risk, nonetheless, the results can be generalized to any other interdependent risk under loss aversion.

Keywords: Cyber risk, Decision theory, Loss aversion, Interdependent risk, Risk controls

JEL classification: D81; D90; H32

1 Introduction

The “Wannacry” ransomware attack in 2017 affected more than 230,000 computers running the Microsoft Windows operating system in 150 countries within one day; the estimated total monetary loss reached almost $4 billion (Berr, 2017). Deep Root Analytics, a data analysis company in the U.S., exposed a database including the personal information of 200 million American voters online in 2017 (Grenoble, 2017), a disclosure very different from the “Wannacry” attack, but a realistic threat and concern for firms, policymakers and individuals.[1] Such extreme events are less likely to occur than the smaller cyber events of daily life (Eling and Wirfs, 2019), but can trigger huge, catastrophic consequences.

One of the key specifics in cyber risk to trigger such a systemic loss is the interconnection in the network environment. For instance, anonymous attackers successfully paralyzed a number of web services operated by Domain Name System (DNS) and hijacked millions of internet-connected household appliances in October 2016 (Perlroth, 2016). This distributed denial of service (DDoS) attack was possible due to the spread of networked home electronics. The success rate of this kind of attack and the size of loss are expected to grow as the Internet of Things (IoT) expands and the interconnection becomes more complicated (EY, 2015).

Cyber events in the interconnected network environment might significantly disrupt businesses in a range of fields, leading to an increase in the awareness of cyber risk among policymakers, regulators and decision-makers in firms.[2] This should motivate decision-makers to invest significant resources in cyber risk management, including the option to buy insurance to transfer this risk. However, several surveys reveal a perception that decision-makers regard cyber events (e.g. hacking, unintended disclosure or intentional breach by an insider) as rather unrealistic to themselves, until they finally experience a loss.[3]

1 Other examples are the data breaches by hacking of Yahoo in 2013 (three billion accounts) and FriendFinder in 2016 (412 million users); see Armerding (2018) for information on biggest data breach events in history.

2 For example, PwC (2017) conducts a survey with 9,500 decision-makers across the globe (122 countries) and documents the increasing awareness of potential consequences by cyber events. Especially increasing awareness is also regularly reported in scenario analysis (see, e.g., Lloyd’s, 2017) and the estimated potential global loss due to cyber risk is in an area of $ 600 billion (see, e.g., CSIS, 2018).

3 For instance, Mersinas et al. (2016) carry out an experiment with 117 participants consisting of 59 IT security professionals and 58 students and observe a behavioral perception in line with Kahneman and Tversky (1979). de Smidt and Botzen (2018) conduct a survey to 1,891 professional decision-makers of Aon Risk Solutions in the Netherlands about cyber risk and analyze the result from 172 respondents that such fatalism exists in their minds, which they call “not-in-my-organization effect” (p. 247). Jalali et al. (2018) also argue that many organizations show a poor performance on cybersecurity management and constantly ignore or underestimate cyber risk. In addition, Schneier (2008) claims that security risk is not only an objective risk experienced in reality, but also a subjective feeling against uncertain, unmeasurable risk. Apart from cyber risk context, Kunreuther and Pauly (2018) find a similar perception that many uninsured individuals decide to take up insurance after suffering from a loss, which made them unhappy with being uninsured.

Some decision-makers also seem to have a fatalistic attitude towards cyber risk; although a threat becomes more realistic, no single party can be fully secure anyway, no matter what cyber risk management strategy it implements, thus decision-makers might perceive that the threat will not happen to them. The survey by the U.K. Government in 2018 (DCMS, 2018)[4] supports this idea by showing that the most common reason not to take up cyber-insurance is that the decision-makers do not see themselves at high risk. We follow Eling and Schnell (2016, p. 37) and call this cognition a "latent fatalism" towards cyber risk.

Extant literature about how agents make decisions facing cyber risk has already documented several results with respect to self-protection and the purchase of cyber-insurance (see, e.g., Gordon and Loeb, 2002; Lelarge and Bolot, 2009; Shetty et al., 2010; Hofmann and Ramaj, 2011; Ogut, Raghunathan and Menon, 2011; Wang, 2017).[5] However, all the above literature assumes rational decision makers under risk aversion, which – as indicated above – might not very well describe the actual behavior. Furthermore, no study has investigated the potential fatalistic behavior in the presence of interdependent risk.

To fill this gap in the literature, we study decision-making on interdependent cyber risk in the context of prospect theory (Kahneman and Tversky, 1979). We assume that agents are more averse against a loss than against a gain by comparing possible outcomes with a reference point. The reference points considered in this paper rely on two possible instruments: self-protection and self-insurance. Particularly, self-protection is the key to describe the status quo of business parties in the network environment to build a realistic set-up in the cyber risk context. It also plays a role of a public good (Lohse, Robledo and Schmidt, 2012), which can affect the loss probability of other agents in the interconnected network environment. In contrast, self-insurance does not affect the risk of others, but rather reduce the loss amount in a form of insurance policy (Ehrlich and Becker, 1972).

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4 The survey consists of 50 in-depth qualitative interviews and a survey among 1,519 UK businesses and 569 UK registered charities.

5 The specific environment that cyber-insurers face is considered to some extent in the literature. For instance, the level of investment in the security system is included to model demand (see, e.g., Lelarge and Bolot, 2009; Shetty et al., 2010; Ogut, Raghunathan and Menon, 2011; Hofmann and Ramaj, 2011; Wang, 2017). Several studies analyze the impact of interdependency between network systems on the loss probability (see, e.g., Lelarge and Bolot, 2009; Shetty et al., 2010; Ogut, Raghunathan and Menon, 2011; Hofmann and Ramaj, 2011). Some studies incorporate the lack to prove the loss occurrence by an insurer, which can be linked with the presence of information asymmetry (see, e.g., Lelarge and Bolot, 2009; Ogut, Raghunathan and Menon, 2011), which is not the focus of this paper. One relevant study to our focus is done by Verendel (2008), where the author attempts to use prospect theory framework to explain decisions on cyber security control. However, the author only considers decision on self-protection measure, but not market insurance and self-insurance. In addition, the possibility of interdependency in risks is not taken into account in the model.

We develop a conceptual model by comparing potential decisions on market insurance, self-insurance and both options in the presence of self-protection with the reference points under interdependent risk.[6] Based on the reference points and the model framework, we find that an agent with the reference point of self-protection as an essential effort against cyber risk is more likely to give up additional risk management tools (market insurance and self-insurance). Latent fatalism against cyber loss can be explained by our finding in that agents stay uninsured despite increasing awareness of cyber risk. We further attempt to provide an empirical evidence on the frequency rate of cyber loss to examine our theoretical findings.

Our findings are important to better understand the nature of cyber risks and their consequences for decision-making on cyber risk management. The results are useful for IT professionals and risk managers in firms and insurance companies that are developing cyber insurance policies. For firms, the results are important not only for decision on internal risk management, but also for new requirements of reporting cyber incidents because the requirement emphasizes the importance of risk control measures (Eling and Wirfs, 2019).[7] Furthermore, this paper contributes not only to the growing literature on cybersecurity in the business and economics domain (e.g., Nagurney and Shukla, 2017), but also to the general business and economics literature in that it applies a descriptive decision model (prospect theory) in the presence of interdependent risk.[8] The remainder of this paper is organized as follows: we provide a brief illustration on a descriptive decision model, prospect theory in Section 2 and develop our model framework by describing interdependent risk, market insurance and reference points in Section 3. This framework is used to show how agents can make decisions on market insurance and risk controls with different reference points in Section 4 and Section 5 concludes the paper.

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6 Table A1 in Appendix A categorizes the models for decision-making under risk into whether they consider risk control measures and loss aversion under the presence of interdependent risk and illustrates the positioning of this study in this categorization. As observed in the table, risk control measures are necessarily considered under the presence of interdependent risk. However, it is hard to observe a study that takes behavioral approach to the decision-making on the cyber security investment and the purchase of insurance. Our study fills this gap in the literature, particularly in the cyber risk context, that the decision-making on cyber risk management can be affected by the propensity for loss aversion in the presence of interdependent risk. We further clarify our contribution in Table A2 by comparing with two studies on insurance decision under loss aversion.

7 In the US, reporting requirements for data breaches have been introduced in many states since 2002 (NCSL, 2016). In the European Union such reporting requirements will apply from 2018 (European Union, 2016). If these are enforced, more data and information will be available. This has happened already in the US with data samples as the Privacy Rights Clearinghouse “Chronology of Data Breaches”.

8 This paper demonstrates the relevance of our results for policymakers, regulators and practitioners to this importance. For academics we present a new avenue for descriptive decision-making model by considering risk controls with interdependency in risks. Our paper thus provides an innovative framework for decision-making on risk transfer and control relevant not only to classical normative decision theory (e.g., Ehrlich and Becker, 1972; Dionne and Eeckhoudt, 1985; Lohse, Robledo and Schmidt, 2012), but also to descriptive decision theory (e.g., Schmidt, 2016).

2 Theoretical background: Prospect theory

The expected utility theory is based on the assumption that an individual makes a decision under risk by maximizing her expected utility with the final states of the wealth (von Neumann and Morgenstern, 1953); the two economic states typically considered in the insurance domain are determined by a loss occurrence. A pre-defined utility function depending on the risk preferences (a concave function) is used to determine the utility level, however, the theory does not explain individual choice between risky prospects with a small number of outcomes (Tversky and Kahneman, 1992).

Prospect theory resolving the violations of the classical theory is based on the assumption that an individual dislikes a loss more than she likes a gain with the comparable size, hence the level of utility in an economic situation is determined by the gain-loss domain. An individual makes a decision on the purchase of insurance based not on the final wealth, but on gains and losses compared to a reference point. This implies that gains and losses count the opportunity cost by comparing a potential decision outcome with the reference point. Tversky and Kahneman (1992) complement their initial model in Kahneman and Tversky (1979) by taking into account the cumulative functional form separately to gains and losses, thereby resolving two possible problems of the first model: lack of the explanation on stochastic dominance and lack of ability to accommodate a large number of outcomes.

We consider an individual facing two possible outcomes in an economic situation, x_1 and x_2. The individual has the loss probability, p, and makes decisions using a value function (hereafter KT value function) and a probability weighting function (hereafter KT weighting function). The probability weighting function is a non-linear form reflecting overweight on less likely events and underweight on more likely events.[9] The function features rank-dependent weighting and the violation that the sum of the subjective probability by the function is not 1. The mathematical expressions of the value function and non-linear weighting functions are (Tversky and Kahneman, 1992):

$$v(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\alpha} & if\ x < 0 \end{cases}, \tag{1}$$

$$\theta^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}, \tag{2}$$

[9] The weighting function in Kahneman and Tversky (1979) was modified in Tversky and Kahneman (1992), since the initial function did not satisfy the first-order stochastic dominance.

$$\theta^{-}(p) = \frac{p^{\xi}}{(p^{\xi} + (1-p)^{\xi})^{1/\xi}}, \tag{3}$$

where $\alpha \in [0,1]$ is a parameter to determine the diminishing sensitivity for concavity in the gain domain and for convexity in the loss domain, λ is a loss aversion parameter and γ and ξ are probability weighting parameters that are differentiated depending on gain or loss state. Tversky and Kahneman (1992) propose the values of α (= 0.88) and λ (= 2.25) with the median parameters found from their experimental study. In this value function (see Figure 1), a loss is defined by a negative value of x, whereas a gain is defined by a positive value. The diminishing sensitivity, α, indicates that an individual is risk averse in the gain state ($x \geq 0$) and risk seeking (i.e. loss averse) in the loss state ($x < 0$).[10] The reference point at the origin in Figure 1 is not differentiable (inflection point) and the higher curvature appears in the loss state than in the gain state.

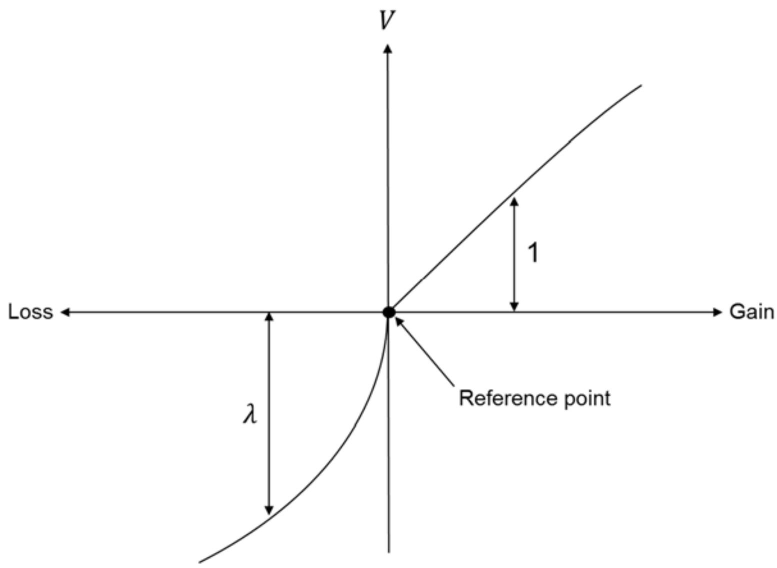

Figure 1. The shape of the value function. The curvature of the gain state is concave, whereas that of the loss state is convex. λ is the parameter of the loss aversion, showing how convex the curvature is.

The expected value of the "prospect" (an economic situation) is determined by the combination of the value function and the weighted probability as follows:

$$V = \theta^{+}(p) \cdot v(x_1) + \theta^{-}(q) \cdot v(x_2), \tag{4}$$

where x_1, x_2 indicate outcomes of two states (gain and loss) and p, q are corresponding probabilities.

[10] This can be rewritten as:

$$v''_{-} > 0, v''_{+} < 0,$$

where v''_{-} is a second order condition in the loss domain and v''_{+} is a second order condition in the gain domain.

The expected value of the “prospect” can be defined as subjective expected value for the decision-making, which is the equivalent concept to the expected utility hypothesis. The probability applied to the states is the subjective probability measure analytically calculated using the probability weighting function. According to the theory, very small probabilities are either overweighted or rounded to 0 (Kunreuther and Michel-Kerjan, 2014; Schmidt, 2016).

3 Model framework

3.1 Modeling interdependency with self-protection

Economic agents[11] nowadays operate their business in an interconnected network environment, represented in Figure 2 with nodes as atomic elements controlled by agents (Böhme and Schwartz, 2010). If one agent were breached in an attack, the rest of agents in the environment would be breached with a high likelihood.[12] This indirect risk to the other agents results in externality and is of importance in the network environment, since a systemic risk by a global hacking attack or virus can take place.

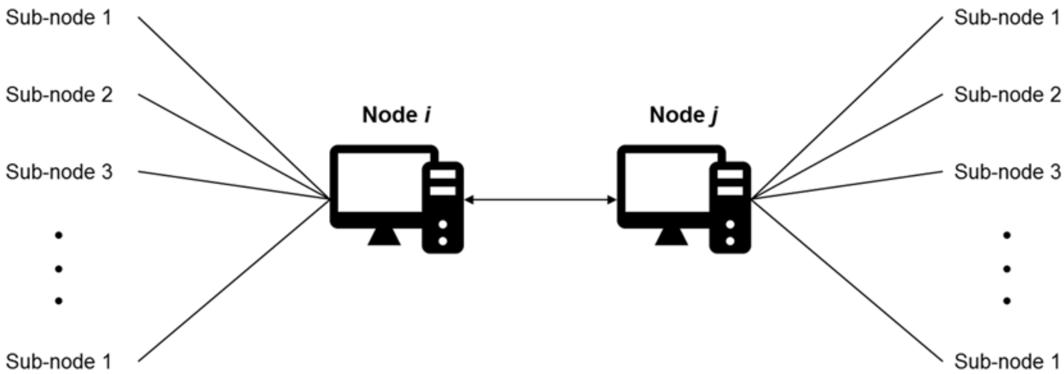

Figure 2. Interconnected network environment between agent i and agent j.

For simplicity, suppose that there are two agents (i and j) in the network. Each agent makes a monetary effort on enhancing the security system denoted by s_i and s_j. These efforts represent all possible management activities by the agents translated monetarily to improve their security levels, for example, the financial investment in updating the security software, cybersecurity training for employees, the production of a manual for possible scenarios and the regular investigation in the security. In the insurance context, this kind of effort is called self-protection, which can reduce the loss probability (Ehrlich and Becker, 1972). The loss probability of each

[11] Following Böhme and Schwartz (2010), we also use “agent” to represent possible economic entities who are looking for cyber-insurance. Potential agents of cyber-insurance incorporate firms, individual clients, governmental and non-governmental institutions as Böhme and Schwartz (2010) define.

[12] Hofmann (2007) and Mürmann and Kunreuther (2008) also parameterize both direct and indirect risks into the model to reflect the possibility of contagion risk from other sources. However, both studies do not consider the degree of interdependency, but only count the presence of interdependency.

agent is characterized by a defense function with the self-protection effort defined by Böhme and Schwartz (2010)[13]

$$p = D(s, G), \tag{5}$$

where G is a function of network topology indicating 0 when the network is connected and 1 when it is disconnected and $D(s, G) \in [0,1]$. In this paper, we consider the interconnected network environment, leading to $D(s) = D(s, G)$. The defense function $D(\cdot)$ satisfies the following assumption:

Assumption 1:

(i) The loss probability decreases with increasing level of the security investment:

$$\frac{\partial D(s)}{\partial s} < 0. \tag{6}$$

(ii) Increasing marginal loss probability over marginal self-protection level is present:

$$\frac{\partial^2 D(s)}{\partial s^2} > 0. \tag{7}$$

(iii) The loss probability cannot reach at zero in spite of substantial capacity of the agent to invest in the security:

$$\lim_{s \to \infty} D(s) > 0. \tag{8}$$

Assumption 1 describes that the defense function is convex and assumes positive externality, however, the risk will not disappear even if the agent is able to spend infinite amount of self-protection measure as described in equation (8). That is, the rate of risk reduction slows down as the level of self-protection is higher, hence no security measure can fully eliminate the risk.

At the core of the model is interdependent risk. Mürmann and Kunreuther (2008) provide a theoretical background on the optimal level of self-protection and insurance in the presence of interdependent risk.[14] Here, following Hofmann (2007) and Mürmann and Kunreuther (2008), we consider the presence of interdependent risk with the non-cooperative setting, but all agents

[13] The loss probability in the cyber risk context is also called *vulnerability*, which indicates the probability that a breach attack is successful (Gordon and Loeb, 2002; Wang, 2017). However, we stay with the term, loss probability, meaning that the loss occurs when a system is breached.

[14] Mürmann and Kunreuther (2008) call the interdependent risk contamination between agents and consider the case of no contamination in comparative statics. However, no contamination between agents is not the case for cyber risk context, especially in the hyper-connected world with Internet of Things. Therefore, we do not take into account the case of no contamination in our model.

invest in the security system for self-protection. The probability of cyber loss for the agent *i* in this setting can be defined as

$$p_i = D_i(s_i) + \big(1 - D_i(s_i)\big) \times D_j\big(s_j\big), \tag{9}$$

where $p_i \in [0,1]$.

The loss probability of the agent *i* (= p_i) is affected by the externality from the interconnected agent *j* with its defense function, $D_j\big(s_j\big)$. This implies that the size of self-protection by the agent *j* influences the loss probability of the agent *i*. Following Mürmann and Kunreuther (2008), we also assume that the externality is perfect in our model, which means that the incurred loss in one policyholder spreads to the other policyholder with certainty. Given Assumption 1, the following assumption also holds:

Assumption 2:[15]

(i) Diminishing loss probability with interdependent risk over increasing self-protection level:

$$\frac{\partial p_i}{\partial s_i} < 0. \tag{10}$$

(ii) Increasing marginal loss probability with interdependent risk over increasing marginal self-protection level:

$$\frac{\partial^2 p_i}{\partial s_i^2} > 0. \tag{11}$$

(iii) The loss probability with interdependent risk does not disappear in spite of substantial capacity of the agent to invest in self-protection:

$$\lim_{s_i \to \infty} p_i(s_i) > 0. \tag{12}$$

3.2 Market insurance design and self-insurance

The cyber-insurance market, particularly in the U.S. where the market is currently most developed, typically provides policies with deductibles, premiums between \$10,000 and \$100,000 and cover limits between \$10 million and \$50 million (Romanosky et al., 2017).[16]

[15] Given that the assumption 1 holds, the assumption (2-i) and (2-ii) can be derived in the following way:

$$\frac{\partial p_i}{\partial s_i} = D_i'(s_i) - D_j\big(s_j\big) \cdot D_i'(s_i) = \Big(1 - D_j\big(s_j\big)\Big) \cdot D_i'(s_i) < 0,$$

$$\frac{\partial^2 p_i}{\partial s_i^2} = \Big(1 - D_j\big(s_j\big)\Big) \cdot D_i''(s_i) > 0,$$

[16] Romanosky et al. (2017) analyze 100 cyber-insurance policies and document that cyber-insurance carriers are mainly concerned about the correlated risk (interdependent risk between network systems) leading to a systemic risk and not so much the possible information asymmetry.

The deductibles are in general as low as \$5,000 and reach at the amount between \$500,000 and \$1 million for insureds, whose asset values are around \$1 billion or more (Romanosky et al., 2017). We denote the deductible with d and additionally consider a loading factor ($= \delta$) proportional to the loss probability to account for cost in underwriting cyber-insurance. The price of cyber-insurance relies on the level of self-protection. We assume that the insurer is fully informed of the implementation of self-protection and its level so that the insurer can price cyber-insurance accordingly. The indemnity I and the insurance premium π with the deductible are defined as (Zweifel and Eisen, 2012, Chapter 3)

$$I = \max[L - d, 0] \cdot \beta = (L - d)^{+} \cdot \beta, \tag{13}$$

$$\pi = (1 + \delta) \times p_i \times I, \tag{14}$$

where L is a loss and β is the coverage of a proportional insurance with the loading factor δ.

Self-insurance does not influence the loss probability (Ehrlich and Becker, 1972) so that the loss probability in the presence of interdependent risk is not affected by the implementation of self-insurance. Following Böhme and Schwartz (2010), we embed self-insurance into our model by denoting the level of self-insurance for the agent *i* by $g_i \in [0,1]$ and a cost function of self-insurance by $K(g_i)$. We assume that the cost function of self-insurance is concave with respect to g_i ($K'(g_i) > 0$, $K''(g_i) < 0$). Table 2 summarizes the parameters of the model.

Table 2. Model Parameters

Notation	Explanation
$\boldsymbol{v}(\cdot)$	Value function
$\boldsymbol{W}$	Initial wealth
$\boldsymbol{L}$	Loss amount
$\boldsymbol{D_i}(\boldsymbol{s_i})$	The defense function for the agent *i* as a function of the self-protection level ($s_i \in [0, W]$)
$\boldsymbol{D_j}(\boldsymbol{s_j})$	The defense function for the agent *j* as a function of the self-protection level ($s_j \in [0, W]$)
$\boldsymbol{p_i}$	The loss probability of the agent *i* in the presence of interdependent risk
$\boldsymbol{d}$	Deductible level ($0 \leq d$)
$\boldsymbol{\delta}$	Proportional loading factor ($\delta \in [0,1]$)
$\boldsymbol{\beta}$	Insurance coverage ($\beta \in [0,1]$)
$\boldsymbol{I}$	Indemnity with deductible
$\boldsymbol{\pi}$	Cyber-insurance premium
$\boldsymbol{g_i}$	The level of self-insurance ($g_i \in [0,1]$)
$\boldsymbol{K}(\boldsymbol{g_i})$	A cost function of self-insurance

3.3 Reference points

A key to analyze decision-making under loss aversion assumption is to set a reference point to compare (Tversky and Kahneman, 1991). A decision-maker makes use of the reference point by comparing with a potential decision outcome, which enables to frame gain and loss of the decision to be made. For example, for insurance decision-making the reference point is compared with an insurance plan that could be underwritten. We consider two possible reference points under prospect theory based on a current position: status quo with self-protection and status quo with both self-protection and self-insurance. The reference point based on self-protection can be said to be cyber-specific, because it becomes basic to working with computerized system in all business areas.

Status quo with self-protection as a public good

Considering self-protection as the status quo is inevitable in the cyber risk context due to the fact that all business parties exposed to cyber risk are based on computerized operation (e.g., Microsoft Windows system or iOS) and network system requiring regular updates on the security system. For this reason, we assume that the agent makes an investment in enhancing its security system to protect itself against a possible cyber-attack. The self-protection effort can reduce the loss probability to raise the confidence in the security. It could also be the case that the government requires all economic parties to enhance their security systems to a certain level. This definition of the reference point is material in that the positive externality from concurrent efforts in the interconnected network environment might matter to increase the social welfare. Therefore, this positive externality plays a role of a public good to reduce the loss probability of all involved party in the interconnected network environment.[17]

We compare three scenarios with this reference point: decision on market insurance with self-protection and decision on self-insurance with self-protection. All scenarios assume that the self-protection measure is constantly implemented in any decision circumstance to build a realistic setting.

Status quo with both self-protection and self-insurance

Here the agent makes an investment in enhancing its security system to protect itself against a possible cyber-attack and conducts a self-insurance plan simultaneously. The self-insurance plan might be a form of a captive insurance or reserves for a potential extreme cyber case, but not a public good because it reduces the size of a loss only for those who invest in self-insurance. Instead, it can be regarded as a private good (Grossklags et al., 2008), which provides a benefit

[17] To our knowledge, there has been no literature to investigate decision-making on self-protection under prospect theory. However, several studies identify the equilibrium under expected utility theory that the interdependent nature of risk leads one to underinvest in its self-protection when there exists a positive externality from other's self-protection measure and a limiting insurance coverage can improve individual or social welfare (Hofmann, 2007; Mürmann and Kunreuther, 2008).

only for the agent implementing the plan. Simultaneous implementations on risk control measures might impose a significant cost to the agent, but reduce the loss probability and size as the maximum effort on risk control. This definition of the reference point is material in that it can show interaction between market insurance and both risk control measures.

In addition, a safe option with the maximum effort as the reference point is plausible for a prospective cyber-insured, which could offer a boundary value to the insured in decision-making. Schmidt (2016) defines the safe option as the purchase of insurance with full coverage, however, a full insurance is not a realistic option in the current cyber-insurance market as found in Section 3.2. Thus, we set the safe option as possible maximum effort on risk controls by agents.[18] We compare one possible scenario with this reference point: decision on market insurance only with self-protection. In this case, we assume that market insurance is the substitute for self-insurance and the agent continues to maintain self-protection effort as a basic risk management tool. We focus on a model with two states of the world, either a loss state or a no-loss state, whose components are illustrated in Table 3.[19]

Table 3. The Model Components

Reference point 1: Status quo with self-protection (public good)		
	Case 1: Comparison with market insurance (+ self-protection)	
State	No-loss	Loss
Probability	$1-p_i$	p_i
Reference point	$W-s_i$	$W-s_i-L$
Final wealth	$W-s_i-\pi$	$W-s_i-\pi-L+I$
Gain / Loss	$-\pi$	$I-\pi$
	Case 2: Comparison with self-insurance (+ self-protection)	
State	No-loss	Loss
Probability	$1-p_i$	p_i
Reference point	$W-s_i$	$W-s_i-L$
Final wealth	$W-K(g_i)-s_i$	$W-K(g_i)-(1-g_i)\cdot L-s_i$
Gain / Loss	$-K(g_i)$	$g_i\cdot L-K(g_i)$
Reference point 2: Status quo with self-protection and self-insurance (public good + private good)		
	Case 3: Comparison with market insurance (+ self-protection)	
State	No-loss	Loss
Probability	$1-p_i$	p_i
Reference point	$W-s_i-K(g_i)$	$W-s_i-K(g_i)-(1-g_i)\cdot L$
Final wealth	$W-s_i-\pi$	$W-s_i-\pi-L+I$
Gain / Loss	$K(g_i)-\pi$	$I-\pi+K(g_i)-L\cdot g_i$

In summary, the following points reflect the features embedded in our framework, which affect the decision-making on cyber risk management and the current cyber-insurance market:

18 As a supporting evidence, Baillon et al. (2016) empirically test with 139 participants what reference points they would choose for decision-making and find that the status quo and the maximum outcome that they can achieve are most selected as the reference point.

19 We apply the structure of tables in Schmidt (2016) to Table 3 for a clear overview of the model framework.

1. *The presence of interdependent risk in the network environment.*
2. *Realistic design for insurance in the current cyber-insurance market: no available full coverage (deductible and partial coverage).*
3. *The reference point of self-protection as an essential under loss aversion.*

However, our model can be also generalized to any decision-making problem in the presence of interdependent risk under loss aversion.

4 Decision-making on cyber risk management under loss aversion

In the expected utility theory, the utility level in any state is determined by the final wealth (endowment), which mostly provides a positive value of utility for the state. On the contrary, the value function in prospect theory consists of a positive value from the gain domain and a negative value from the loss domain in accordance with a reference point. Thus, a decision-making problem can be solved by determining the sign of the value function. In other words, if the value function turns out to be positive, a potential decision scenario can be said to be preferred over a reference point, whereas the reference point can be said to be preferred over the decision scenario if the value function is negative.

1. $V > 0$ *A decision scenario is preferred.*
2. $V \leq 0$ *A reference point is preferred.*

However, as in the expected utility theory we assume that the decision maker on cyber risk management is a value maximizer with the KT value function to find an optimal decision on risk controls and market insurance and derive the slope of an indifference curve, thereby solving the following optimization problem:

$$\max_{z \in \mathcal{H}} V(z), \tag{18}$$

where $z = (p, 1-p, x_1, x_2)$ in equation (4) and $\mathcal{H}$ is a decision space under prospect theory.

4.1 Case 1: Comparison with the decision on market insurance

The reference point is set as the status quo with self-protection in this case, hence positive externality works in the interconnected network environment. A public good potentially functions for all relevant parties in the network and the agent *i* intends to evaluate the optimal decision on self-protection and the purchase of insurance by comparing two options. In this case, the agent *i* recognizes the state of no loss as the occurrence of a loss due to the premium payment, whereas it recognizes the loss state as a gain because of the indemnity payment for the actual loss. With the gain and loss values in table 3, the expected value under prospect theory is defined as

$$V = (1 - p_i) \cdot v(-\pi) + p_i \cdot v(I - \pi). \tag{19}$$

Clearly, it needs to hold that the indemnity is larger than the premium to derive a gain and a loss of the decision on market insurance and the loss is bigger than the deductible level, leading to the condition, $p_i < \frac{1}{1+\delta}$. If purchasing market insurance is a preferred scenario, the value function should be positive. Apply the KT value function to equation (19), the preferred decision on market insurance is defined as

$$V = -\lambda \cdot (1 - p_i) \cdot \pi^{\alpha} + p_i \cdot (I - \pi)^{\alpha} > 0. \tag{20}$$

Equation (20) can be rewritten as follows (see Appendix B.2.):

$$\left[\frac{1 - (1 + \delta) \cdot p_i}{(1 + \delta) \cdot p_i}\right]^{\alpha} > \lambda \cdot \frac{1 - p_i}{p_i}. \tag{21}$$

Inequality (21) holds only if the loading factor is small enough, KT parameters (= λ, α) are also small enough and the loss probability is large enough. It implies that market insurance with self-protection effort is only preferred for agents, who are less loss averse, sensitive to the reference point, when a loss is most likely expected to occur.[20] This result is in line with the finding in Schmidt (2016), where the author sets the status quo as the reference point. To provide a clearer idea on these conditions, we derive a numerical example with particular values of parameters, shown in Table 4. We set 10% of the loading factor and diversify KT parameters to identify the criteria to satisfy inequality (21).[21] We find that an agent with higher loss aversion (higher λ) and higher sensitivity to the reference point (higher α) tends not to make a decision on purchasing cyber-insurance. Even for an agent with the opposite propensity (lower loss aversion and lower sensitivity), market insurance is an attractive option for risk transfer only when the loss probability is high enough.

In the following, we investigate how the self-protection measure affects the value of the decision. The optimization problem for the agent with equation (19) can be resolved with

[20] The parameter for diminishing sensitivity (= α) determines the size of curvature of the value function, where lower level of the parameter leads to higher curvature around the reference point in both gain and loss states as described in the left panel of figure 3.

[21] Inequality (21) with no loading factor becomes identical to Proposition 1 in Schmidt (2016), where the value of the decision is positive if $p_i > \lambda^{1/(1-\alpha)}/(1 + \lambda^{1/(1-\alpha)})$. As Schmidt (2016) concludes, purchasing market insurance is not preferred (negative value of the decision) for most realistic cases that have low probability of a loss.

respect to the level of self-protection. The following proposition reveals the decision on self-protection effort when the agent is considering an insurance option in the decision-making process.

Table 4. Criteria for Market Insurance in Case 1

$\delta = 0.1$		α				
		0.1	0.3	0.5	0.7	0.88
λ	1.25	T ($p_i \geq 0.57$)	T ($p_i \geq 0.61$)	T ($p_i \geq 0.69$)	F	F
	1.75	T ($p_i \geq 0.66$)	T ($p_i \geq 0.73$)	F	F	F
	2.25	T ($p_i \geq 0.72$)	T ($p_i \geq 0.81$)	F	F	F
	2.75	T ($p_i \geq 0.76$)	F	F	F	F
	3.00	F	F	F	F	F

Note: The table shows the result with 10% loading factor as an example. However, as the loading factor is bigger, criteria for market insurance shrink further. "T" addresses that inequality (21) holds with the level of the loss probability in the parentheses, whereas "F" indicates nonsatisfaction of inequality (21). The loss probability with the validity of inequality (21) should be lower than 91% (=1/(1+δ)).

Proposition 1: Suppose that an agent in the interdependent network environment is a value maximizer with the expected value in equation (19). The marginal value of the agent decreases over the level of self-protection if the following condition is satisfied:

$$\frac{\alpha}{1+\alpha} < p_i < \frac{1}{(1+\alpha)(1+\delta)}, \tag{22}$$

where α indicates the parameter of diminishing sensitivity homogeneous in both gain and loss spaces as shown by Tversky and Kahneman (1992). Conversely, increasing marginal value is observed over the level of self-protection if the following condition is satisfied:

$$\frac{1}{(1+\alpha)(1+\delta)} < p_i < \frac{\alpha}{1+\alpha}. \tag{23}$$

Proof. See Appendix B.1. ■

Proposition 1 leads to the link between the level of self-protection and insurance premium that decreasing marginal value with the level of self-protection holds when the loading factor is smaller than $\frac{1-\alpha}{\alpha}$, whereas increasing marginal value holds with higher loading factor (see Appendix B.1). For instance, applying the KT parameter ($\alpha = 0.88$), we identify that increasing self-protection effort results in declining marginal value if the loading on the insurance premium is lower than 13.6%. Intuitively, it describes a relationship that by comparing the insurance decision with the reference point of self-protection the agent in the KT prospect theory world is more incentivized to increase self-protection if a highly loaded premium is given. It supports the finding in Table 4 in that inequality (21) does not hold as the loading factor is higher, showing that the value of the decision on market insurance becomes negative.

It is illustrated in the right panel of Figure 3, showing that the area above the line indicates positive marginal value reflecting an incentive to increase self-protection with highly loaded insurance scenarios. Proposition 1 also holds when the first order condition is investigated with respect to the level of self-protection by the interconnect agent j.[22] Thus, it states that the self-protection measure and the insurance decision are substitutes for an agent with the reference point of self-protection, which is counter to Ehrlich and Becker (1972) under expected utility theory. In addition, positive externality from self-protection as a public good is more effective particularly in a situation where agents cannot find affordable insurance policy in the market.

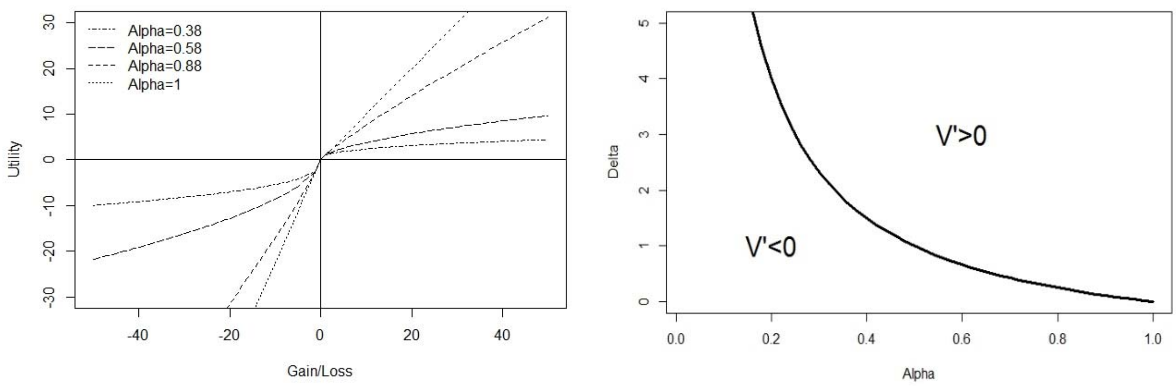

Figure 3. The description on varying diminishing sensitivity and the marginal productivity of self-protection measure when considering market insurance. The left panel displays the KT value function with different sizes of the parameter for diminishing sensitivity and the right panel shows the sign of the marginal value with respect to the effort on security to describe the incentive for self-protection.

The finding in this section demonstrates latent fatalism that the agent is inclined to stay with self-protection, which is an essential measure in the interconnected business environment. The agent only considers market insurance unfairly priced in the current market if a loss occurrence is nearly certain with the amount above the deductible level. The following proposition summarizes this finding.

Proposition 2: Suppose an agent with the KT value function ($\alpha \in [0,1]$ and $\lambda \geq 1$), who makes a decision on implementing self-insurance in the presence of self-protection by comparing it to its status-quo only with self-protection. The agent is willing to purchase market insurance if both conditions are satisfied; inequality (21) holds and a loading factor is lower than $\frac{1-\alpha}{\alpha}$.

[22] The result of the first order condition with respect to the level of self-protection by the interconnected agent is identical to that with respect to the agent i's self-protection level.

Proof. See Appendix B.2. ■

Following Schmidt (2016), we graphically describe indifference curves of both decision outcomes: maintaining the status quo and purchasing market insurance. We find the marginal productivity of the self-insurance coverage, by solving the optimization problem with regard to β as follows:

$$\frac{\partial V}{\partial \beta} = (1 - p_i) \cdot v_0' \cdot (-\pi') + p_i \cdot v_1' \cdot [(1 - (1 + \delta) \cdot p_i) \cdot I'] = 0, \tag{24}$$

where v_0' and v_1' indicate the marginal values in loss and gain domains respectively.

The optimization problem in equation (27) can be solved by satisfying the following first order condition:

$$-\frac{(1 - p_i) \cdot v_0'}{p_i \cdot v_1'} = \frac{(1 - (1 + \delta) \cdot p_i) \cdot I'}{-\pi'} = \frac{\mathrm{d}x_1}{\mathrm{d}x_0} < 0, \tag{25}$$

where $\mathrm{d}x_0$ and $\mathrm{d}x_1$ are marginal values in the loss and gain domains respectively and $I', \pi' > 0$.

In both loss and gain domains, the marginal utilities, v_0' and v_1', are positive as observed in Figure 1. The right hand side of equation (25) addresses the slope of the indifference curve maximizing the expected value, thus, the above condition implies a negative slope of the indifference curve in a (x_0, x_1)-space when considering the decision on market insurance in the existence of self-protection. According to Lemma 1 in Schmidt (2016), the curvature of the KT value function can be determined by the sign of the value, V, showing the opposite sign between the value function and the second order condition. Thus, upon the assumption that $p_i < \frac{1}{1+\delta}$ holds, the indifference curve is concave (linear or convex respectively) if $V > 0$ (= or < 0 respectively), meaning that inequality (21) holds.

We depict indifference curves of both cases, where staying only with self-protection is optimal and purchasing market insurance is preferred, in Figure 4. The (x_0, x_1)-space lies in the second quadrant, since the loss state (x_0) is negative and the gain state (x_1) is positive to achieve Proposition 2. In the left panel, staying with the reference point is optimal so that the indifference curve through the origin is either linear (V = 0) or convex (V < 0) above the point indicating the preference for market insurance. The right panel explains the optimality of the preference for market insurance with the concave indifference curve (V > 0), which lies above the origin.

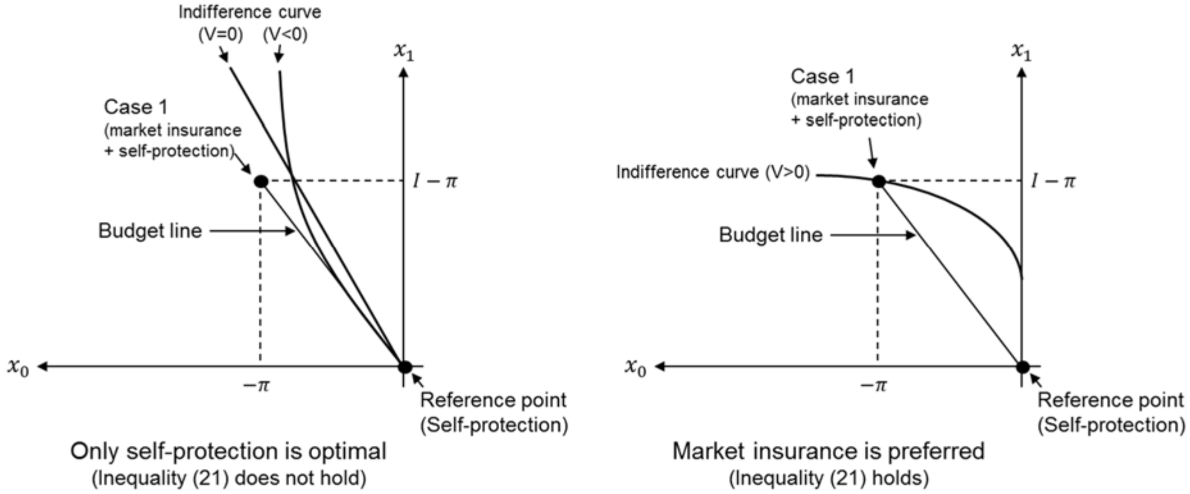

Figure 4. Indifference curves based on Proposition 2. x_0 is the outcome in the loss state and x_1 is the outcome in the gain state. The left panel displays indifference curves describing the optimality of staying on the reference point, whereas the right panel shows the indifference curve of the preference for market insurance. We follow Schmidt (2016) for these plots.

4.2 Case 2: Comparison with the decision on self-insurance as a private good

In this case, we compare the decision on self-insurance on top of self-protection measure with the reference point. The agent i recognizes the state of no loss as the occurrence of a loss due to the cost for self-insurance. The expected value of this case is defined as

$$V = (1 - p_i) \cdot v\big(-K(g_i)\big) + p_i \cdot v\big(g_i \cdot L - K(g_i)\big). \tag{26}$$

In order that the self-insurance plan is effective in this decision-making, $g_i \geq \frac{K(g_i)}{L}$ needs to hold so that the expected value is determined in both loss and gain states (see Appendix B.3.). That is, this condition implies that the self-insurance coverage above a certain level is required to achieve the validity of decision-making problem in self-insurance under loss aversion. We evaluate the necessary condition on the positive value addressing the tendency for self-insurance. The preferred decision on self-insurance is defined as

$$V = -\lambda \cdot (1 - p_i) \cdot K(g_i)^\alpha + p_i \cdot \big(g_i \cdot L - K(g_i)\big)^\alpha > 0. \tag{27}$$

We rewrite equation (27) as follows (see Appendix B.3.):

$$p_i > \frac{1}{\left(\frac{g_i \cdot L}{K(g_i)} - 1\right)^\alpha / \lambda + 1}, \tag{28}$$

where $\frac{g_i \cdot L}{K(g_i)}$ accounts for the ratio between the self-insurance endowment and its cost so that $\frac{g_i \cdot L}{K(g_i)} - 1$ can be said to be net margin of self-insurance. Under the assumption that self-insurance is effective in this case (i.e. $g_i \geq \frac{K(g_i)}{L}$), net margin should be positive.

Net margin of self-insurance depends on the magnitude of a loss so that a larger loss results in a bigger net margin.[23] As can be observed in Table 5, a more loss averse, sensitive agent to the reference point is more likely to give up self-insurance and this propensity is stronger as net margin of self-insurance becomes lower (i.e. the criterion on the loss probability in inequality (28) is higher), however, the criterion on the loss probability to prefer self-insurance is still high in any case (at least above 0.5). This finding is clarified in Proposition 3.

Table 5. Criteria for Self-insurance in Case 2

		α				
Net margin=10%		0.1	0.3	0.5	0.7	0.88
	1.25	0.611	0.714	0.798	0.862	0.905
	1.75	0.688	0.777	0.847	0.898	0.930
λ	2.25	0.739	0.818	0.877	0.919	0.945
	2.75	0.776	0.846	0.897	0.932	0.954
	3.00	0.791	0.857	0.905	0.938	0.958
Net margin=50%		0.1	0.3	0.5	0.7	0.88
	1.25	0.573	0.606	0.639	0.670	0.697
	1.75	0.652	0.683	0.712	0.740	0.763
λ	2.25	0.707	0.735	0.761	0.785	0.805
	2.75	0.747	0.772	0.795	0.817	0.835
	3.00	0.763	0.787	0.809	0.830	0.847
Net margin=100%		0.1	0.3	0.5	0.7	0.88
	1.25	0.556	0.556	0.556	0.556	0.556
	1.75	0.636	0.636	0.636	0.636	0.636
λ	2.25	0.692	0.692	0.692	0.692	0.692
	2.75	0.733	0.733	0.733	0.733	0.733
	3.00	0.750	0.750	0.750	0.750	0.750

Note: The table shows the critical values of the loss probability to achieve inequality (26) with varying possible values of net margin as examples (the condition holds if the loss probability is higher than the critical value for each case). The bigger net margin is, the lower the loss probability level is in inequality (26). However, the level of the loss probability to satisfy a positive value of the decision on self-insurance is considerably high for all three cases.

Proposition 3: Suppose an agent with the KT value function ($\alpha \in [0,1]$ and $\lambda \geq 1$), who makes a decision on implementing self-insurance in the presence of self-protection by comparing it to its status-quo only with self-protection. The agent is willing to implement self-insurance if both conditions are satisfied; inequality (28) holds and $g_i \geq \frac{K(g_i)}{L}$.

Proof. See Appendix B.3. ■

[23] Let us denote the net margin function by $A(L, g_i) = \frac{g_i \cdot L}{K(g_i)} - 1$. The first order condition of net margin with respect to the loss variable is

$$\frac{\partial A(L, g_i)}{\partial L} = \frac{g_i}{K(g_i)} > 0.$$

It shows that self-insurance can be more effective when the size of a loss is substantial.

In order to find the marginal productivity of the self-insurance coverage, an optimization problem with regard to g_i reveals as follows:

$$\frac{\partial V}{\partial g_i} = (1 - p_i) \cdot v_0' \cdot [-K(g_i)'] + p_i \cdot v_1' \cdot (L - K(g_i)') = 0, \tag{29}$$

where v_0' and v_1' indicate the marginal values in loss and gain domains respectively.

The optimization problem in equation (29) can be solved by satisfying the following first order condition:

$$-\frac{(1 - p_i) \cdot v_0'}{p_i \cdot v_1'} = \frac{L - K(g_i)'}{-K(g_i)'} = \frac{\mathrm{d}x_1}{\mathrm{d}x_0} < 0, \tag{30}$$

where $\mathrm{d}x_0$ and $\mathrm{d}x_1$ are marginal values in the loss and gain domains respectively.

In both loss and gain domains, the marginal values, v_0' and v_1', are positive as observed in Figure 1 and the marginal cost of self-insurance ($= K(g_i)'$) is also positive due to the concavity. Thus, upon the assumption that $g_i \geq \frac{K(g_i)}{L}$ holds, the indifference curve is concave (linear or convex respectively) if $V > 0$ (= or < 0 respectively), meaning that inequality (28) holds.

As in Figure 4, we again depict indifference curves of both cases, where staying only with self-protection is optimal and implementing self-insurance is preferred, in Figure 5. We only focus on the second quadrant, where the loss state (x_0) is negative and the gain state (x_1) is positive to achieve Proposition 3. The reference point is optimal in the left panel with either linear (V = 0) or convex (V < 0) indifference curve above the point indicating the preference for self-insurance. In contrast, self-insurance is preferred in the right panel with the concave indifference curve (V > 0), which lies above the origin. The finding in this section also supports the evidence of latent fatalism that the agent is inclined to stay with self-protection as a basic effort required in the interconnected business environment. The agent takes into consideration a self-insurance coverage above a certain level only if the loss probability is substantial.

Figure 6 illustrates inequality (28) with varying the diminishing sensitivity parameter ($= \alpha$) and the self-insurance level ($= g_i$) under the assumption that $g_i \geq \frac{K(g_i)}{L}$ is satisfied. We isolate the effects of other parameters by setting examples; panel A: $L = 100{,}000$, $K(g_i) = 10{,}000$, $\lambda = 2.25$; panel B: $L = 100{,}000$, $K(g_i) = 10{,}000$, $g_i = 0.2$; panel C: $L = 100{,}000$, $K(g_i) = 10{,}000$, $\alpha = 0.88$. For each example, we focus on the interaction of varying parameters with the loss probability. It supports the finding that below a certain level of the loss probability an agent is not willing to implement self-insurance plan, the level that varies with the parameters. In panel

B and C, we vary the loss aversion parameter (= λ) above one to reflect a loss averse agent, that is, if $\lambda < 1$, the agent is less averse against a loss than against a gain.

For instance, we find in the panel A that the probability range to be less motivated to self-insure for an agent with higher aversion represented by smaller α is larger than that for a less-averse agent. This finding is also observed in the panel B and C that an agent with higher loss aversion (higher λ) tends more to give up self-insurance on controlling risk, thereby showing a more distinct fatalistic behavior. Here, higher loss aversion addresses higher risk seeking in the loss domain as described in Section 2.

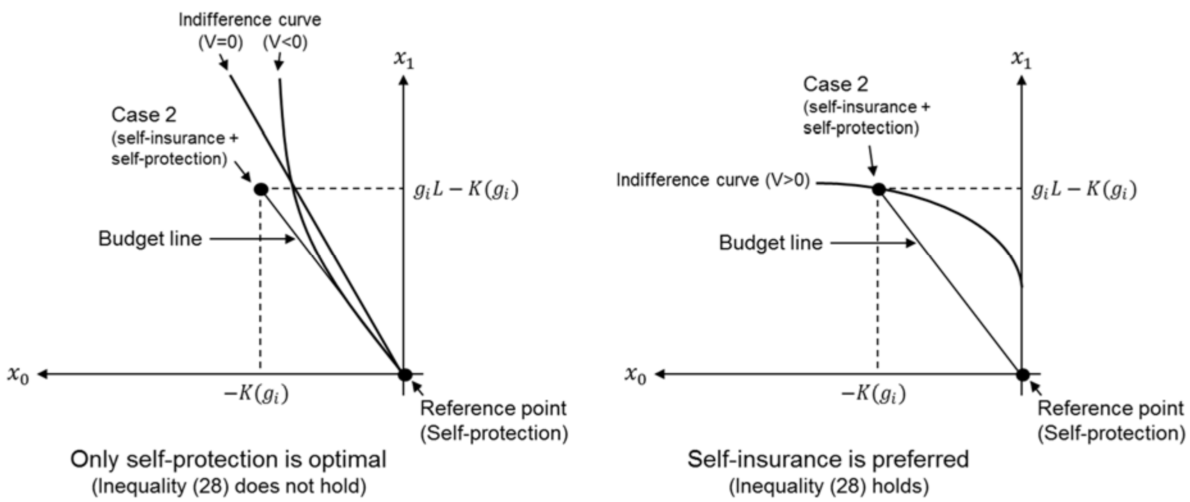

Figure 5. Indifference curves based on Proposition 3. The left panel displays indifference curves describing the optimality of staying on the reference point, whereas the right panel shows the indifference curve of the preference for self-insurance. We follow Schmidt (2016) for these plots.

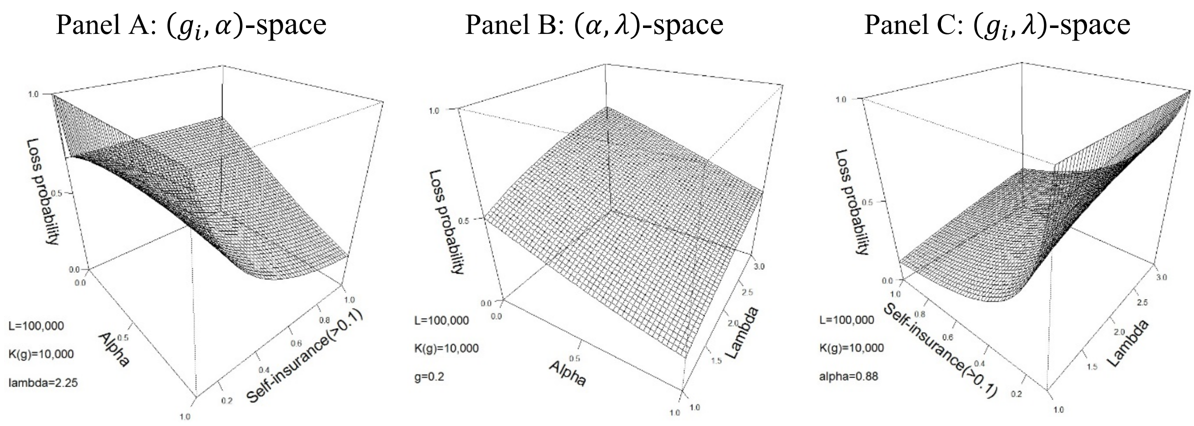

Figure 6. Three-dimensional plots representing Proposition 2.

4.3 Case 3: Status quo with self-protection and self-insurance

An agent could make the maximum effort by itself, which can provide a safe option without any transfer measure. Based on this possible maximum effort, the agent evaluates decision-making outcome by switching from self-insurance to market insurance when self-protection continues to be implemented as a basic management option. This evaluation is relevant under the assumption that market insurance and self-insurance are substitutes for each other, hence the agent might take into account the difference in the cost and the value of the decision between two options. The expected value of this case is defined as

$$V = (1 - p_i) \cdot v(K(g_i) - \pi) + p_i \cdot v(I - g_i \cdot L + K(g_i) - \pi). \tag{31}$$

To see the indifference curve in the second quadrant, the signs of the gain and the loss should be opposite to each other, which splits equation (31) into two cases: 1) $K(g_i) - \pi \leq 0$, $I - g_i \cdot L + K(g_i) - \pi > 0$ and 2) $K(g_i) - \pi > 0$, $I - g_i \cdot L + K(g_i) - \pi \leq 0$.

The first case indicates that market insurance charges higher premium with higher coverage for loss than the cost of self-insurance with lower coverage.[24] The second case is the opposite instance, where self-insurance with higher coverage costs higher than market insurance with lower coverage does. The following proposition summarizes decision-making outcomes of two possible cases in this scenario.

Proposition 4: Suppose an agent with the KT value function ($\alpha \in [0,1]$ and $\lambda \geq 1$), who makes a decision on purchasing market insurance in the presence of self-protection by comparing it to the maximum effort (self-protection and self-insurance). The agent is willing to purchase market insurance instead of self-insurance if the following conditions are satisfied:

For market insurance	
i) Positive net effect of the transition from self-insurance to market insurance:	$\left[\frac{I - g_i \cdot L}{\pi - K(g_i)} - 1\right]^{\alpha} > \lambda \cdot \frac{1 - p_i}{p_i}$
ii) Positive net effect of staying with self-insurance:	$\left[\frac{g_i \cdot L - I}{K(g_i) - \pi} - 1\right]^{\alpha} < \frac{1}{\lambda} \cdot \frac{p_i}{1 - p_i}$

Proof. See Appendix B.4. ■

The equations in the parentheses of the left hand side can explain the net effect of choosing a particular insurance type; the equation in the first condition addresses the net effect of the transition from self-insurance to market insurance and the one in the second condition indicates the net effect of staying with self-insurance. Those net effects are all positive with the inequality assumptions from equation (31).

[24] To prove this, the following inequality should hold

$$|I - g_i \cdot L| > |K(g_i) - \pi|,$$

where $I - g_i \cdot L > 0$ since $K(g_i) - \pi \leq 0$ and $I - g_i \cdot L + K(g_i) - \pi > 0$. Then, the right hand side of the above inequality can be rewritten

$$I - g_i \cdot L = (\beta - g_i) \cdot L - d \cdot \beta > 0.$$

Thus, $\beta - g_i > 0$ should hold.

Table 6 shows the criteria for the loss probability that makes the conditions of Proposition 4 hold. For the first condition in panel A, the numerical results turn out to be identical to the results in Table 5 (Case 2), implying that a more loss averse, sensitive agent to the reference point is more likely to stay with self-insurance and this tendency is stronger as the net effect of the transition from self-insurance to market insurance becomes lower. The results in panel B of Table 6 from the second condition are exactly in line with it by showing the reverse outcomes in that a more loss averse, sensitive agent to the reference point is more likely to take up market insurance instead of self-insurance and it tends to be stronger as the net effect of staying with self-insurance becomes lower.

In Figure 7, we again draw indifference curves of both decision outcomes: maintaining self-insurance and replacing it by market insurance. The marginal productivity of market insurance can be obtained by solving the optimization problem in terms of β with two cases in loss and gain domains as follows:

1) $K(g_i) - \pi \leq 0$ and $I - g_i \cdot L + K(g_i) - \pi > 0$:

$$\frac{\partial V}{\partial \beta} = (1 - p_i) \cdot v_0' \cdot (-\pi') + p_i \cdot v_1' \cdot [(1 - (1 + \delta) \cdot p_i) \cdot I'] = 0, \tag{32}$$

2) $K(g_i) - \pi > 0$ and $I - g_i \cdot L + K(g_i) - \pi \leq 0$:

$$\frac{\partial V}{\partial \beta} = (1 - p_i) \cdot v_0' \cdot [(1 - (1 + \delta) \cdot p_i) \cdot I'] + p_i \cdot v_1' \cdot (-\pi') = 0, \tag{33}$$

where v_0' and v_1' indicate the marginal values in loss and gain domains respectively.

The above optimization problems are solved by satisfying the following first order conditions:

1) $K(g_i) - \pi \leq 0$ and $I - g_i \cdot L + K(g_i) - \pi > 0$:

$$-\frac{(1 - p_i) \cdot v_0'}{p_i \cdot v_1'} = \frac{(1 - (1 + \delta) \cdot p_i) \cdot I'}{-\pi'} = \frac{\mathrm{d}x_1}{\mathrm{d}x_0} < 0, \tag{34}$$

2) $K(g_i) - \pi > 0$ and $I - g_i \cdot L + K(g_i) - \pi \leq 0$:

$$-\frac{(1 - p_i) \cdot v_0'}{p_i \cdot v_1'} = \frac{-\pi'}{(1 - (1 + \delta) \cdot p_i) \cdot I'} = \frac{\mathrm{d}x_1}{\mathrm{d}x_0} < 0, \tag{35}$$

where $\mathrm{d}x_0$ and $\mathrm{d}x_1$ are marginal values in the loss and gain domains respectively and $I', \pi' > 0$.

Based on Lemma 1 in Schmidt (2016), we have the curvature of the KT value function in Figure 7 determined by the sign of the value, V, showing that the indifference curve is concave (linear or convex respectively) if $V > 0$ (= or < 0 respectively) with satisfying Proposition 4.

Table 6. Criteria for Replacing Self-insurance by Market Insurance in Case 3

Panel A: The critical values of the loss probability in the first condition of Proposition 4

		α				
Net effect=10%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.61$)	T($p_i \geq 0.71$)	T($p_i \geq 0.80$)	T($p_i \geq 0.86$)	T($p_i \geq 0.91$)
	1.75	T($p_i \geq 0.69$)	T($p_i \geq 0.78$)	T($p_i \geq 0.85$)	T($p_i \geq 0.90$)	T($p_i \geq 0.93$)
λ	2.25	T($p_i \geq 0.74$)	T($p_i \geq 0.82$)	T($p_i \geq 0.88$)	T($p_i \geq 0.92$)	T($p_i \geq 0.95$)
	2.75	T($p_i \geq 0.78$)	T($p_i \geq 0.85$)	T($p_i \geq 0.90$)	T($p_i \geq 0.93$)	T($p_i \geq 0.95$)
	3.00	T($p_i \geq 0.79$)	T($p_i \geq 0.86$)	T($p_i \geq 0.91$)	T($p_i \geq 0.94$)	T($p_i \geq 0.96$)
Net effect=50%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.57$)	T($p_i \geq 0.61$)	T($p_i \geq 0.64$)	T($p_i \geq 0.67$)	T($p_i \geq 0.70$)
	1.75	T($p_i \geq 0.65$)	T($p_i \geq 0.68$)	T($p_i \geq 0.71$)	T($p_i \geq 0.74$)	T($p_i \geq 0.76$)
λ	2.25	T($p_i \geq 0.71$)	T($p_i \geq 0.74$)	T($p_i \geq 0.76$)	T($p_i \geq 0.79$)	T($p_i \geq 0.81$)
	2.75	T($p_i \geq 0.75$)	T($p_i \geq 0.77$)	T($p_i \geq 0.80$)	T($p_i \geq 0.82$)	T($p_i \geq 0.84$)
	3.00	T($p_i \geq 0.76$)	T($p_i \geq 0.79$)	T($p_i \geq 0.81$)	T($p_i \geq 0.83$)	T($p_i \geq 0.85$)
Net effect=100%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)
	1.75	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)
λ	2.25	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)
	2.75	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)
	3.00	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)

Panel B: The critical values of the loss probability in the second condition of Proposition 4

		α				
Net effect=10%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.50$)	T($p_i \geq 0.39$)	T($p_i \geq 0.28$)	T($p_i \geq 0.20$)	T($p_i \geq 0.14$)
	1.75	T($p_i \geq 0.58$)	T($p_i \geq 0.47$)	T($p_i \geq 0.36$)	T($p_i \geq 0.26$)	T($p_i \geq 0.19$)
λ	2.25	T($p_i \geq 0.64$)	T($p_i \geq 0.53$)	T($p_i \geq 0.42$)	T($p_i \geq 0.31$)	T($p_i \geq 0.23$)
	2.75	T($p_i \geq 0.69$)	T($p_i \geq 0.58$)	T($p_i \geq 0.47$)	T($p_i \geq 0.35$)	T($p_i \geq 0.27$)
	3.00	T($p_i \geq 0.70$)	T($p_i \geq 0.60$)	T($p_i \geq 0.49$)	T($p_i \geq 0.37$)	T($p_i \geq 0.28$)
Net effect=50%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.54$)	T($p_i \geq 0.50$)	T($p_i \geq 0.47$)	T($p_i \geq 0.44$)	T($p_i \geq 0.40$)
	1.75	T($p_i \geq 0.62$)	T($p_i \geq 0.59$)	T($p_i \geq 0.55$)	T($p_i \geq 0.52$)	T($p_i \geq 0.49$)
λ	2.25	T($p_i \geq 0.68$)	T($p_i \geq 0.65$)	T($p_i \geq 0.61$)	T($p_i \geq 0.58$)	T($p_i \geq 0.55$)
	2.75	T($p_i \geq 0.72$)	T($p_i \geq 0.69$)	T($p_i \geq 0.66$)	T($p_i \geq 0.63$)	T($p_i \geq 0.60$)
	3.00	T($p_i \geq 0.74$)	T($p_i \geq 0.71$)	T($p_i \geq 0.68$)	T($p_i \geq 0.65$)	T($p_i \geq 0.62$)
Net effect=100%		0.1	0.3	0.5	0.7	0.88
	1.25	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)	T($p_i \geq 0.56$)
	1.75	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)	T($p_i \geq 0.64$)
λ	2.25	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)	T($p_i \geq 0.69$)
	2.75	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)	T($p_i \geq 0.73$)
	3.00	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)	T($p_i \geq 0.75$)

Note: The table shows the criteria for the loss probability to achieve the conditions of Proposition 4 with varying possible values of net effect as examples (the conditions hold if the loss probability is higher than the critical value for each case). "T" addresses that two conditions of Proposition 4 in panel A and B respectively hold with the level of the loss probability in the corresponding parentheses The net effect in panel A indicates the effect of the transition from self-insurance to market insurance and the one in panel B accounts for the effect of staying with self-insurance.

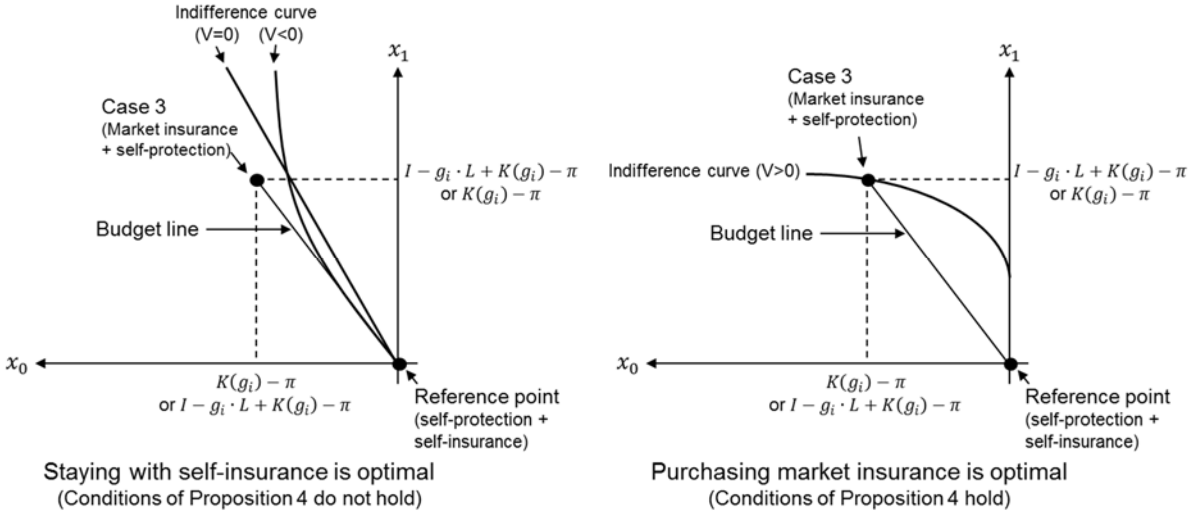

Figure 7. Indifference curves based on Proposition 4. The left panel displays indifference curves describing the optimality of staying on the reference point, whereas the right panel shows the indifference curve of the preference for market insurance instead of self-insurance. We follow Schmidt (2016) for these plots.

4.4 Empirical evidence on loss probability and an implication for findings

We find necessary conditions in each case to show that a decision scenario considered is preferred to the reference point. These conditions require a certain level of loss probability that can make an agent decide to implement additional risk management plan (either market insurance or self-insurance). Although the level of probability leading the agent to do additional action on risk management is significantly high in all cases, the frequency of cyber loss events is dramatically increasing over the last decade (World Economic Forum, 2016), which possibly increases the likelihood to achieve the conditions of propositions. To obtain a relevant evidence, we empirically estimate appropriate loss probability using a publicly available database for cyber risk provided by Private Rights Clearinghouse (PRC).[25]

PRC collects information on data breach events and categorizes losses by eight types of breach (PRC, 2019). Among them, we only focus on four types of breach (Hacking, Insider breach, Payment card fraud and unknown) as a malicious type of risk by following the categorization in Edwards et al. (2016). The malicious type of risk is more relevant to our model framework, particularly with regard to the presence of interdependent risk. The definition of cyber risk is generally broader than data breach risk (Eling and Wirfs, 2019), but here we restrict it to data breach loss, which is one of main coverages in most cyber-insurance policies (Romanosky et

[25] PRC continuously expands its database and updates the loss events based on new information that it keeps finding. With this effort, it forms the largest public database for cyber loss, the dataset that incorporates information on the breach date, the event location, the entity level, the loss type and the total number of breached records.

al., 2017). Over the entire data period from 2005 to 2018, we find 4,053 observations about malicious risk (as of February 5, 2019), among which we use 2,774 of non-zero records.

In Table 7, we estimate the number of days experiencing at least one loss event per year over the data period between 2005 and 2018. From the insurer perspective, a loss frequency can be estimated by the ratio between the number of claims and the number of risks in the pool. However, from the insured perspective, it is difficult to measure a possible rate of loss frequency without the availability of loss data and information on the risk pool. As a proxy for the rate of loss frequency from an insured perspective, we attempt to estimate the number of days when at least one breach loss is publicly announced over the period. The rate for each year is measured upon different deductible levels ($ 5,000 as lowest to $ 1 million with $ 250,000 increment) subject to possible cyber-insurance coverages, the deductibles that are based on the finding in Romanosky et al. (2017) as described in Section 3.2.

Importantly, the observations in the PRC database account for only the number of breach records, thus we need to translate them to the monetary unit to apply the deductibles. There has been lack of studies on the measurement of economic cost for data breach, but Ponemon Institute (2018) reports its analysis on the cost of data breach on an annual basis. Thus, we adapt the average economic cost year-by-year estimated by Ponemon Institute to obtain approximate measures of economic loss from data breach events. Table 7 shows the frequency rate per year from 2005 to 2018 with five deductible levels. Although it does not exactly reflect the actual loss probability, we can observe that the rate is significantly increasing, showing, for example, a threefold increase over the decade (from 2008 to 2018 across all deductible categories). It implies that the dynamic landscape of cyber risk with a dramatic increase in loss frequency can lift up the demand for additional risk management tools (market insurance or self-insurance), the demand that is higher by a less averse agent against a loss.

For small-sized companies mostly with a lower level of deductible in the coverage, the rate is much higher than that for large companies with a higher level of deductible, which can induce small-sized companies to demand additional risk management tool by satisfying the conditions of propositions. Overall, our finding implies that although a fatalistic behavior against cyber risk still exists in the market, it becomes more inevitable to implement additional plans for risk management, thereby leading the cyber-insurance market to be on track.

Table 7. Empirical Frequency Rate of Data Breach Loss (per year)

	Deductible				
Year	\$ 5,000	\$ 250,000	\$ 500,000	\$ 750,000	\$ 1,000,000
2005	14.0%	11.5%	11.0%	10.4%	9.3%
2006	19.7%	14.5%	13.2%	11.0%	9.6%
2007	18.1%	12.3%	11.5%	11.0%	9.9%
2008	18.9%	13.9%	12.3%	10.1%	9.8%
2009	13.7%	8.2%	7.4%	7.4%	7.4%
2010	28.2%	16.2%	12.9%	11.8%	11.8%
2011	37.5%	20.0%	15.1%	13.7%	12.9%
2012	42.1%	25.1%	19.7%	16.4%	15.3%
2013	32.9%	24.7%	21.1%	17.8%	15.6%
2014	32.1%	21.6%	17.8%	16.7%	15.9%
2015	20.0%	16.2%	14.5%	13.4%	12.3%
2016	36.6%	29.5%	24.6%	23.5%	22.4%
2017	36.4%	31.0%	28.5%	24.9%	22.7%
2018	59.7%	44.7%	38.9%	34.8%	32.3%

Note: This table shows a frequency rate per year of data breach loss over the period between 2005 and 2018. The data is derived from Privacy Rights Clearinghouse (PRC), which is a non-profit organization collecting and updating information on data breach events published in media.

5 Conclusion

We propose a conceptual framework for decision-making in cyber risk management with a descriptive decision model under loss aversion. The framework is constructed under the assumption of interdependent risk in the interconnected network environment. We consider market insurance and risk control measures (self-protection and self-insurance) and investigate their interaction. It is vital for any decision-making problem under loss aversion to set a relevant, realistic reference point. In this regard, we postulate self-protection as the status quo of the reference point, since self-protection as a form of enhancing (or maintaining) cyber security system is an essential management tool in the interconnected business world. We compare the reference point with different scenarios under the assumption that self-protection continues to be implemented, the scenarios that are 1) market insurance and 2) self-insurance. We also set self-protection and self-insurance as the second reference point, which is interpreted as the maximum effort on controlling cyber risk (public good + private good), and compare it to the decision on market insurance only with the self-protection measure under the assumption that both insurance plans are substitutes for each other.

In the different scenarios, we come to the consistent conclusion that an agent with the reference point of self-protection as a basic management tool against cyber risk is more likely to avoid additional risk management measures (market insurance and self-insurance). Particularly, a more loss averse agent is more likely to maintain the status quo (self-protection), and the higher net margin of insurance policies is for the agent, the less the agent is likely to give up additional measures. These findings address latent fatalism observed in the current market that the agent tends not to choose any additional risk management tool, but stay only with self-protection.

We find conditions on the range of loss probability, the loading factor for market insurance and the level of coverage for self-insurance, where the agent are willing to decide on market insurance and self-insurance. Based on the findings, it can be concluded that market insurance and self-insurance are substitutes of self-protection as the reference point under loss aversion, which is counter to the finding by Ehrlich and Becker (1972). We further estimate the frequency rate per year as a proxy of the loss probability from the insured perspective using an empirical dataset (PRC database). We find that it is more likely to implement additional risk management tools particularly for small-sized companies as time goes on. A critical remark on our model framework is that it can be not only applied to the cyber risk context, but also generalized to any interdependent risk model under loss aversion, which particularly considers measures to control risk.

Self-protection as the status quo for potential cyber-insureds is significant for a regulatory implication as well as for the reflection of the current market situation. Self-protection facilitates to reinforce the security system of all interconnected parties (positive externality), hence it can function as a public good in the interconnected network environment. The government can utilize this characteristic to increase the overall security level against cyber risk in the society by forcing all business agents to meet a certain level of security requirement. This regulation can also help cyber-insurers reduce information asymmetry and underwrite cyber-insurance with an optimal deductible and premium possibly containing a comprehensive risk management offer.[26] Therefore, in the cyber risk context we can base the status quo on the implementation of self-protection.

Although this study firstly opens up a discussion of a descriptive decision model on market insurance and risk control measures in the presence of interdependent risk, several limitations are still inherent in our model. Particularly, some assumptions can be relaxed to adapt other possibilities in the model. For instance, a likelihood of successful contagion between nodes can be affected by other factors (e.g., heterogeneous security systems) so that the perfect contagion could not occur in the network system. Furthermore, we do not take into account a possibility of probability distortion due to the modeling of interdependent risk; however, it could be a

[26] Some cyber-insurance policies in the current market offer a so-called risk management package, which incorporates from building a detection system and a reporting process to the monetary coverage (e.g., Allianz Cyber Protect and HSB Cyber Insurance by Munich Re).

possible avenue to combine both features in a model. Co-insurance or reinsurance portfolio can be considered as alternative insurance design possibly embedded in the model.

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Appendix A. Review of Literature and position of the paper

A.1. The literature on risk control measures and interdependent risk

In Table A1, we categorize the references dealing with risk control measures in decision-making framework. The categorization is defined upon three criteria: interdependency in risks, propensity under risk and relation to cyber risk. We consider 20 academic papers including the present study and find the following trends; the studies in the presence of interdependent risk tend to model the impact of self-protection, but not of self-insurance. This might be due to the fact that self-protection affects the loss probability, which is of importance to model interdependent risk. In contrast, the studies in the absence of interdependent risk take into account both self-protection and self-insurance to see decision-making in risk control measures and market insurance. Additionally, all cyber risk-related papers in our categorization analyze only self-protection in relation to cyber-insurance by accounting for cyber security enhancement. In this regard, our paper contributes to the literature by considering both self-protection and self-insurance in decision-making framework against cyber risk under interdependent risk.

Table A1. The Classification on Risk Control and Loss Aversion under Interdependency in Risks

		Risk control measures			
		No interdependent risk		Interdependent risk	
Risk-aversion (EU)	Non-cyber	・EB72 (SP, SI) ・DE85 (SP, SI) ・S90 (SP, SI) ・KS93 (SP, SI)	・S11 (SP, SI) ・LRS12 (SP, SI) ・AGT13 (SP, SI)	・KH03 (SP) ・H07 (SP) ・MK08 (SP)	
	Cyber-related	・KNL18 (SP)		・LB09 (SP) ・SSFW10 (SP) ・HR11 (SP) ・ORM11 (SP)	・ZXW13 (SP) ・NL14 (SP) ・HS18 (SP)
Loss-aversion (PT)	Non-cyber	・NA		NA	
	Cyber-related	・V08 (SP)		**Present paper (SP, SI)**	

Note: "SP", "SI", and "NA" stand for self-protection, self-insurance and not applicable, respectively. The literature categorized above is listed in the following; EB72: Ehrlich and Becker (1972); DE85: Dionne and Eeckhoudt (1985); S90: Shogren (1990); KS93: Konrad and Skaperdas (1993); KH03: Kunreuther and Heal (2003); H07: Hofmann (2007); MK08: Mürmann and Kunreuther (2008); V08: Verendel (2008); LB09: Lelarge and Bolot (2009); SSFW10: Shetty et al. (2010); S11: Snow (2011); HR11: Hofmann and Ramaj (2011); ORM11: Ogut, Raghunathan and Menon (2011); LRS12: Lohse, Robledo and Schmidt (2012); AGT13: Alary, Gollier and Treich (2013); ZXW13: Zhao, Xue and Whinston (2013); NL14: Naghizadeh and Liu (2014); HS18: Hota and Sundaram (2018); KNL18: Khalili, Naghizadeh and Liu (2018).

A.2. The comparison with the literature on insurance decision under prospect theory

Although we can find a great number of studies on insurance demand under the classical decision theory, there has been very little literature on insurance demand under prospect theory. One possible explanation on lack of study in this context is a difficulty in the choice of a reference point and the vagueness of how to examine the decision-making under the descriptive model. However, we attempt to overcome these obstacles by focusing on a specific context (cyber risk management). Nevertheless, our framework can be also generalized to the context about risk control measures and interdependent risk under loss aversion. In this regard, we compare this paper with two relevant studies under loss aversion in Table A2. All studies in the table lie in the insurance context by commonly investigating the optimal decision on insurance.

Table A2. The Comparison with Two Relevant References under Loss Aversion

	Eckles and Wise (2011)	Schmidt (2016)	Present paper
Focus of study	Insurance demand under prospect theory	Insurance demand under prospect theory	Interaction between risk control measures and market insurance under prospect theory
Risk controls	No	No	Yes (self-protection and self-insurance)
Insurance design	Deductible and loaded premium	Full insurance	Deductible and loaded premium
Interdependent risk	No	No	Yes
Reference point	· Status quo · Take-up insurance · Initial wealth	· Status quo · Take-up full insurance	· Status quo with self-protection · Self-protection and self-insurance (Full self-control)
Decision scenario	· Optimal coverage	· Optimal coverage	· Optimal coverage · Self-insurance · Optimal coverage and self-insurance
Main results	· Prospect theory explains 1) the preference for low deductibles for mandatory insurance, 2) the lack of demand for non-mandatory insurance and 3) the over-demand to insure small losses.	· An agent will demand either full coverage or no insurance, depending on the loss probability. · The indifference curve of the KT value function relies on the sign of the value function.	· An agent with the reference point of self-protection as a basic management tool against cyber risk is more likely to avoid additional risk management measures (market insurance and self-insurance), addressing latent fatalism observed in the market.

Appendix B. Proofs

B.1. Proof of Proposition 1

The agent with the value function in equation (1) optimizes equation (19) with respect to the level of self-protection as follows:

$$
\begin{aligned}
\frac{\partial V}{\partial s_i} &= \frac{\partial[-\lambda \cdot (1-p_i) \cdot \pi^{\alpha} + p_i (I-\pi)^{\alpha}]}{\partial s_i} \\
&= -\lambda \cdot \alpha \cdot \pi^{\alpha-1} \cdot (1+\delta) \cdot I \cdot p_i' + \lambda \cdot (\alpha+1) \cdot \pi^{\alpha} \cdot p_i' + (I-\pi)^{\alpha} \cdot p_i' - \alpha \cdot \pi \cdot (I-\pi)^{\alpha-1} \cdot p_i' \\
&= \lambda \cdot \pi^{\alpha} \cdot p_i' \cdot \left[(\alpha+1) - \frac{\alpha}{p_i}\right] + (I-\pi)^{\alpha-1} \cdot p_i' \cdot (I - \pi - \alpha\pi)
\end{aligned}
$$

We know that the loss probability is a convex function with respect to the level of self-protection according to assumption 2. It leads the above equation to rely on $\left[(\alpha+1) - \frac{\alpha}{p_i}\right]$ and $(I - \pi - \alpha\pi)$ to determine the first order condition. If both cases are all positive, the marginal value for the agent decreases over the level of self-protection, leading to the following condition:

$$\frac{\alpha}{1+\alpha} < p_i < \frac{1}{(1+\alpha)(1+\delta)}.$$

Conversely, if both cases are all negative, the marginal value increases over the level of self-protection with the following condition:

$$\frac{1}{(1+\alpha)(1+\delta)} < p_i < \frac{\alpha}{1+\alpha}.$$

The above conditions determine the loading factor by equating the upper and the lower bounds such that

$$\delta = \frac{1-\alpha}{\alpha}.$$

Decreasing marginal value over the level of self-protection holds when the loading factor is smaller than $\frac{1-\alpha}{\alpha}$, whereas increasing marginal value holds with higher loading factor.

B.2. Proof of Proposition 2

We derive inequality (21) with a simple mathematical process as follows; first, the KT value function is applied to the expected value in equation (19) and the expected value should be positive when market insurance is preferred.

$$V = -\lambda \cdot (1 - p_i) \cdot \pi^\alpha + p_i \cdot (I - \pi)^\alpha > 0.$$

The premium ($= \pi$) is a function of the loss probability ($= p_i$), decomposed to $(1 + \delta) \times p_i \times I$. Then, the expected value is decomposed to

$$V = -\lambda \cdot (1 - p_i) \cdot [(1 + \delta) \cdot p_i \cdot I]^\alpha + p_i \cdot [(1 - (1 + \delta) \cdot p_i) \cdot I]^\alpha > 0.$$

It can be rewritten as

$$p_i \cdot (1 - (1 + \delta) \cdot p_i)^\alpha > \lambda \cdot (1 - p_i) \cdot [(1 + \delta) \cdot p_i]^\alpha.$$

Since $p_i \geq 0$ and $(1 + \delta) \cdot p_i \geq 0$,

$$\left[\frac{1 - (1 + \delta) \cdot p_i}{(1 + \delta) \cdot p_i}\right]^\alpha > \lambda \cdot \frac{1 - p_i}{p_i}.$$

Appendix B.1. proves the condition $\delta \leq \frac{1-\alpha}{\alpha}$.

B.3. Proof of Proposition 3

We clarify loss and gain domains by ensuring the positive value of the size of gain to understand the decision under prospect theory. The necessary condition to achieve this is $g_i \geq \frac{K(g_i)}{L}$, which is solved from $g_i \cdot L - K(g_i) \geq 0$ with regard to g_i.

Inequality (26) can be derived with a simple mathematical process as follows; as in Appendix B.2., the KT value function is first applied to the expected value in equation (24) and the expected value should be positive when self-insurance is preferred

$$V = -\lambda \cdot (1 - p_i) \cdot K(g_i)^\alpha + p_i \cdot \left(g_i \cdot L - K(g_i)\right)^\alpha > 0.$$

It can be rewritten as

$$p_i \cdot \left(g_i \cdot L - K(g_i)\right)^\alpha > \lambda \cdot (1 - p_i) \cdot K(g_i)^\alpha.$$

Since $p_i \geq 0$ and $K(g_i) \geq 0$,

$$\left(\frac{g_i \cdot L - K(g_i)}{K(g_i)}\right)^{\alpha} > \lambda \cdot \frac{1-p_i}{p_i}.$$

Solving it with respect to p_i yields

$$p_i > \frac{1}{\left(\frac{g_i \cdot L}{K(g_i)} - 1\right)^{\alpha} / \lambda + 1}.$$

B.4. Proof of Proposition 4

To prove Proposition 4, we consider two cases in the following.

i) $\boldsymbol{K(g_i) - \pi \leq 0}$ ***and*** $\boldsymbol{I - g_i \cdot L + K(g_i) - \pi > 0}$ *(Positive net effect of the transition from self-insurance to market insurance)*

The expected value with the KT value function in this case can be written as

$$V = -\lambda \cdot (1-p_i) \cdot [-(K(g_i) - \pi)]^{\alpha} + p_i \cdot [I - g_i \cdot L + K(g_i) - \pi]^{\alpha}.$$

To determine the preference for market insurance instead of self-insurance, we postulate a positive expected value, then the above equation can be rewritten as

$$p_i \cdot [I - g_i \cdot L + K(g_i) - \pi]^{\alpha} > \lambda \cdot (1-p_i) \cdot [\pi - K(g_i)]^{\alpha}.$$

Since $p_i \geq 0$ and $K(g_i) - \pi \leq 0$,

$$\left[\frac{I - g_i \cdot L + K(g_i) - \pi}{\pi - K(g_i)}\right]^{\alpha} > \lambda \cdot \frac{1-p_i}{p_i}.$$

It ends up with

$$\left[\frac{I - g_i \cdot L}{\pi - K(g_i)} - 1\right]^{\alpha} > \lambda \cdot \frac{1-p_i}{p_i}.$$

ii) $\boldsymbol{K(g_i) - \pi > 0}$ ***and*** $\boldsymbol{I - g_i \cdot L + K(g_i) - \pi \leq 0}$ *(Positive net effect of staying with self-insurance)*

Now the signs of two outcomes are switched. The expected value with the KT value function in this case is

$$V = -\lambda \cdot (1-p_i) \cdot [-(I - g_i \cdot L + K(g_i) - \pi)]^{\alpha} + p_i \cdot [K(g_i) - \pi]^{\alpha}.$$

To determine the preference for market insurance instead of self-insurance, we need to have a positive expected value, leading to the following equation

$$p_i \cdot [K(g_i) - \pi]^\alpha > \lambda \cdot (1 - p_i) \cdot [g_i \cdot L - I - (K(g_i) - \pi)]^\alpha.$$

Since $p_i \geq 0$ and $I - g_i \cdot L + K(g_i) - \pi \leq 0$,

$$\left[\frac{K(g_i) - \pi}{g_i \cdot L - I - (K(g_i) - \pi)}\right]^\alpha > \lambda \cdot \frac{1 - p_i}{p_i}.$$

It ends up with

$$\left[\frac{g_i \cdot L - I}{K(g_i) - \pi} - 1\right]^\alpha < \frac{1}{\lambda} \cdot \frac{p_i}{1 - p_i}.$$