Optimal Insurance Demand – Low Probability, High Consequence versus High Probability, Low Consequence

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Abstract

Contrary to expected utility theory empirical studies document a rather low insurance demand for rare catastrophic risks (LPHC – low probability high consequence) and a rather high insurance demand for small but frequent risks (HPLC – high probability low consequence). We explain this puzzle by mental accounting in a basic insurance model with two independent risks. To do so, we consider two optimization problems in which one risk (either LPHC or HPLC) is insurable while the other risk is seen as (uninsurable) background risk. We find that the optimal insurance demand against LPHC risks (HPLC is background risk) can be larger or smaller than the optimal insurance against HPLC risks (LPHC is background risk). This depends on, i.e. the risk preferences or initial wealth of the decision-maker. The loss probability affects this puzzle as well.

Keywords: Basic insurance model, high probability low consequence (HPLC) risk, low probability high consequence (LPHC) risk, optimal insurance demand, background risk, mental accounting

JEL: D81, D91, G22

1 Introduction

Empirical studies find a rather low insurance demand for "catastrophic risk" characterised by occasional but severe losses.[1] in contrast, one observes a high insurance demand for risks which give rise to small losses only but occur often such as an insurance of mobile phones or against bicycle thefts.[2] These observations are not in line with expected utility theory which predicts a high demand for disaster insurance compared to mobile phone or bicycle insurance, i.e. individuals do not behave in line with expected utility theory.

In this paper, we explain this puzzle or behavioral bias in a basic insurance model by mental accounting. We basically consider two types of risks. One risk occurs with low probability and has high consequences (LPHC), whereas the other risk occurs with high probability and has low consequences (HPLC). We show that the investor has a higher demand for insuring LPHC-risks than HPLC-risks when only one of these risks is present. In the more realistic case that both risks are present, however, one may also observe the opposite result in that the investor has a higher demand for insuring HPLC-risk than LPHC-risk.

In a first step, we reconsider the basic insurance model. The decision-maker is endowed with an initial wealth w_0 which is threatened by a possible loss. A loss of size $L > 0$ can occur with probability p while there is no loss with probability $1 - p$. Investing the amount y into insurance reduces the loss size from L to $l(y, p)$ (if a loss occurs). The insurance premium includes a loading factor δ such that it is equal to $(1 + \delta)$ times the actuarially fair premium (given by the expected payout of the insurance contract).

Our explanation of the behavioral bias takes the standard results within the basic insurance model as a starting point. Hence, we briefly review some well known results. In particular, we consider the case where the expected impact of the loss is constant, i.e. $p \cdot L = \text{constant}$, and show that the optimal investment into insurance is decreasing in the loss probability/increasing in the loss size. This implies that the investor should invest more to insure LPHC-risks than he invests to insure HPLC-risks. Furthermore, we show that the optimal investment into insurance decreases

[1]For example, Botzen and van den Bergh (2012) observe that a significant proportion of home-owners neglect the low-probability flood risk.

[2]Empirical examples are e.g. given in Browne et al. (2015)

in initial wealth (for decreasing absolute risk aversion).

In a second step, we turn to mental accounting. We introduce a second (independent) risk into the model. One risk occurs with a rather low probability $p^{(l)}$, while the other risk occurs with a rather high probability $p^{(h)}$ ($p^{(h)} > p^{(l)}$). The decision maker is thus, at the same time, confronted with an LPHC-risk and an HPLC-risk. We assume that the two risks have the same expected impact c. The loss sizes $L^{(l)}$ ($L^{(h)}$) of the LPHC (HPLC)-risk is then given by $L^{(x)} = \frac{c}{p^{(x)}}$ ($x \in \{l, h\}$). There are no differences in pricing, i.e. both risks share the same insurance loading factor δ.[3] Mental accounting then amounts to stating two optimization problems with an insurance demand either for the LPHC- or the HPLC-risk, while the other risk is considered to be uninsurable background risk.

We then compare the optimal investment into insurance of the LPHC-risk (with HPLC-risk as background risk) to the switched situation in which the investor insures against HPLC-risk (with LPHC-risk as background risk). When comparing the resulting optimal insurance demands, it turns out that both relations are possible: we may find a higher or a lower insurance demand for LPHC compared to HPLC. As we show in an example, the higher demand for HPLC-risk is observed when the initial wealth is small relative to the loss sizes and/or when the loss size of the LPHC-risk is very large.

Our contribution to the literature can be summarized as follows. Starting from the basic insurance model we give an easy explanation of the insurance puzzle which is posed by a rather low demand for catastrophe insurance compared to mobile phone or bicycle insurance. The results are intuitively explained in the standard insurance model (with one insurable risk and one background risk). The results are illustrated for CRRA-utility (which implies decreasing absolute risk aversion (DARA)) and CARA-utility.

Our paper is related to several strands of the literature which aim to explain the empirically observed insurance demand, in particular with respect to LPHC- and HPLC-risks. A complete overview is beyond the scope of the paper such that we restrict ourselves to a subset of papers. We start with some references to laboratory studies and experiments which document the puzzle mentioned above (namely

[3]The interested reader is referred to Kousky and Cooke (2012) who show that when insuring risks with loss distributions characterised by fat tails, micro-correlations or tail dependence, insurers need to charge a price that is many times the expected loss in order to meet their solvency constraint.

that insurance for HPLC is preferred). Then, we turn to the literature tackling the insurance demand in a basic insurance model and to some extensions, in particular regrading background risk and mental accounting.

Slovic et al. (1977) are one of the first papers using laboratory studies to show that decision-makers prefer to buy insurance against HPLC-risks.[4] Subsequently, a lot of experiments were conducted in the field of research. The following papers concentrate on the insurance demand for LPHC-risks. McClelland et al. (1993) argue that decision-makers either do not pay attention to LPHC-risks at all or overestimate the risk. By overestimating the risk they are willing to pay more then the expected value for an insurance against such a risk. Ganderton et al. (2000) show that decision-makers are more sensitive towards the loss probability than the loss size.[5] Laury et al. (2009) point out that decision-makers underinsure against LPHC-risks because they can not distinguish between low- and zero-probabilities or because the insurance is too expensive.[6]

Experiments regarding the behavior of the decision-maker are e.g. provided by Schoemaker and Kunreuther (1979). They find that the decision-makers act more risky if the loss probability is small.[7] Kunreuther et al. (2001) point out that the decision-maker needs a lot of information to differentiate between low probabilities. Common probabilities or insurance premiums do not reflect the feeling of riskiness properly for the decision-maker.

Interestingly, there exists no model to explain the anomaly that decision-makers prefer buying insurance against HPLC- over LPHC-risks if both risks are priced along the lines of the same loading factor. The literature discussing the optimal insurance demand dates back to Mossin (1968). He shows that a risk averse decision-maker prefers full insurance coverage if the insurance is priced actuarially fair and partial insurance coverage if there is a positive loading. A lot of theories

[4]Slovic et al. (1977) point out that their results are in line with a field study by Kunreuther (1977). The authors mention two explanations for their result: The decision-maker has a convex utility function over losses (instead of a (risk averse) concave function) or she does not care about very small probabilities.

[5]Ganderton et al. (2000) also point out that many of their results are in line with expected utility theory, e.g. the decision-maker buys less insurance when the insurance is not priced actuarially fair.

[6]Laury et al. (2009)In an experiment they derive the result that decision-makers do not underinsure LPHC-risks when the incentives are real

[7]Schoemaker and Kunreuther (1979) point out that this results in the limited sensitivity of the decision-makers towards occurrences with low probabilities

developed over the years extend this famous result of the author and try to explain the demand for insurance. Hence, our work is also related to the literature dealing with background risk and mental accounting.

The literature regarding background risk and the explanation for the insurance demand dates back to Eeckhoudt and Kimball (1992). They argue that the presence of an uninsurable background risk lead to a higher optimal insurance amount for the insurable risk. Schlesinger (2000) confirms the findings of Mossin (1968) as argued above for an independent background risk.[8] For a non-independent background risk the author argues that the value of the optimal insurance demand depends on different circumstances, e.g. if the non-independent background risk exists in the loss and the no-loss state. Fei and Schlesinger (2008) show that a higher uninsurable background risk in the loss state leads to an increase in the insurance demand for any prudent decision-maker while this is the opposite for a higher background risk in the no-loss state.[9]

The concept of mental accounting as another approach to explain the demand for insurance dates back to Thaler (1999). Mental accounting is comparable to financial accounting, i.e. recording and analyzing financial transactions. The author argues that *expenditures are grouped into budgets* and that these budgets are allocated to different mental accounts which are not substitutable. Consequently, insurance decisions for different types of risks as HPLC- and LPHC-risks are evaluated separate in various mental accounts.

The outline of the paper is as follows. In Section 2, we reconsider the basic insurance model and give a brief review of the results which are needed subsequently. In addition, we show that the decision-maker prefers to buy insurance for LPHC-risks over HPLC-risks if the expected loss sizes coincide and the loading factor is the same in both cases. In Section 3, we introduce two risks, LPHC- and HPLC-risk, into the model, i.e. they exist at the same time. We introduce mental accounting by stating two optimization problems with an insurance demand for one risk only (either for the LPHC- or the HPLC-risk) while taking the other risk as (uninsurable)

[8]Schlesinger (2000) adds that in case of an actuarially unfair insurance and a utility function that exhibits standard risk aversion (meaning that absolute risk aversion and prudence are decreasing in wealth) results in a higher optimal insurance amount.

[9]Fei and Schlesinger (2008) point out that for a higher uninsurable background risk in the loss state and an actuarially fair insurance more then full insurance coverage is optimal. This result is contrary the the one of Mossin (1968).

background risk. We analyze the first order conditions and compare the resulting optimal insurance demands. The main result which explains the insurance puzzle is numerically illustrated. Section 4 concludes the paper.

2 Basic insurance model and its implications for optimal insurance demand

We start with the basic model of insurance demand and show how to determine the optimal investment into insurance. We then briefly review some well known results which will turn out to be useful afterwards. The section closes with a proof of the result that in the conventional setup, the insured prefers a higher insurance level for LPHC- than HPLC-risks.

2.1 Basic model of insurance demand

We assume that the decision maker is endowed with an initial wealth w_0 ($w_0 > 0$). In the basic model of insurance demand, a loss of L ($0 < L < w_0$) can occur with probability p while there is no loss with probability $1 - p$. Buying insurance can reduce the loss size L, i.e. investing the amount y reduces the loss (if it occurs) from L to $l(y,p)$ where $l(0,p) = L$. The reduced loss size $l(y)$ and the costs for insurance y are given by

$$l(y,p) = L - \frac{y}{(1+\delta)p} \qquad \text{and} \qquad y = p(1+\delta)(L - l(y,p)),$$

where δ denotes the loading factor which is proportional to the expected liability of the insurance. We assume that $\delta \geq 0$. For $\delta = 0$, the insurance is priced actuarially fair, while $\delta > 0$ implies that the insurer requires a premium above the actuarially fair one. In addition, we assume that $p(1+\delta) < 1$, so that a finite optimal loss size exists.[10]

The terminal wealth of the decision-maker depends on his investment into insurance and on whether a loss occurs. It is equal to $w_0 - y$ in case there is no loss, and equal to $w_0 - y - l(y,p)$ in case a loss occurs.

[10]For a proof see Appendix A.1

The decision-maker maximizes her expected utility of terminal wealth. Her utility function is u, and we assume that she is risk averse ($u' > 0, u'' < 0$). The optimal insurance demand then follows from

$$y^*(p, w_0) = \arg\max_y \{p \cdot u(w - y - l(y,p)) + (1-p) \cdot u(w-y)\}$$

The following proposition summarizes the well known optimality condition for the insurance demand.

Proposition 1 (Optimal insurance) *The first order condition for the optimal investment into insurance $y^*(p, w_0)$ is given by*

$$\underbrace{\frac{u'(w-y-l(y,p))}{u'(w-y)}}_{=:f(y,p,w_0)} = \underbrace{\frac{1-p}{\frac{1}{1+\delta} - p}}_{=:g(p)},$$

where f denotes the ratio of marginal utilities in the loss and the no-loss state. Given p and L, the optimal investment $y^*(p, w_0)$ into insurance thus solves

$$f(y^*(p, w_0), p, w_0) = g(p).$$

Proof: For the sake of completeness, a proof is given in Appendix A.2.

2.2 Optimal Investment into Insurance

We now review and illustrate some well known results which turn out to be useful in the following analyses. If not mentioned otherwise, we assume CRRA preferences with $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ and $u(x) = \ln x$ for $\gamma = 1$.

2.2.1 Impact of risk aversion

As already stated by Mossin (1968), a risk averse decision maker will optimally choose full coverage if the insurance is actuarially fair, i.e. if $\delta = 0$. For non actuarially fair insurance, i.e. for $\delta > 0$, partial insurance coverage is optimal. In this case, the optimal investment in insurance is the larger the higher the risk aversion of the decision-maker (see e.g. Schlesinger (2000)).

The impact of risk aversion is illustrated in Figure 1. It gives the certainty equivalent wealth as a function of the investment into insurance for a relative risk

Benchmark parameter constellation

w_0	p	L	c (expected loss)	δ	ymax
20	0.14706	6.8	0.1	0.1	0.11

Table 1: Benchmark parameter constellation.

aversion of $\gamma = 3$ (dashed red line) and $\gamma = 8$ (solid black line).[11] In line with intuition, the certainty equivalent is lower for the high risk aversion $\gamma = 8$ than for $\gamma = 3$. Furthermore, the optimal investment into insurance – shown by the vertical dashed lines – is larger for the high risk aversion than for the low risk aversion.

2.2.2 Impact of initial wealth

For $\delta > 0$, the optimal investment into insurance is below the investment for full coverage and in general depends on the risk aversion of the decision-maker (see Schlesinger (2000)). For constant absolute risk aversion (with utility function $u(x) = -e^{-\gamma x}$), the optimal $y(p, w_0)$ is independent of the initial wealth. For a decreasing absolute risk aversion (which is the case for the CRRA decision-maker), it is decreasing in the initial wealth. The higher the wealth level of the decision-maker, the more willing he is to accept some risk, and the less he aims at reducing the risk of a loss by buying insurance. Finally, an increasing absolute risk aversion would imply that the optimal investment into insurance increases in the initial wealth.

2.2.3 Impact of an independent background risk

Uninsurable background risk will in general also have an impact on the optimal investment into insurance (see Schlesinger (2000)). We assume that the risk that can be insured and the background risk are independent. It then again holds that full insurance is optimal for $\delta = 0$, while partial coverage is optimal for $\delta > 0$. For increasing relative risk aversion, it furthermore holds that the optimal investment into insurance is increasing in the amount of zero-mean background risk.

[11] The certainty equivalent as a function of the insurance demand y is defined by the value where the utility of the decision maker is equal to the expected terminal wealth if the insurance amount is y.

Optimal insurance y^* for different risk aversion

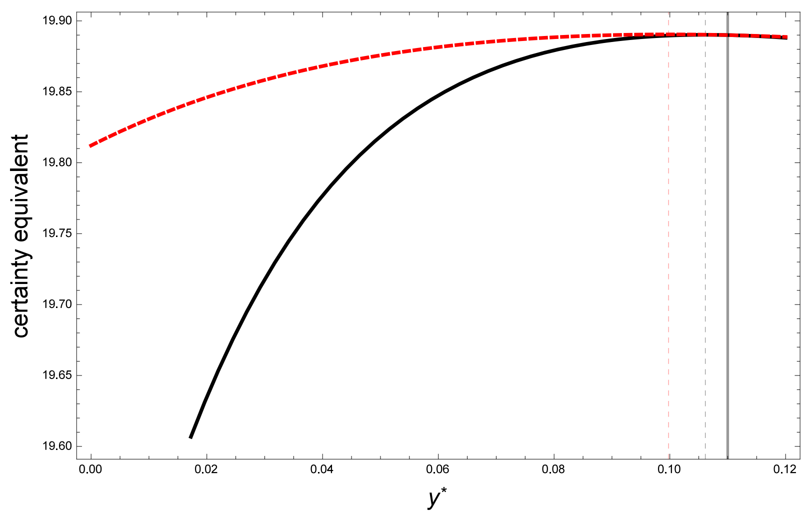


Figure 1: The figure gives the certainty equivalent wealth as a function of the investment into insurance. The relative risk aversion of the decision-maker is equal to $\gamma = 3$ (dashed red line) and $\gamma = 8$ (black solid line). The solid vertical line gives the insurance investment y_{max} which corresponds to full coverage, while the dashed vertical lines give the optimal insurance investments for the two decision-maker.

2.3 Optimal insurance demand for LPHC and HPLC risks

We now take a first look at the difference in the insurance demand for LPHC- (low probability for a high loss) and HPLC-risks (high probability for a small loss). To do so, we set $p \cdot L(p) = c$, i.e. assume that the expected loss is equal for all risks $(p, L(p))$.

Proposition 2 (Optimal insurance as a function of loss probability) *Assume that the expected impact of the loss is constant, i.e. $p \cdot L(p) = c$. For the optimal investment $y^*(p, L(p))$, it holds true that*

$$\frac{\partial y^*(p, L(p))}{\partial p} < 0.$$

The optimal investment into insurance is thus larger for LPHC than for HPLC.

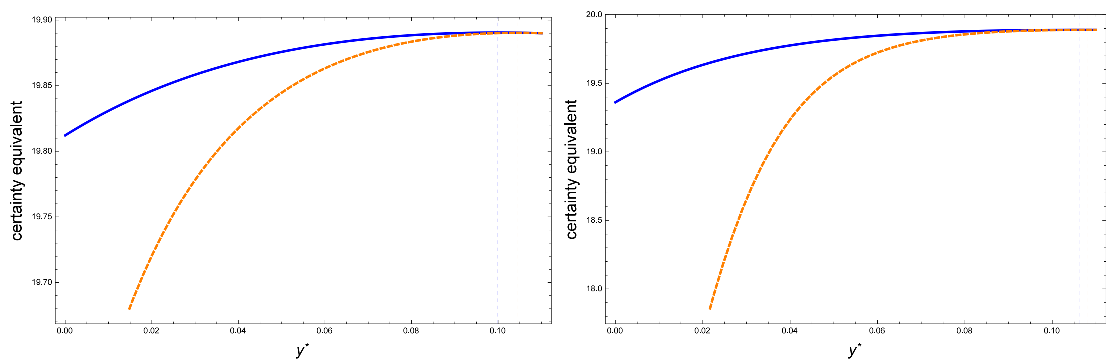
Proof: The proof is given in Appendix A.3 and B.2.

Benchmark parameter constellation

w_0	p^{low}	L^{high}	p^{high}	L^{low}	c (expected loss)	δ	$ymax$
20	0.00784	12.75	0.01471	6.8	0.1	0.1	0.11

Table 2: Benchmark parameter constellation.

Optimal insurance y^* with two risks for different risk aversion



(a) Less risk averse decision-maker ($\gamma = 3$)

(b) More risk averse decision-maker ($\gamma = 8$)

Figure 2: The figure gives the certainty equivalent as a function of the investment into insurance for LPHC-risk (dashed orange line) and HPLC-risk (solid blue line). The left graphs shows a decision maker with $\gamma = 3$, the right graph shows a decision maker with $\gamma = 8$.

The result is illustrated in Figure 2, which compares the certainty equivalents for LPHC-risk and HPLC-risk. For both risk aversion levels, it holds that the certainty equivalent is smaller in case of LPHC-risk than in case of HPLC-risk. The decision-maker is thus more afraid of the risk of large but rare losses than of the risk of small but frequent losses.

In line with this intuition, the optimal investment into insurance is larger for LPHC-risk ($y = 0.10459$ for the less risk averse decision-maker, $y = 0.10795$ for the more risk averse decision-maker) than for HPLC-risk ($y = 0.09978$ for the less risk averse decision-maker, $y = 0.10613$ for the more risk averse decision-maker).

In the following, we consider the difference between HPLC and LPHC when both risks are present simultaneously. In particular, we want to know whether the decision-maker is always more anxious to insure against LPHC-risk than against HPLC-risk.

3 Background Risk

3.1 Model Setup

In the base case, the optimal investment into insurance is larger for LPHC-risk than for HPLC-risk (cf. Proposition (2)). The investor is thus more afraid of rare but large losses than of small but frequent losses, i.e. if the expected loss sizes coincide and the loading factor is the same in both cases. In order to explain the puzzle that one observes the opposite direction of insurance demand, i.e. there is a higher demand for e.g. mobile phone insurance than for e.g. flood insurance, we consider mental accounting. The following reasonings about background risk are motivated by the assumption that the insured does not simultaneously decide about different insurance contracts. Rather, she chooses her optimal demand for a particular insurance coverage while having the other risks in the background.

Thus, we shed further light on the optimal insurance demand under the impact of background risk, i.e. we assume that there is an additional risk which the decision-maker can not insure. Instead of general background risk, we consider the special case that the background risk is similar to the insurable risk for which a loss of size L_b happens with probability p_b. Furthermore, we assume that the insurable risk and the background risk are independent of each other.

We are interested in the impact of LPHC and HPLC background risks on the insurance decision for the (p, L)-risk. We assume that $p_b \cdot L_p = p \cdot L = c$, i.e. the expected losses coincide for the insurable risk and the background risk.

The expected utility of the insured is given by

$$E\left[u\left(W\right)\right] = E\left[u\left(w_0 - 1_{Lb} \cdot l(0, p_b) - y - 1_L \cdot l(y, p)\right)\right]$$

where 1_L and 1_{Lb} are the indicator functions for the insurable loss event and for the background loss event. We can rewrite this expected utility as

$$
\begin{aligned}
& E\left[u\left(W\right)\right] \\
&= E\left[E\left[u\left(w_0 - 1_{Lb}\cdot l(0,p_b) - y - 1_L\cdot l(y,p)\right)\Big| 1_{Lb}\right]\right] \\
&= p_b E\left[u\left(w_0 - l(0,p_b) - y - 1_L\cdot l(y,p)\right)\right] + (1-p_b)E\left[u\left(w_0 - y - 1_L\cdot l(y,p)\right)\right].
\end{aligned}
$$

In the presence of an independent background risk, the expected utility can thus be stated as a weighted average of the expected utilities when only the insurable risk is present, where we account for the background loss by a reduction in initial wealth from w_0 to $w_0 - l(0, p_b)$ in case it occurs. The first order conditions are then also a weighted average of the respective first order conditions in the basic insurance model.

3.2 Dependence of Optimal Investment into Insurance on Loss Probabilities and Initial Wealth

In the basic model, the optimal investment into insurance is decreasing in wealth (for decreasing absolute risk aversion), (cf. Section (2.1)). This result together with the assumption that the decision-maker is risk averse, i.e. her utility function is concave, give the following bounds on the optimal investment into insurance:

Proposition 3 (Optimal insurance with background risk: bounds) *The optimal investment $y(p, p_b, w_0)$ into insurance in case of uninsurable background risk solves the optimization problem*

$$\arg\max_y E\left[u\left(w_0 - 1_{Lb}\cdot l(0,p_b) - y - 1_L\cdot l(y,p)\right)\right].$$

It holds that

1. $y(p, p_b, w_0) > y(p, 0, w_0)$
2. $y(p, p_b, w_0) > y(p, 0, w_0 - p_b l(0, p_b))$
3. $y(p, p_b, w_0) < y(p, 0, w_0 - l(0, p_b))$

For a proof, see Appendix C.1.

The first inequality states that, compared to the optimal insurance demand without background risk $y(p, 0, w_0)$, the presence of background risk implies a larger optimal investment into insurance. This is due to (i) the lower expected initial wealth (which drops from w_0 to $w_0 - p_b \cdot l(0, p_b)$) and (ii) the additional variance in wealth. At the same time, the optimal investment in case of a deterministic reduction of initial wealth by $l(0, p_b)$ is an upper bound for the investment into insurance (second inequality).

It is also interesting to consider the dependence of the optimal investment on the initial amount of wealth, the loss probability of the insurable event, and the loss probability of the background loss.

Proposition 4 *For the optimal investment $y(p, p_b, w_0)$ in case of background risk, it holds that*

1. *$y(p, p_b, w_0)$ is a decreasing function of wealth*
2. *$y(p, p_b, w_0)$ is a decreasing function of the loss probability p*
3. *$y(p, p_b, w_0)$ is a decreasing function of the background loss probability p_b*

Proof: see Appendix C.3

The first two results are inherited from the basic insurance model. Under a decreasing absolute risk aversion (DARA), the decision maker is willing to take a higher risk if her wealth increases. Thus, she is willing to hold on to a larger part of the risk in terms of the tuple of loss probability and loss size $\left(p, \frac{c}{p}\right)$ and thereby reduces her investment into insurance.

The dependence on the loss probability p reflects that the decision-maker is more afraid of large but rare losses than of small but frequent losses. She invests more into insurance for LPHC-risks than for HPLC-risks.

The dependence on the background loss probability is more involved. The decision-maker suffers from the expected reduction in her initial wealth and from the additional riskiness of her terminal wealth. The first effect is the same for LPHC-risks and HPLC-risks, since we assume that the expected loss is kept constant. However, LPHC-risks are again more severe for the decision-maker than HPLC-risks, so that the presence of the disastrous LPHC background risk increases the optimal insurance by more than the presence of the less severe HPLC background risk.

3.3 Comparison of HPLC and LPHC in the Switch-Case

We now compare the optimal insurance demand of HPLC- and LPHC-risks by fixing a low probability p_l and a high probability p_h. Then we consider the following two situations where we switch the roles of the two risks from background risk to insurable risk and vice versa:

1. The LPHC-risk is insurable, while the HPLC-risk is the uninsurable background risk. The optimal amount of insurance is $y(p_l, p_h, w_0)$.
2. The HPLC-risk is insurable, while the LPHC-risk is the uninsurable background risk. The optimal amount of insurance is $y(p_h, p_l, w_0)$.

Consider the switch from the second case (LPHC is background risk) to the first case (LPHC is insurable). For the insurable risk, the loss probability decreases from p_h to p_l. If the decision-maker faces rare but large losses, she increases her optimal investment into insurance. At the same time, the probability of the background risk increases from p_l to p_h. Background risk, which is now constituted of small but frequent losses, is thus less severe, and the optimal investment into insurance drops. Overall, there are thus two opposing directional effects on the optimal investment into insurance. The optimal insurance demand for HPLC-risk with LPHC background risk may thus be higher than the optimal insurance demand for the LPHC-risk in the switched case or vice versa.

In most cases, the optimal investment into insurance is larger when rare but large losses (LPHC) can be insured than in the switched case where small but frequent losses (HPLC) can be insured, i.e. it holds that $y(p_l, p_h, w_0) > y(p_h, p_l, w_0)$. There are, however, also cases where the optimal investment into insurance drops when the insurable risk switches from HPLC to LPHC.

To get the intuition, we consider a numerical example. The basic parameters for the following figures are given in Table 2. The utility function belongs to the class of utility functions with constant relative risk aversion (CRRA) implying decreasing absolute risk aversion (DARA). The constant level of relative risk aversion is denoted by γ.

Figure 3 depicts the optimal amount of insurance as a function of the relative risk aversion. It shows that the decision-maker chooses a higher insurance for large

Comparison of $y(p_l, p, w_0)$ and $y(p_h, p_l, w_0)$ for different risk aversion

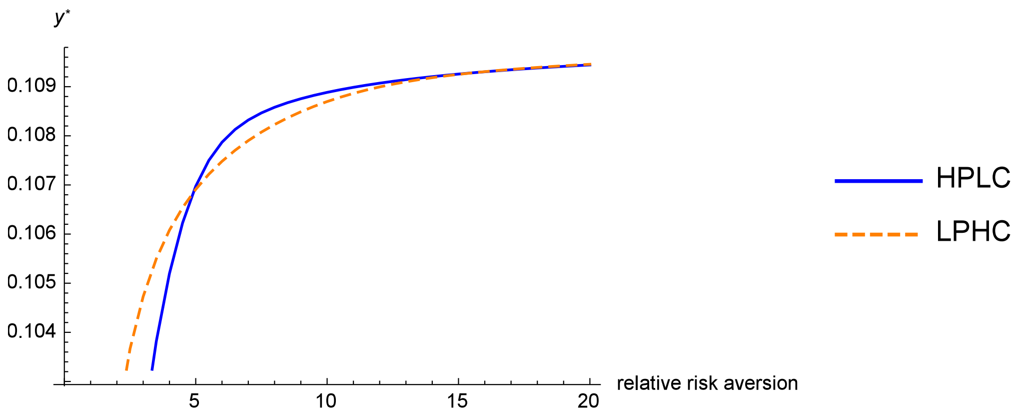


Figure 3: The figure gives the optimal investment into insurance as a function of the relative risk aversion. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 2.

but rare losses (the standard result) if she is described by a small relative risk aversion. If her relative risk aversion increases, however, the result changes, and she now buys more insurance for small but frequent losses. For a high risk aversion, finally, the investment into insurance approaches its maximum for both types of risk, since the decision-maker decides to eliminate the insurable risk nearly completely.

Figure 4 gives the optimal amount of insurance as a function of the probability p_l of large but rare losses. Again, the standard result holds if the probability p_l is not too small. The more extreme the LPHC-risk gets, however, i.e. the smaller the loss probability and the larger the loss size, the larger the insurance of HPLC-risk (with LPHC background risk) than the insurance of LPHC risk (with HPLC background risk).

Figure 5 gives the optimal amount of insurance as a function of the initial wealth w_0. For high levels of initial wealth, we find the standard result that large but rare losses demand a higher investment into insurance then small but frequent losses. However, the result changes again for low levels of initial wealth. Now, the

Comparison of $y(p_l, p, w_0)$ and $y(p_h, p_l, w_0)$ for different values of p_l

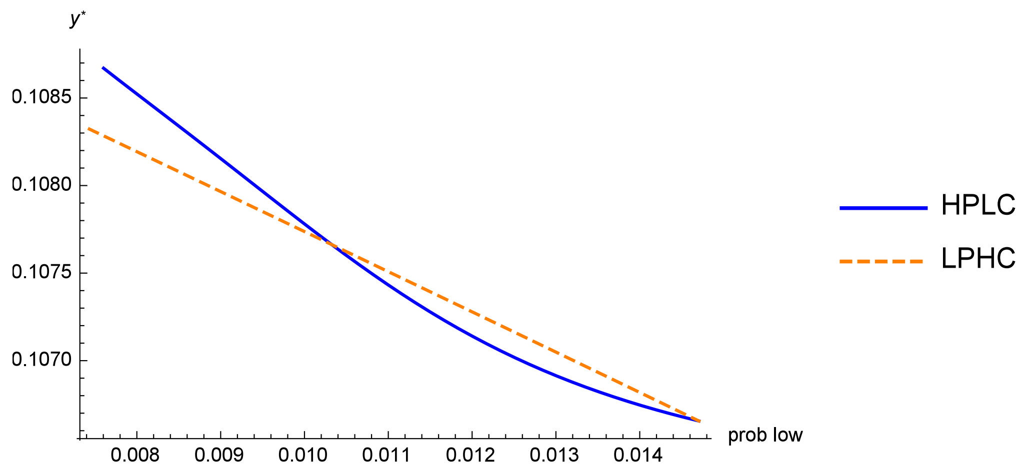


Figure 4: The figure gives the optimal investment into insurance as a function of the low probability p_l. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 2.

optimal amount of insurance is larger for HPLC than for LPHC.

To get the intuition, it is convenient to have a look at the level of terminal wealth. The lowest value of terminal wealth realizes if both loss events occur. Since the decision-maker suffers most from the large losses, she is particularly interested in achieving a somehow higher wealth in the worst case. She can do so by insuring the risk. If she can buy insurance for LPHC-risk, the increase of wealth in the worst case is larger than when she can only buy insurance for HPLC-risk. Since the loading is the same for both types of risk, this leads to the standard result that she invests more to protect herself against insurable LPHC-risk than against insurable HPLC-risk (where she is still left with the uninsurable LPHC-risk).

However, this result can also change. If she has a small initial wealth w_0 or if the probability of the LPHC-risk p_l is very small (and the loss size $\frac{c}{p_b}$ thus very large), the worst case terminal wealth is very small and rather close to zero. In this case, the decision-maker urgently wants to improve her worst case wealth level.

Comparison of $y(p_l, p, w_0)$ and $y(p_h, p_l, w_0)$ for different levels of initial wealth

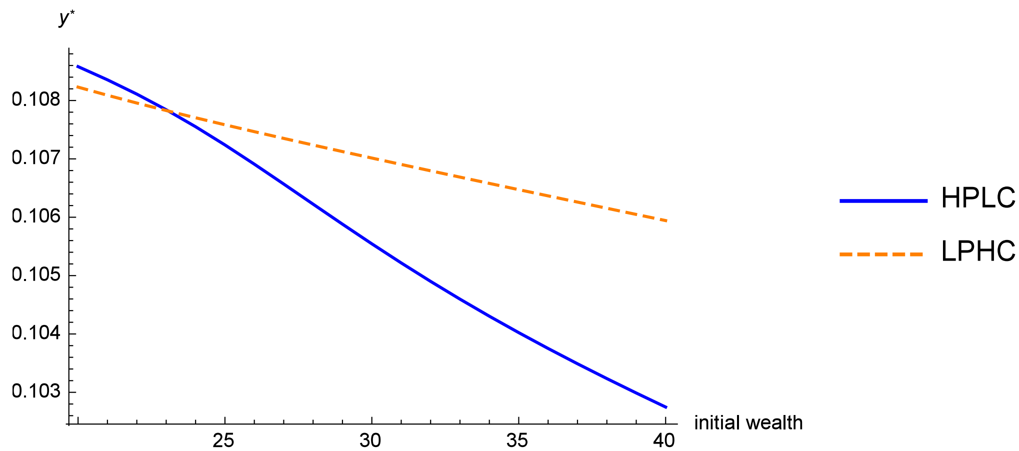


Figure 5: The figure gives the optimal investment into insurance as a function of the initial wealth w_0. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 2.

Buying insurance against HPLC-risk can then become very attractive. Even if the decision-maker still has to cope with the remaining uninsurable risk, the insurance against HPLC-risk increases the wealth in the worst case at least a bit. If this state is disastrous, the decision-maker would even be willing to accept a higher loading then the given one, and heavily buys HPLC insurance. When she buys LPHC insurance, a lower investment into insurance helps her to bring the worst case level of wealth to a comparable level.

In summary, it is thus possible to explain the insurance puzzle posed by a rather low demand for catastrophe insurance compared to the demand for HPLC-risk as e.g. given by mobile phones or bicycle thefts in a basic insurance model with two independent risk by introducing mental accounting.

4 Conclusion

Expected utility theory predicts a high insurance demand for LPHC-risks. Contrary to this result empirical observations and experiments find a rather low insurance demand for LPHC-risks and a high demand for HPLC-risks. To the best of our knowledge, there is yet no theoretical model which explains the anomaly that decision-makers prefer buying insurance for HPLC-risks over buying insurance for LPHC-risks if these risks are comparably priced with the same loading factor. In this paper, we explain this puzzle or behavioral bias by the concept of mental accounting in a basic insurance model with two independent risks.

We introduce a second independent risk so that the decision-maker is confronted with two risks, LPHC and HPLC, at the same time. Assuming the concept of mental accounting the decision-maker decides about her two insurance demands separately while considering the other risk as an uninsurable background risk. On the one hand she thus chooses her optimal insurance amount for the LPHC-risk (HPLC-risk is an uninsurable background risk) and on the other hand the optimal insurance demand for the HPLC-risk (LPHC-risk is an uninsurable background risk). Comparing the optimal insurance demands for both risks we find that both relations are possible: there can be a higher or a lower insurance demand for LPHC-risk compared to HPLC-risk. We provide an intuitive explanation for this finding by disentangling the opposing directional effects.

A Basic Insurance Model with one Insurable Risk

A.1 Restriction $p(1+\delta) < 1$

We denote the terminal wealth in the static insurance model in the no-loss and in the loss state as

$$\begin{aligned} w^{NL} &= w_0 - y \\ w^{L} &= w_0 - y - l(y) \end{aligned}$$

where the loss size is given by

$$l(y) = L - \frac{y}{(1+\delta)p} \qquad y = p(1+\delta)\left(L - l(y)\right).$$

The loss size $l(y)$ remains positive for

$$y \leq p(1+\delta)L =: y_{max}.$$

We assume $\delta \geq 0$ and

$$p(1+\delta) < 1.$$

To see the reason for the latter restriction, consider the wealth as a function of $l(y)$:

$$\begin{aligned} w^{NL} &= w_0 - p(1+\delta)L + p(1+\delta)l(y) \\ w^{L} &= w_0 - p(1+\delta)L + [p(1+\delta) - 1]\, l(y) \end{aligned}$$

If $p(1+\delta) > 1$, both w^{NL} and w^L increase in $l(y)$. In this case, it would be optimal for the investor to choose an infinite loss size. This choice would also be optimal for $p(1+\delta) = 1$. For a finite optimal loss size, we thus need $p(1+\delta) < 1$.

A.2 Proof of Proposition 1

The expected utility is

$$\begin{aligned} E[u(W)] &= (1-p)u(w^{NL}) + p\,u(w^L) \\ &= (1-p)u(w_0 - y) + p\,u(w_0 - y - l(y)) \\ &= (1-p)u(w_0 - y) + p\,u\left(w_0 - y - L + \frac{y}{(1+\delta)p}\right) \end{aligned}$$

The investor wants to choose the optimal investment into insurance, i.e. the optimal $y^*(p, L)$. The first order condition is

$$\frac{\partial E[U(W)]}{\partial y} = 0$$

This can be rewritten as

$$\begin{aligned} (1-p)u'(w^{NL})(-1) + p\,u'(w^L)\left(-1 + \frac{1}{(1+\delta)p}\right) &= 0 \\ p\,u'(w^L)\left(-1 + \frac{1}{(1+\delta)p}\right) &= (1-p)u'(w^{NL}) \\ u'(w^L)\left(-p + \frac{1}{1+\delta}\right) &= (1-p)u'(w^{NL}) \\ \underbrace{\frac{u'(w^L)}{u'(w^{NL})}}_{f(y,p,L)} &= \underbrace{\frac{1-p}{\frac{1}{1+\delta} - p}}_{g(p)} \end{aligned}$$

Given p and L, the optimal investment $y^*(p, L)$ into insurance thus solves

$$f(y^*(p, L), p, L) = g(p).$$

A.3 Proof of Proposition 2

With CRRA utility, the ratio of marginal utilities is

$$f(y, p, L) = \frac{u'(w^L)}{u'(w^{NL})} = \left(\frac{w_0 - y - l(y)}{w_0 - y}\right)^{-\gamma} = \left(1 - \frac{l(y)}{w_0 - y}\right)^{-\gamma} = \left(1 - \frac{L - \frac{y}{(1+\delta)p}}{w_0 - y}\right)^{-\gamma}.$$

The assumption $pL = c$ allows to eliminate L in the relation between $l(y)$ and y:

$$l(y) = L - \frac{y}{(1+\delta)p} = \frac{c}{p} - \frac{y}{(1+\delta)p} = \frac{1}{p}\left(c - \frac{y}{1+\delta}\right)$$

The function f then becomes

$$f(y, p, L(p)) = \left(1 - \frac{\frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\gamma} = \left(\frac{w_0 - y - \frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\gamma}$$

The partial derivatives of $f(y, p, L(p))$ w.r.t. p and y are

$$\frac{\partial f(y, p, L(p))}{\partial p} = -\gamma \left(\frac{w_0 - y - \frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\gamma-1} \frac{\frac{1}{p^2}(c - \frac{y}{1+\delta})}{w_0 - y}$$

$$\begin{aligned}
\frac{\partial f(y, p, L(p))}{\partial y} &= -\gamma \left(\frac{w_0 - y - \frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\gamma-1} \\
&\quad \cdot \frac{\left(\frac{1}{(1+\delta)p} - 1\right)(w_0 - y) - (w_0 - y - \frac{1}{p}(c - \frac{y}{1+\delta}))(-1)}{(w_0 - y)^2} \\
&= -\gamma \left(\frac{w_0 - y - \frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\gamma-1} \frac{w_0 - y - c(1+\delta) + y}{(1+\delta)p(w_0 - y)^2} \\
&= -\gamma \left(\frac{w_0 - y - \frac{1}{p}\left(c - \frac{y}{(1+\delta)}\right)}{w_0 - y}\right)^{-\frac{1}{\gamma}-1} \frac{w_0 - c(1+\delta)}{(w_0 - y)^2(1+\delta)p}
\end{aligned}$$

The optimal investment into insurance depends on the loss probability p, i.e. $y^* = y^*(p, L(p))$. The derivative of f (evaluated for the optimal y) w.r.t. p is then given by

$$\frac{\partial f(y(p, L(p)), p, L(p))}{\partial p} = -\gamma \left(\frac{w_0 - y - \frac{1}{p}(c - \frac{y}{1+\delta})}{w_0 - y} \right)^{-\gamma-1} \left[\frac{\frac{1}{p^2}(c - \frac{y}{1+\delta})}{w_0 - y} + \frac{w_0 - c(1+\delta)}{(w_0 - y)^2(1+\delta)p} \cdot y'(p) \right]$$

For the optimal y^*, it holds that f coincides with g. The derivative of g w.r.t. p is

$$\frac{\partial g(p)}{\partial p} = \frac{-1\left(\frac{1}{1+\delta} - p\right) + 1(1-p)}{\left(\frac{1}{1+\delta} - p\right)^2} = \frac{1 - \frac{1}{1+\delta}}{\left(\frac{1}{1+\delta} - p\right)^2}$$

The equality of the function values $f(y(p, L(p)), p, L(p))$ and $g(p)$ for all p implies the equality of the partial derivatives w.r.t. p. Plugging the derivatives in gives

$$-\gamma \left(\frac{w_0 - y - \frac{1}{p}(c - \frac{y}{1+\delta})}{w_0 - y} \right)^{-\gamma-1} \left[\frac{\frac{1}{p^2}(c - \frac{y}{1+\delta})}{w_0 - y} + \frac{w_0 - c(1+\delta)}{(w_0 - y)^2(1+\delta)p} \cdot y'(p) \right] = \frac{1 - \frac{1}{1+\delta}}{(\frac{1}{1+\delta} - p)^2}$$

In the following, it will turn out to be useful to use that

$$\begin{aligned} f(p) &= \left(\frac{w_0 - y - \frac{1}{p}(c - \frac{y}{1+\delta})}{w_0 - y} \right)^{-\gamma} \\ f(p)^{-\frac{1}{\gamma}} &= \left(\frac{w_0 - y - \frac{1}{p}(c - \frac{y}{1+\delta})}{w_0 - y} \right) \end{aligned}$$

In the following, we will also use that $f(p) = g(p)$. Plugging in gives

$$\begin{aligned} -\gamma f(p)^{-\frac{1}{\gamma}(-\gamma-1)} \left[\frac{\frac{1}{p^2}(c - \frac{y}{1+\delta})}{w_0 - y} + \frac{w_0 - c(1+\delta)}{(w_0 - y)^2(1+\delta)p} \cdot y'(p) \right] &= \frac{1 - \frac{1}{1+\delta}}{(\frac{1}{1+\delta} - p)^2} \\ -\gamma f(p)^{1+\frac{1}{\gamma}} \left[\frac{\frac{1}{p^2}(c - \frac{y}{1+\delta})}{w_0 - y} + \frac{w_0 - c(1+\delta)}{(w_0 - y)^2(1+\delta)p} \cdot y'(p) \right] &= \frac{1 - \frac{1}{1+\delta}}{(\frac{1}{1+\delta} - p)^2} \end{aligned}$$

We can now solve for $y'(p)$:

$$\begin{aligned}
y'(p) &= \left[\frac{1-\frac{1}{1+\delta}}{(\frac{1}{1+\delta}-p)^2}\cdot\frac{-1}{\gamma}\cdot f(p)^{-1-\frac{1}{\gamma}} - \frac{1}{p}\cdot\frac{c-\frac{y}{1+\delta}}{p(w_0-y)}\right]\cdot\frac{(w_0-y)^2(1+\delta)p}{w_0-c(1+\delta)}\\
&= \left[\frac{1-\frac{1}{1+\delta}}{(\frac{1}{1+\delta}-p)^2}\cdot\frac{-1}{\gamma}\cdot f(p)^{-\frac{1}{\gamma}}\cdot\frac{\frac{1}{1+\delta}-p}{1-p} + \frac{1}{p}\left(f(p)^{-\frac{1}{\gamma}}-1\right)\right]\cdot\frac{(w_0-y)^2(1+\delta)p}{w_0-c(1+\delta)}\\
&= \frac{1}{p}\left[\left(1-\frac{1}{\gamma}\cdot\frac{\left(1-\frac{1}{1+\delta}\right)p}{(\frac{1}{1+\delta}-p)(1-p)}\right)f(p)^{-\frac{1}{\gamma}}-1\right]\cdot\frac{(w_0-y)^2(1+\delta)p}{w_0-c(1+\delta)}\\
&= \frac{1}{p}\left[\left(1-\frac{1}{\gamma}\cdot\frac{\left(1-\frac{1}{1+\delta}\right)p}{(\frac{1}{1+\delta}-p)(1-p)}\right)\left(\frac{1-p}{\frac{1}{1+\delta}-p}\right)^{-\frac{1}{\gamma}}-1\right]\cdot\frac{(w_0-y)^2(1+\delta)p}{w_0-c(1+\delta)}
\end{aligned}$$

From this relation, we can deduce the sign of the relation between p and y. It holds that

$$y'(p)<0 \Leftrightarrow \underbrace{1-\frac{1}{\gamma}\cdot\frac{\left(1-\frac{1}{1+\delta}\right)p}{(\frac{1}{1+\delta}-p)(1-p)}}_{<1} < \underbrace{\left(\frac{1-p}{\frac{1}{1+\delta}-p}\right)^{\frac{1}{\gamma}}}_{>1}$$

To conclude that the left-hand (right-hand) side of the second inequality is smaller (larger) than one, we have assumed that

$$p<\frac{1}{1+\delta}<1$$

i.e. that

$$\delta>0 \text{ and } p(1+\delta)<1$$

Overall, we thus have

$$\begin{aligned}
y'(p)&<0\\
y'(L)&>0
\end{aligned}$$

This shows that the investment into insurance is larger for LPHC (small p) than for HPLC (large p).

B Basic Insurance Model with one Insurable Risk: Optimal insurance $y^* = y(p, w_0)$

The expected utility of the insuree is

$$E\left[u\left(w_0 - y - 1_L l(y,p)\right)\right] = p \cdot u\left(w_0 - y - l(y,p)\right) + (1-p) \cdot u\left(w_0 - y\right),$$

where 1_L is the indicator event for the loss to be insured and the loss size after insurance is given by

$$l(y,p) = \frac{c}{p} - \frac{y}{(1+\delta)p}.$$

The decision maker has to choose the investment into insurance y. The optimality condition is

$$p \cdot u'\left(w_0 - y - l(y,p)\right)\left(-1 + \frac{1}{(1+\delta)p}\right) + (1-p) \cdot u'\left(w_0 - y\right)(-1) = 0.$$

Simplifying gives that the left hand side is equal to

$$g(y,p,w_0) = \left(\frac{1}{1+\delta} - p\right) u'\left(w_0 - y - l(y,p)\right) - (1-p) \cdot u'\left(w_0 - y\right).$$

We want to determine the dependence of the optimal $y(p, w_0)$ on the loss probability and on the initial wealth. To do so, we start from the optimality condition

$$g(y(p,w),p,w) = 0$$

Taking the derivative of both sides w.r.t. the loss probability p gives

$$\frac{\partial g(y(p,w),p,w)}{\partial y} \cdot \frac{\partial y(p,w)}{\partial p} + \frac{\partial g(y(p,w),p,w)}{\partial p} = 0.$$

This equation can be solved for the partial derivative of y w.r.t. p:

$$\frac{\partial y(p,w)}{\partial p} = -\frac{\frac{\partial g(y(p,w),p,w)}{\partial p}}{\frac{\partial g(y(p,w),p,w)}{\partial y}}$$

Similarly, one gets

$$\frac{\partial y(p,w)}{\partial w} = -\frac{\frac{\partial g(y(p,w),p,w)}{\partial w}}{\frac{\partial g(y(p,w),p,w)}{\partial y}}.$$

To determine these expressions, we need the partial derivatives of g w.r.t. its arguments y, p, and w_0, which we then evaluate for $y = y(p, w_0)$.

B.1 Partial derivatives of g

B.1.1 Partial derivative of g w.r.t. y

The partial derivative of g w.r.t. y is given by

$$\frac{\partial g(y,p,w_0)}{\partial y} = \left(\frac{1}{1+\delta} - p\right) u''\left(w_0 - y - l(y,p)\right)\left(-1 + \frac{1}{(1+\delta)p}\right) + (1-p)\cdot u''\left(w_0 - y\right)$$

Using the definition $ARA(x) = -\frac{u''(x)}{u'(x)}$ and thus the relation $u''(x) = -u'(x)ARA(x)$ gives

$$\begin{aligned}
\frac{\partial g(y,p,w_0)}{\partial y} =& -\frac{1}{p}\left(\frac{1}{1+\delta} - p\right)^2 u'\left(w_0 - y - l(y,p)\right) ARA\left(w_0 - y - l(y,p)\right) \\
& - (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right) \\
=& -\frac{1}{p}\left(\frac{1}{1+\delta} - p\right)\left[g(y,p,w_0) + (1-p)\cdot u'\left(w_0 - y\right)\right] ARA\left(w_0 - y - l(y,p)\right) \\
& - (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right) \\
=& -\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right) \\
& + (1-p)u'\left(w_0 - y\right)\left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right]
\end{aligned}$$

For $y = y(p,w_0)$, the optimality condition $g(y(p,w_0),p,w_0) = 0$ holds true. The partial derivative simplifies to

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial y} = (1-p)u'\left(w_0 - y\right)\left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right]$$

and it holds that

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial y} < 0.$$

B.1.2 Partial derivative of g w.r.t. p

The partial derivative of g w.r.t. p is given by

$$
\begin{aligned}
\frac{\partial g(y,p,w_0)}{\partial p} &= -\, u'\left(w_0 - y - l(y,p)\right) + \left(\frac{1}{1+\delta} - p\right) u''\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p} + u'\left(w_0 - y\right) \\
&= -\, u'\left(w_0 - y - l(y,p)\right) + u'\left(w_0 - y\right) \\
&\quad - \left(\frac{1}{1+\delta} - p\right) u'\left(w_0 - y - l(y,p)\right) ARA\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p} \\
&= -\, \frac{1}{\frac{1}{1+\delta} - p} \left[g(y,p,w_0) + (1-p)\cdot u'\left(w_0 - y\right)\right] + u'\left(w_0 - y\right) \\
&\quad - \left[g(y,p,w_0) + (1-p)\cdot u'\left(w_0 - y\right)\right] ARA\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p} \\
&= \left[-\frac{1}{\frac{1}{1+\delta} - p} - ARA\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p}\right] g(y,p,w_0) \\
&\quad + u'(w_0 - y) \left[-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - y - l(y,p)\right) l(y,p)\right]
\end{aligned}
$$

For $y = y(p, w_0)$, the optimality condition $g(y(p,w_0), p, w_0) = 0$ holds true. The partial derivative simplifies to

$$
\frac{\partial g(y(p,w_0),p,w_0)}{\partial p} = u'(w_0 - y) \left[-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - y - l(y,p)\right) l(y,p)\right]
$$

It then holds that

$$
\frac{\partial g(y(p,w_0),p,w_0)}{\partial p} < 0.
$$

B.1.3 Partial derivative of g w.r.t. w_0

The partial derivative of g w.r.t. w_0 is given by

$$\begin{aligned}
\frac{\partial g(y,p,w_0)}{\partial w_0} &= \left(\frac{1}{1+\delta} - p\right) u''\left(w_0 - y - l(y,p)\right) - (1-p)\cdot u''\left(w_0 - y\right) \\
&= -\left(\frac{1}{1+\delta} - p\right) u'\left(w_0 - y - l(y,p)\right) ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right) \\
&= -\left[g(y,p,w_0) + (1-p)\cdot u'\left(w_0 - y\right)\right] ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right) \\
&= -\, g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + (1-p)\cdot u'\left(w_0 - y\right)\left[ARA\left(w_0 - y\right) - ARA\left(w_0 - y - l(y,p)\right)\right]
\end{aligned}$$

For $y = y(p, w_0)$, the optimality condition $g(y(p,w_0),p,w_0) = 0$ holds true. The partial derivative simplifies to

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0} = (1-p)\cdot u'\left(w_0 - y\right)\left[ARA\left(w_0 - y\right) - ARA\left(w_0 - y - l(y,p)\right)\right]$$

For a decreasing absolute risk aversion, it then holds that

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0} < 0.$$

B.2 Dependence of $y(p, w_0)$ on p and w_0

For the partial derivative of $y(p, w_0)$ w.r.t. the initial wealth w, it holds that

$$\frac{\partial y(p,w)}{\partial w} = -\frac{\frac{\partial g(y(p,w),p,w)}{\partial w}}{\frac{\partial g(y(p,w),p,w)}{\partial y}}.$$

Since both the nominator and the denominator of the fraction are negative, we get that

$$\frac{\partial y(p,w)}{\partial w} \begin{cases} < 0 & \text{decreasing ARA} \\ = 0 & \text{constant ARA} \end{cases}$$

For a decreasing absolute risk aversion, the optimal insurance is a decreasing function of wealth. For a constant absolute risk aversion, it is independent of wealth (the proof for that special case could be done much easier!).

For the partial derivative of $y(p, w_0)$ w.r.t. the loss probability p, it holds that

$$\frac{\partial y(p,w)}{\partial p} = -\frac{\frac{\partial g(y(p,w),p,w)}{\partial p}}{\frac{\partial g(y(p,w),p,w)}{\partial y}}.$$

Again, both the nominator and the denominator of the fraction are negative, we get that

$$\frac{\partial y(p,w)}{\partial p} < 0.$$

The optimal insurance is thus a decreasing function of the loss probability. In particular, it is larger for LPHC than for HPLC.

C Two risks: HPLC versus LPHC

C.1 Bounds on Optimal Insurance Demand

The expected utility of the insuree is given by

$$p_b E\Big[u\Big(w_0 - \frac{c}{p_b} - y - 1_L l(y,p)\Big)\Big] + (1-p_b)E\Big[u\Big(w_0 - y - 1_L l(y,p)\Big)\Big],$$

where 1_L is the indicator event for the loss to be insured. The first order condition for the investment into insurance y is

$$h(y, p, p_b, w_0) = p_b g(y, p, w_0 - c/p_b) + (1-p_b) g(y, p, w_0) = 0.$$

For $0 < y < y(p, w_0)$, both $g(y, p, w_0 - c/p_b)$ and $g(y, p, w_0)$ are positive. For $y(p, w_0) < y < y(p, w_0 - c/p_b)$, $g(y, p, w_0)$ is negative, while $g(y, p, w_0 - c/p_b)$ is still positive. For $y(p, w_0 - c/p_b) < y$, both $g(y, p, w_0 - c/p_b)$ and $g(y, p, w_0)$ are negative. This implies that

$$y(p, p_b, w_0) \in (y(p, w_0), y(p, w_0 - c/p_b))$$

For the optimal $y(p, p_b, w_0)$, it thus holds that

$$g(y(p, p_b, w_0), p, w_0) < 0$$
$$g(y(p, p_b, w_0), p, w_0 - c/p_b) > 0$$

To determine the dependence of $y(p, p_b, w_0)$ on p and p_b, we proceed as in the preceding section. From the optimality condition, we get that

$$\frac{\partial h(y(p, p_b, w), p, p_b, w)}{\partial y} \cdot \frac{\partial y(p, p_b, w)}{\partial p} + \frac{\partial g(y(p, p_b, w), p, w)}{\partial p} = 0.$$

Solving for the partial derivative of y w.r.t. the loss probability p gives

$$\frac{\partial y(p, p_b, w)}{\partial p} = -\frac{\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial p}}{\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial y}}.$$

Similarly, we get for the partial derivative of y w.r.t. the probability p_b of a background loss

$$\frac{\partial y(p, p_b, w)}{\partial p_b} = -\frac{\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial p_b}}{\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial y}}.$$

C.2 Partial derivatives of h

C.2.1 Partial derivative of h w.r.t. p

The partial derivative of h w.r.t. p is given by

$$
\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p} \\
&= p_b \frac{\partial g(y,p,w_0 - c/p_b)}{\partial p} + (1-p_b)\frac{\partial g(y,p,w_0)}{\partial p} \\
&= p_b \Bigg[\left(-\frac{1}{\frac{1}{1+\delta} - p} - ARA\left(w_0 - c/p_b - y - l(y,p)\right) \frac{l(y,p)}{p} \right) g(y,p,w_0 - c/p_b) \\
&\qquad\qquad + u'(w_0 - c/p_b - y) \left(-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - c/p_b - y - l(y,p)\right) l(y,p) \right) \Bigg] \\
&\quad + (1-p_b) \Bigg[\left(-\frac{1}{\frac{1}{1+\delta} - p} - ARA\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p} \right) g(y,p,w_0) \\
&\qquad\qquad + u'(w_0 - y) \left(-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - y - l(y,p)\right) l(y,p) \right) \Bigg] \\
&= p_b \left(-\frac{1}{\frac{1}{1+\delta} - p} - ARA\left(w_0 - c/p_b - y - l(y,p)\right) \frac{l(y,p)}{p} \right) g(y,p,w_0 - c/p_b) \\
&\quad + (1-p_b) \left(-\frac{1}{\frac{1}{1+\delta} - p} - ARA\left(w_0 - y - l(y,p)\right) \frac{l(y,p)}{p} \right) g(y,p,w_0) \\
&\quad + p_b u'(w_0 - c/p_b - y) \left(-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - c/p_b - y - l(y,p)\right) l(y,p) \right) \\
&\quad + (1-p_b) u'(w_0 - y) \left(-\frac{\delta}{1-(1+\delta)p} - \frac{1-p}{p} ARA\left(w_0 - y - l(y,p)\right) l(y,p) \right)
\end{aligned}
$$

In the optimum, it holds that

$$
\begin{aligned}
&\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial p} \\
=\ & -(1-p_b)\left(-\frac{1}{\frac{1}{1+\delta}-p}-ARA\left(w_0-c/p_b-y-l(y,p)\right)\frac{l(y,p)}{p}\right)g(y,p,w_0) \\
&+(1-p_b)\left(-\frac{1}{\frac{1}{1+\delta}-p}-ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right)g(y,p,w_0) \\
&+p_b u'(w_0-c/p_b-y)\left(-\frac{\delta}{1-(1+\delta)p}-\frac{1-p}{p}ARA\left(w_0-c/p_b-y-l(y,p)\right)l(y,p)\right) \\
&+(1-p_b)u'(w_0-y)\left(-\frac{\delta}{1-(1+\delta)p}-\frac{1-p}{p}ARA\left(w_0-y-l(y,p)\right)l(y,p)\right) \\
=\ &(1-p_b)\left(ARA\left(w_0-c/p_b-y-l(y,p)\right)-ARA\left(w_0-y-l(y,p)\right)\right)\frac{l(y,p)}{p}g(y,p,w_0) \\
&+p_b u'(w_0-c/p_b-y)\left(-\frac{\delta}{1-(1+\delta)p}-\frac{1-p}{p}ARA\left(w_0-c/p_b-y-l(y,p)\right)l(y,p)\right) \\
&+(1-p_b)u'(w_0-y)\left(-\frac{\delta}{1-(1+\delta)p}-\frac{1-p}{p}ARA\left(w_0-y-l(y,p)\right)l(y,p)\right)
\end{aligned}
$$

For decreasing absolute risk aversion, it holds that

$$
\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial b}<0
$$

C.2.2 Partial derivative of h w.r.t. p_b

The partial derivative of h w.r.t. p_b is given by

$$
\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p_b} \\
=\ &g(y,p,w_0-c/p_b)+p_b\frac{\partial g(y,p,w_0-c/p_b)}{\partial p_b}-g(y,p,w_0)+(1-p_b)\frac{\partial g(y,p,w_0)}{\partial p_b} \\
=\ &g(y,p,w_0-c/p_b)-g(y,p,w_0)+p_b\frac{\partial g(y,p,w_0-c/p_b)}{\partial w}\frac{c}{p_b^2} \\
=\ &g(y,p,w_0-c/p_b)-g(y,p,w_0) \\
&+\frac{c}{p_b}\Big[-g(y,p,w_0-c/p_b)ARA\left(w_0-c/p_b-y-l(y,p)\right) \\
&\qquad+(1-p)\cdot u'\left(w_0-c/p_b-y\right)\left[ARA\left(w_0-c/p_b-y\right)-ARA\left(w_0-c/p_b-y-l(y,p)\right)\right]\Big]
\end{aligned}
$$

In the optimum, we get that

$$
\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p_b} \\
=&\frac{g(y,p,w_0-c/p_b)}{1-p_b} \\
&+\frac{c}{p_b}\Big[-g(y,p,w_0-c/p_b)ARA\left(w_0-c/p_b-y-l(y,p)\right) \\
&\qquad+(1-p)\cdot u'\left(w_0-c/p_b-y\right)\left[ARA\left(w_0-c/p_b-y\right)-ARA\left(w_0-c/p_b-y-l(y,p)\right)\right]\Big] \\
=&\frac{g(y,p,w_0-c/p_b)}{1-p_b} \\
&+\frac{c}{p_b}\Big[-g(y,p,w_0-c/p_b)ARA\left(w_0-c/p_b-y-l(y,p)\right) \\
&\qquad+\frac{\partial g(y,p,w_0-c/p_b)}{\partial w}+g(y,p,w_0-c/p_b)ARA\left(w_0-c/p_b-y-l(y,p)\right]\Big] \\
=&\frac{g(y,p,w_0-c/p_b)}{1-p_b}+\frac{c}{p_b}\frac{\partial g(y,p,w_0-c/p_b)}{\partial w}<0
\end{aligned}
$$

C.2.3 Partial derivative of h w.r.t. y

The partial derivative of h w.r.t. y is given by

$$
\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial y} \\
&= p_b \frac{\partial g(y,p,w_0 - c/p_b)}{\partial y} + (1-p_b)\frac{\partial g(y,p,w_0)}{\partial y} \\
&= \; -\frac{p_b}{p}\left(\frac{1}{1+\delta} - p\right) g(y,p,w_0 - c/p_b) ARA\left(w_0 - c/p_b - y - l(y,p)\right) \\
&\quad + p_b(1-p)u'\left(w_0 - c/p_b - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - c/p_b - y - l(y,p)\right) - ARA\left(w_0 - c/p_b - y\right)\right] \\
&\quad - \frac{1-p_b}{p}\left(\frac{1}{1+\delta} - p\right) g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + (1-p_b)(1-p)u'\left(w_0 - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right] \\
&= \; -\frac{p_b}{p}\left(\frac{1}{1+\delta} - p\right) g(y,p,w_0 - c/p_b) ARA\left(w_0 - c/p_b - y - l(y,p)\right) \\
&\quad - \frac{1-p_b}{p}\left(\frac{1}{1+\delta} - p\right) g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + p_b(1-p)u'\left(w_0 - c/p_b - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - c/p_b - y - l(y,p)\right) - ARA\left(w_0 - c/p_b - y\right)\right] \\
&\quad + (1-p_b)(1-p)u'\left(w_0 - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta} - p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right]
\end{aligned}
$$

In the optimum, i.e. for $y = y(p, p_b, w)$, it holds that

$$
\begin{aligned}
&\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial y} \\
&= \frac{1-p_b}{p}\left(\frac{1}{1+\delta}-p\right) g(y,p,w_0) ARA\left(w_0 - c/p_b - y - l(y,p)\right) \\
&\quad - \frac{1-p_b}{p}\left(\frac{1}{1+\delta}-p\right) g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right) \\
&\quad + p_b(1-p)u'\left(w_0 - c/p_b - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta}-p\right) ARA\left(w_0 - c/p_b - y - l(y,p)\right) - ARA\left(w_0 - c/p_b - y\right)\right] \\
&\quad + (1-p_b)(1-p)u'\left(w_0 - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta}-p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right] \\
&= \frac{1-p_b}{p}\left(\frac{1}{1+\delta}-p\right) g(y,p,w_0)\left[ARA\left(w_0 - c/p_b - y - l(y,p)\right) - ARA\left(w_0 - y - l(y,p)\right)\right] \\
&\quad + p_b(1-p)u'\left(w_0 - c/p_b - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta}-p\right) ARA\left(w_0 - c/p_b - y - l(y,p)\right) - ARA\left(w_0 - c/p_b - y\right)\right] \\
&\quad + (1-p_b)(1-p)u'\left(w_0 - y\right) \\
&\quad \left[-\frac{1}{p}\left(\frac{1}{1+\delta}-p\right) ARA\left(w_0 - y - l(y,p)\right) - ARA\left(w_0 - y\right)\right]
\end{aligned}
$$

For decreasing absolute risk aversion, it holds that

$$\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial y} < 0$$

C.3 Dependence of $y(p, p_b, w_0)$ on p and p_b

With the signs of the partial derivatives of h in the optimum, we get

$$\frac{\partial y(p,p_b,w)}{\partial p} < 0$$

$$\frac{\partial y(p,p_b,w)}{\partial p_b} < 0$$

In line with intuition, the optimal investment into insurance is thus decreasing in the loss probability of the event to be insured (again, HPLC needs less insurance than LPHC) and decreasing in the probability of the background loss (the worse the background risk, i.e. the more the investor has to deal with rare but large background losses, the higher the insurance needed for the insurable event).

C.4 Proof of Proposition 3

The expected utility in case of uninsurable risk is given by

$$p_b E\left[u\left(w_0 - l(0,p_b) - y - 1_L \cdot l(y,p)\right)\right] + (1-p_b) E\left[u\left(w_0 - y - 1_L \cdot l(y,p)\right)\right].$$

It is a weighted average of the expected utilities when only the insurable risk is present, where we account for the background loss by a reduction in initial wealth from w_0 to $w_0 - l(0,p_b)$ in case it occurs.

For the solutions of the two basic insurance problems, it holds that

$$y(p,w_0) < y(p,w_0 - l(0,p_b))$$

where we use that the optimal investment is decreasing in wealth. For the solution of the optimizátion problem with background risk, it thus holds that

$$y(p,w_0) < y(p,p_b,w_0) < y(p,w_0 - l(0,p_b)).$$

We can also use that the utility is concave. This gives that

$$\begin{aligned} E\left[u\left(W\right)\right] &= E\left[E\left[u\left(w_0 - 1_{Lb} \cdot l(0,p_b) - y - 1_L \cdot l(y,p)\right) \middle| 1_{Lb}\right]\right] \\ &< E\left[u\left(E\left[w_0 - 1_{Lb} \cdot l(0,p_b) - y - 1_L \cdot l(y,p) \middle| 1_{Lb}\right]\right)\right] \\ &= E\left[u\left(w_0 - p_b l(0,p_b) - y - 1_L \cdot l(y,p)\right)\right]. \end{aligned}$$

The utility with background risk is thus smaller than the utility in case background risk is replaced by a reduction in initial wealth equal to the expected background loss. This furthermore implies that

$$y(p,w_0) < y(p,w_0 - p_b \cdot l(0,p_b)) < y(p,p_b,w_0) < y(p,w_0 - l(0,p_b)).$$

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