Risk Attitudes with State-Dependent Indivisibilities in Consumption

7th February, 2019

Abstract

Some consumption opportunities, e.g. medical treatments, are both indivisible and only valuable in particular states of nature. The existence of such state-dependent indivisible consumption opportunities influences a person's risk attitudes. In general, people are not risk averse anymore even if utility from divisible consumption is concave. I propose a definition of insurance in the context of state-dependent preferences and investigate the different motives underlying insurance demand. The same reasons that rule out risk aversion turn out to be the basis of a desire to insure. This calls into question the standard approach that bases insurance demand on risk aversion with important implications for policy and research.

JEL Classification: D01,D81

Keywords: Risk Preferences, Indivisible Consumption, Insurance, Gambling

1 Introduction

People's attitudes toward risk and uncertainty are an important subject of economic theory because these attitudes are vital in shaping individual behavior in many markets. Indeed, the very existence of several markets, in particular insurance markets, is a consequence of such preferences as these markets are valuable in allowing to achieve a superior allocation of risk between market participants with varying risk preferences (Borch 1962; Gollier 1992).

It is standard to assume that economic agents have risk-averse preferences, i.e., that they prefer a lottery's expected value over the lottery itself. This assumption is typically justified with the plausibility of assuming diminishing marginal utility of wealth. Despite this, it is long known that indivisibilities in consumption can motivate risk-seeking behavior. Building on prior work by Ng (1965), Jones (2008), and Vasquez (2017) I investigate people's risk attitudes if wealth can be allocated into divisible and indivisible consumption opportunities. I extend this prior work by allowing indivisible consumption opportunities to vary across states of nature.

State-dependent indivisibilities in consumption result in a state-dependent indirect utility function of wealth. In general, people cannot be classified as risk-averse, -neutral, or -seeking anymore. This is due to the fact that an individual's risk attitudes are no longer depending only on the moments of a distribution. Instead, the identity of the states, in which different payoffs occur, matters. In addition, indivisibilities result in the marginal utility of wealth to differ across states even if the utility from divisible consumption is identical across states. Hence, state-dependent indivisibilities can be the root cause of a previously identified source of state-dependence: differences in marginal utility of wealth across states. However, indivisibilities in consumption also mean that the (state-dependent) marginal utility of wealth is less informative about people's risk attitudes. I argue that instead of redefining the notion of risk in a setting with state-dependent preferences (Karni 1985), we need to understand the new motives that state-dependence produces for gambling and insurance. In particular, insurance is no longer valuable only to reduce risk exposure. Instead, the major purpose of insurance is to allow a *targeted* redistribution of wealth across states of nature to address conditional needs.

As before, this can be valuable to redistribute wealth from states with lower marginal utility to states with higher marginal utility of wealth (consumption smoothing). I investigate how indivisible consumption opportunities create a consumption-smoothing motive in insurance that is

distinct from the classic consumption-smoothing motive in response to potential losses. In addition to consumption-smoothing, insurance is valuable to realize indivisible consumption opportunities that are otherwise either not feasible (compare the “access motive” of Nyman 1999) or only desirable if financed “across states”. I investigate the conditions under which this access motive (Nyman 1999) dominates the consumption-smoothing motive. The distinction between different motives for insurance is of particular relevance in insurance contexts that are not (primarily) characterized by financial losses but specific needs, such as health, long-term care, or old-age insurance.
Gambling, in contrast to insurance, is valuable for optimally allocating wealth into indivisible consumption opportunities that are state-independent (Ng 1965; Jones 2008; Vasquez 2017). Notably, this understanding of insurance and gambling views the two activities no longer as opposites. Instead, these activities have a far more complex relationship, sometimes complementing each other, sometimes being imperfect (or even perfect) substitutes.

The separation of insurance motives from risk aversion and the insight that the latter constitutes an exception rather than the rule have important implications for both policy and research. First, policy recommendations with regard to the optimal design of insurance assume almost exclusively that insurance derives its value from consumption-smoothing. These recommendations can change fundamentally if other motives are accounted for (Fels 2016). In addition, measures of risk preferences derived from insurance choices cannot be generalized beyond the context in which they are derived since they incorporate the specific conditional needs that these insurances address. On the positive side, this means that the common finding of differing risk attitudes across context or time does not need to be a sign of unstable preferences (Barseghyan et al. 2011; Schildberg-Hörisch 2018) but of differing conditional needs. On the other hand, risk attitudes derived in one context might have little value informing insurance design in other contexts. Similarly, risk attitudes derived in a laboratory setting might have limited value in informing actual insurance policy as the former abstracts from the conditional needs that govern actual insurance choices.

The paper proceeds as follows. After introducing the analysis of indivisible consumption opportunities in a simple framework close to Vasquez (2017) in section 2, I investigate the implications of allowing these opportunities to differ across states in section 3. In particular, I discuss the different role of insurance and gambling in a context of state-dependent preferences. In section 4, I assume utility from divisible consumption to take the popular concave functional form, investigating the two roles of insurance - consumption-smoothing and access provision - in this setup. In conclude with a short discussion of the implications for policy and research. All proofs are in the Appendix.

2 Preliminaries

Suppose that an individual can divide his wealth w between consumption opportunities that are perfectly divisible and consumption opportunities that are indivisible. Let the (indirect) utility from consuming divisible consumption opportunities be described by a utility function $u_d(w_d)$, with $u_d(w_d) = w_d$ for now[0], where w_d denotes the share of wealth allocated to divisible consumption. Suppose that, in addition, there is a finite list I of indivisible consumption items that are each described by a cost c_i and utility v_i. Following Vasquez (2017), I define the utility density of an item $i \in I$ as $d_i = v_i/c_i$ and make the following assumption:

Assumption 1. *For all $i \in I$,*
(i) $c_i > 0$, $v_i > 0$,
(ii) $d_{i+1} < d_i$,
(iii) there is some $0 < C < \infty$, such that $c_i \leq C$.

Let the sequence $a = (a_i)_{i \in I}$ of 0s and 1s denote the decision which indivisible consumption opportunities are bought. The optimal allocation of wealth maximizes

$u_d(w_d) + \sum_i a_i v_i$ subject to $w_d + \sum_i a_i c_i \leq w$.

Define the utility of wealth accordingly by

$$U(w) = \sup_{w_d, a} \left\{ u_d(w_d) + \sum_{i \in I} a_i v_i : w_d + \sum_{i \in I} a_i c_i \leq w \right\} \tag{1}$$

where $a_i \in \{0, 1\}$. This utility of wealth exhibits several discontinuities at wealth levels at which it is optimal to change the optimal sequence a.

Define the solution of the linear relaxation of the problem (treating the consumption opportunities

[0]This assumption leads to utility being unbounded from above, resulting in the famous St. Petersburg paradox. It is chosen here for its convenience to introduce the implications of indivisible consumption opportunities, and replaced by the familiar assumption of a concave $u_d(w_d)$ in section 4.

as if they were perfectly divisible) as

$$\bar{U}(w) = \sup_{w_d, \bar{a}} \left\{ u_d(w_d) + \sum_{i \in I} \bar{a}_i v_i : w_d + \sum_{i \in I} \bar{a}_i c_i \leq w \right\} \tag{2}$$

where $\bar{a} = (\bar{a}_i)_{i \in I}$ and $\bar{a}_i \in [0, 1]$. The solution to the latter turns out to be simpler as the greedy algorithm delivers the optimal solution to the relaxed knapsack problem. Define ι as the highest i such that $d_i \geq 1$. For a well-defined solution, assume $d_i \neq 1, \forall i$.

Proposition 1. *The optimal consumption plan for a wealth level w - given that consumption opportunities $i \in I$ are perfectly divisible - is described by*

$$w_d^* = \max \left\{ 0, w - \sum_{i=1}^{\iota} c_i \right\}, \tag{3}$$

$$\bar{a}_i^* = \begin{cases} \max \left\{ \min \left\{ (w - \sum_{j=1}^{i-1} c_j)/c_i, 1 \right\}, 0 \right\} & \text{if } d_i > 1 \\ 0 \text{ if } d_i < 1 \end{cases} . \tag{4}$$

Define $w_i^* = \sum_{j=1}^{i} c_j$ and $w_0^* = 0$. The utility $\bar{U}$ of the relaxed problem is a sequence of adjoining line segments with slope d_i for wealth levels $w : \ w_{i-1}^* < w < w_i^*, \ i \leq \iota$ and with slope 1 for wealth levels $w > w_\iota^*$. $\bar{U}(w)$ constitutes the concave envelope of $U(w)$ which is the maximal utility derived in the original problem with $a_i \in \{0, 1\}$. What can be said about $U(w)$? $U(w) = \bar{U}(w)$ if $w = w_i^*$ for some $i \leq \iota$ and for all $w > w_\iota^*$, and $U(w) < \bar{U}(w)$ for all other wealth levels w. $U(w)$ exhibits jump discontinuities at $w_i^*, i \leq \iota$ and possibly at several points in between. The slope equals $u_d'(w) = 1$ everywhere but at the points of discontinuities.

Example 1. *Suppose there are four indivisible consumption opportunities with*

$$v_1 = 12, c_1 = 4,$$

$$v_2 = 10, c_2 = 5,$$

$$v_3 = 3, c_2 = 2,$$

$$v_4 = 1, c_4 = 2.$$

Figure 1 depicts the utility of wealth $U(w)$, and the utility $\bar{U}(w)$.

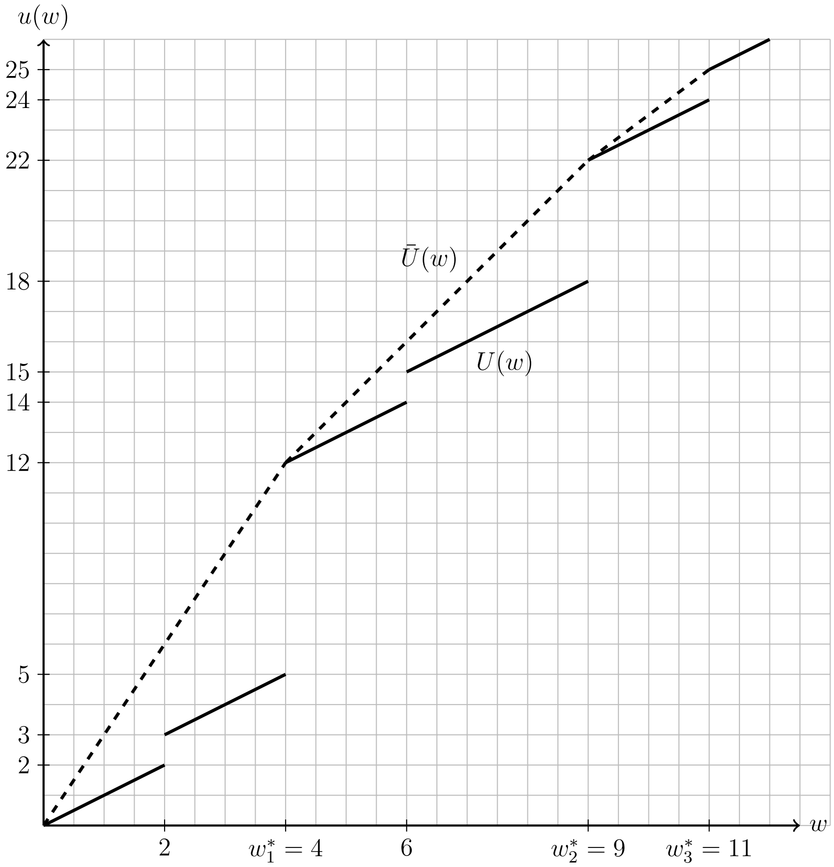

Figure 1: Example 1. The black solid line denotes $U(w)$. The black dashed line denotes $\bar{U}(w)$ if it exceeds $U(w)$.

Vasquez (2017) provides several important insights for this setting.

First, at almost all wealth levels $w < w_\iota^*$ there is an incentive to gamble as $\bar{U}(w) > U(w)$. Suppose $w = pw_i^* + (1-p)w_{i+1}^*$ for some $i < \iota$, $p \in (0,1)$. Then $pU(w_i^*) + (1-p)U(w_{i+1}^*) = \bar{U}(w) > U(w)$. Simultaneously, there is a willingness to pay for insurance. Consider wealth levels w_1, w_2 with $w_1 < w_i^* < w_2$ for some $i \leq \iota$ and consider the gamble $(w_1, p; w_2, 1-p)$ with $w = pw_1 + (1-p)w_2 \geq w_i^*$. Here $U(w) > pU(w_1) + (1-p)U(w_2)$. If, at the outset, the individual faces an undesirable lottery with expected wealth above a threshold w_i^*, $i \leq \iota$ and some wealth realizations below said threshold, that individual seeks to rid itself of this undesirable risk by buying insurance. Afterwards, unless the certain wealth equals exactly one of the thresholds w_i^*, the individual seeks to acquire a desirable risk that incorporates wealth realizations at thresholds w_i^*.

Second, the desire to gamble and/or insure are independent of the marginal utility of wealth $U'(w)$ that is 1 (almost) everywhere.[1]

[1] Vasquez (2017) argues that the Arrow-Pratt measure of risk aversion is not well-defined for a knapsack utility function. However, this is due to his implicit assumption of $u_d'(w) = 0$. Here, the Arrow-Pratt measure of risk aversion is well-defined almost everywhere. However, it remains meaningless for understanding risk attitudes.

Third, $\bar{U}(w)$ is concave. Hence, if a person already possesses an optimal wealth distribution, i.e., the person holds a wealth lottery $W = (w_i^*, p; w_{i+1}^*, 1-p)$ with expected value w, then that person rejects any further gamble.

The analyses by Ng (1965), Jones (2008), and Vasquez (2017) already show that with indivisibilities in consumption, people cannot be classified as risk-averse/risk-seeking in the sense that for *any* lottery, they prefer the expected value of that lottery to the lottery itself or vice versa.[2] With indivisible consumption opportunities, some lotteries are desirable, others are not. Hence, people's risk attitudes are more nuanced than seeing all risk as either bad or good. There is, however, no reason to believe that consumption opportunities - more specifically their value v_i and their cost c_i - should be the same in all states. If they differ across states, the (indirect) utility associated with a particular wealth level must also vary across states. This links the idea of indivisibilities in consumption to the literature on state-dependent preferences. Notably, this literature offers another reason why risk attitudes should be more complex than the typical risk aversion/ risk love dichotomy. If marginal utility of wealth differs across states, it is optimal to deviate from an equal distribution of wealth across states.[3] To distinguish the analysis of state-dependent indivisibilities from the literature on irreplaceable commodities, I maintain the assumption that marginal utility from divisible consumption is state-independent.

3 State-dependent indivisibilities

Suppose that there exists a state-dependence of the indivisible opportunities. It may derive from differences in cost or in value of these opportunities across states. For example, the price of an indivisible good may differ across states leading to a lower density $d_i = v_i/c_i$ in states with a higher price. Alternatively, there might be some opportunities that only have value in a subset of states. Aggressive medical treatments are a prominent example.

I assume that preferences differ across states only in the indivisible consumption opportunities

[2] Indeed, Ng (1965) argues that indivisibilities might be a source of local convexities in the utility function as hypothesized by Friedman and Savage (1948).

[3] Cook and Graham (1977), Shioshansi (1982), Schlesinger (1984) and Huang and Tzeng (2006) discuss optimal insurance in a setting with state-dependent preferences. Their results indicate that optimal insurance coverage can fall short of covering the loss, exceed the loss, or even cover the event in which the loss does not happen, depending how marginal utility varies across states. Curiously, none of them - to my knowledge - draws the appropriate conclusion from these results that, contrary to conventional wisdom, insurance is not a means to mitigate losses. I discuss this point in more detail in the following section.

that each state offers. Denote by S the finite set of states s that can be distinguished by the available indivisible consumption opportunities $I(s)$. p_s denotes the probability of the state s.

Given that we are not interested in the identity of the consumption opportunities that are optimally consumed at a particular wealth level, but in the shape of the utility function that results from optimal consumption, I assume that in each state s consumption opportunities are ordered according to density. This means that $i(s)$ refers to the indivisible consumption opportunity that has the i-highest density in state s, while I suppress the dependence whenever this is possible.

Define the state-dependent utility function over wealth as

$$U_s(w) = \sup_{w_d, a} \left\{ u_d(w_d) + \sum_{i(s) \in I(s)} a_{i(s)} v_{i(s)} : w_d + \sum_{i(s) \in I(s)} a_{i(s)} c_{i(s)} \leq w \right\} \tag{5}$$

where $a_{i(s)} \in \{0, 1\}$. Define the solution of the linear relaxation of the problem as

$$\bar{U}_s(w) = \sup_{w_d, \bar{a}} \left\{ \sum_{i(s) \in I(s)} u_d(w_d) + \bar{a}_{i(s)} v_{i(s)} : w_d + \sum_{i(s) \in I(s)} \bar{a}_{i(s)} c_{i(s)} \leq w \right\} \tag{6}$$

where $\bar{a}_{i(s)} \in [0, 1]$. Let $U(w) = \sum_{s \in S} p_s U_s(w)$ be the expected utility of consuming wealth w optimally in each state $s \in S$ and let $\bar{U}(w) = \sum_{s \in S} p_s \bar{U}_s(w)$. As before, $\bar{U}_s(w) \geq U_s(w)$ and $\bar{U} \geq U(w)$.

In this environment, it can be optimal to transfer wealth across states s in order to realize opportunities with large densities d_i that only occur in a particular state and are not feasible or desirable in this state given the current allocation of wealth.[4] It is easy to see that it is only a highly special case that the optimal allocation of wealth across states s is an equal distribution. Thus, people cannot be considered risk-averse.[5] Similar to the case of state-independence, insurance can be viewed as a means to transfer wealth into states in which wealth achieves a larger

[4]This is reminiscent of what Nyman (1999) calls the *access value* of insurance in the context of health insurance. Insurance is valuable as it allows to gain access to treatment that is otherwise unaffordable. Note that this requires the consumption of an indivisible item to be infeasible without insurance. However, insurance can also be valuable for financing an opportunity that would be undesirable to consume without insurance. A discussion of the different motives underlying insurance is deferred to section 4.

[5]Karni (1985) depicts the steps that are necessary to still call people "risk averse" in a framework with state-dependent preferences. Essentially, it requires a redefinition of the term "risk" from being a deviation from certainty to being a deviation from optimality. In consequence, if the optimal wealth distribution is unique, people are risk-averse *by definition*. I am not willing to redefine the concept of risk and insist on the definition by Rothschild and Stiglitz (1970). In consequence, state-dependence simply rules out risk aversion.

increase in utility. Under state-independence, differences in wealth are the only possible reason for such a transfer to be desirable. In the context of state-dependence, wealth transfers across states are desirable even without any risk in wealth. Thus, state-dependence requires a different understanding of insurance. It is no longer solely a means to decrease risk exposure, but a means for directed wealth transfers across states.

Definition 1. *Insurance is a targeted transfer of wealth across states in order to address conditional needs.*[6]

Gambling thus differs from insurance in that the *identity* of the states, in which the different realizations of the lottery occur, do not matter. In contrast, insurance can be understood as a *targeted* redistribution of wealth across states of which the identity matters. This definition differs from previous ones in that it no longer regards insurance solely as a tool to rid the decision-maker of risk. This is still possible under the above definition: if marginal utility of wealth is not constant in wealth, then redistributing wealth from states with higher wealth into states with lower wealth constitutes insurance according to the above definition, because marginal utility is a measure of needs addressed through the consumption of divisible consumption opportunities. A larger marginal utility (compared to other states) thus reflects a larger conditional need. In addition, the above definition allows insurance to be a tool to acquire favorable risks that redistribute wealth into states in which the DM reaps a larger utility from wealth even if these states are not characterized by a lower wealth level in comparison to other states. The most obvious case is longevity risk. Living longer than expected does not constitute a loss in wealth. If at all, it constitutes a positive shock to lifetime wealth as it prolongs earnings potential. However, given that the need for (and thus utility of) money is arguably larger in the state in which one is alive, there is a desire to transfer wealth into the state in which this need occurs. Insurance against longevity risk is thus an insurance that exposes us to more risk in our wealth distribution. This is desirable, however, as it allows us to acquire a *desirable* risk. The existence of state-dependent preferences thus either requires to redefine certain forms of insurance as gambling; or it requires us to give up the idea that insurance is solely a means to rid oneself of risk. I argue for the latter. Gambling, in contrast, always means an increase in the risk of one's wealth distribution.

What is the maximal utility that can be derived from directed wealth transfers, i.e., insurance?

[6] The idea that insurance addresses conditional needs is, to my knowledge, first proposed by Braess (1960, p.14). Braess' definition, however, requires a loss to be the cause of a conditional need.

Let $\tilde{w} = (w_s)_{s \in S}$ be a distribution of wealth across states $s \in S$ and $\mathbb{E}\tilde{w}$ its expected value. Define

$$U_I(w) = \max_{\tilde{w}:\mathbb{E}\tilde{w}=w} \sum_{s \in S} p_s U_s(w_s) \tag{7}$$

as the maximal utility from insuring. It is clear that $U_I(w) \geq U(w)$ since $U(w)$ is the expected utility of a given wealth level that is identically distributed across states $s \in S$ while $U_I(w)$ is the utility that is attainable by optimally distributing wealth w across states $s \in S$. U_I determines the expected utility that is attainable by allocating the available wealth across states given that in each state the allocated wealth is optimally consumed. I thus call $U_I(w)$ the utility from *insuring*. It derives from transferring wealth optimally across states given the consumption opportunities that present themselves in each state, and thus, the *conditional needs* that an individual faces.[7]

However, neither $\bar{U}(w)$ nor $U_I(w)$ constitute the maximum utility of wealth attainable. Define

$$U^*(w) = \max_{\tilde{w}:\mathbb{E}\tilde{w}=w} \sum_{s \in S} p_s \bar{U}_s(w_s). \tag{8}$$

Clearly, $U^*(w) \geq \bar{U}(w)$ by the same argument as $U_I(w) \geq U(w)$. In addition, $U^*(w) = \sum_s p_s \bar{U}_s(w_s) \geq U_I(w) = \sum_s p_s U_s(w_s)$ since $\bar{U}_s(w_s) \geq U_s(w_s)$ for any w_s. Hence, $U^*(w)$ is the maximum expected utility that can be derived from a wealth level w. It is achieved by optimally distributing wealth across states $s \in S$ given that in each of these states the DM gambles optimally with the allocated wealth w_s to achieve a wealth level $w^*_{i(s)}$.

It is important to recognize that both insurance and gambling are used here to achieve an *unequal* distribution of wealth across states of nature. Hence, if (and only if) $U^*(w) > U(w)$, then the DM profits from assuming particular risks and is, thus, not risk-averse. Note that $U^*(w) = U(w)$ only at wealth levels $w \geq \max_{s \in S} w_{\iota(s)}$ and at wealth levels at which $w = w^*_{i(s)}$ for all states $s \in S$. Thus, even diffidence, the dislike of all mean-zero lotteries at a specific wealth level w, is a highly special case.

It is informative to investigate possible difference between $U(w)$, $U_I(w)$, $\bar{U}(w)$, and $U^*(w)$. Differences between the first and the following two indicate gains from insuring and gambling,

[7] The case of state-dependent preferences reveals a common misconception about insurance. People buy insurance to address conditional needs, and not to mitigate losses. While losses can be the source of greater needs, the occurrence of a loss is not necessary for insurance to be desirable.

respectively. Positive differences between U^* and both U_I and $\bar{U}$ indicate a gain from both gambling and insurance if there are also strictly positive differences between U_I, $\bar{U}$, and $U(w)$.

Example 2. *Consider two equiprobable states s_1, s_2. There is one indivisible consumption opportunity $v = 8, c = 6$ that is available in both states, and another indivisible consumption opportunity with $v = 8, c = 4$ that is unique to state s_2. Suppose there is also a utility loss of 10 associated with state 2.*[8] *Figure 2 depicts the setup.*

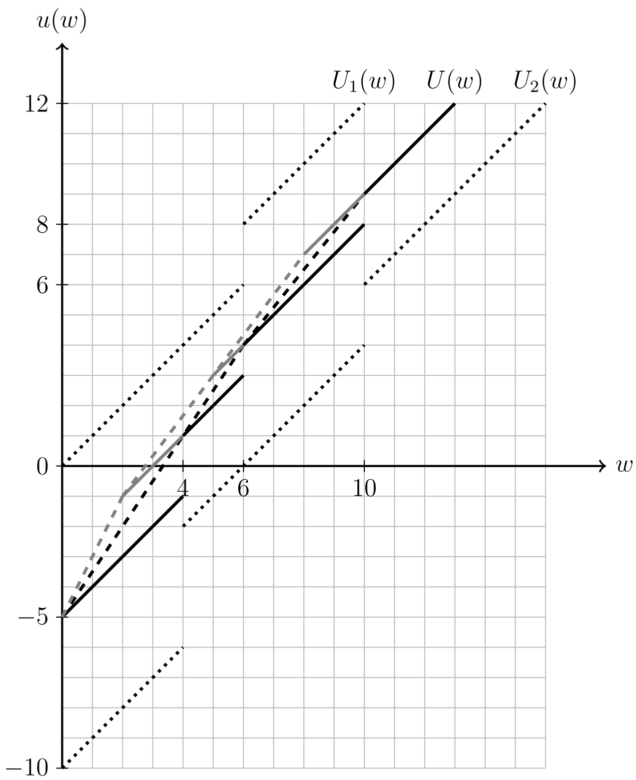

Figure 2: Example 2. The black dotted lines denote state-dependent utility $U_s(w)$. The black solid line denotes $U(w)$. The black dashed line denotes $\bar{U}(w)$. The gray solid line denotes $U_I(w)$ if it exceeds $U(w)$. The gray dashed line denotes $U^*(w)$.

Consider the wealth level $w = 3$. At this level of wealth, none of the indivisible consumption opportunities is affordable. Hence, $U_1(3) = 3$, $U_2 = 3 - 10 = -7$, and $U(3) = 0.5 * 3 + 0.5 * (-7) = -2$. Is there a gain from gambling? Consider the wealth lottery $W = (4, 0.75; 0, 0.25)$ with $\mathbb{E}W = 3$.

[8]The utility loss is of no importance for behavior as such a shock simply shifts the utility function. Its sole purpose is distinguish the state-dependent utility functions more clearly in the following figure.

Taking the gamble yields the utility

$$\begin{aligned}\bar{U}(3) =& \frac{3}{4}\left[0.5 * u_1(4) + 0.5 * u_2(4)\right] + \frac{1}{4}\left[0.5 * u_1(0) + 0.5 * u_2(0)\right] \\ =& \frac{3}{4}\left[0.5 * 4 + 0.5 * (8 - 10)\right] + \frac{1}{4}\left[0.5 * 0 + 0.5 * (0 - 10)\right] \\ =& -0.5 > U(3)\end{aligned}$$

Hence, there is a gain from gambling. Intuitively, gambling allows to reach the wealth level $w = 4$ *that is particularly valuable in state 2, when a desirable indivisible consumption opportunity becomes available. However, it is even better to insure than to gamble. Consider the insurance contract that transfers one unit of wealth from state 1 to state 2.*[9] *Buying such a contract yields the utility*

$$\begin{aligned}U_I(3) =& 0.5 * u_1(3 - 1) + 0.5 * u_2(3 + 2 - 1) \\ =& 0.5 * 2 + 0.5 * (8 - 10) \\ =& 0 > U(3).\end{aligned}$$

Hence, it is worthwhile to buy such an insurance contract. Also, given that $U_I(3) > \bar{U}(3)$, *it is strictly better to insure rather than to gamble. This is intuitive as the insuree is able to transfer the one unit of wealth that is required to afford the indivisible good in state 2 directly from state 1. In contrast, the gambler must hope for the coincidence of winning the gamble when state 2 occurs. Yet, it is possible to show that* $U_I(3) < U^*(3)$, *i.e., that there is an even better way to allocate wealth than to solely insure. Consider the following combination of insurance and gambling: the individual buys a lottery ticket that pays a prize of 3 with probability* $1/3$ *at a cost of* -1. *This yields the wealth lottery* $W = (5, \frac{1}{3}; 2, \frac{2}{3})$. *Conditional on winning the lottery, he buys insurance that transfers 1 unit of wealth from state 2 to state 1. Conditional on losing the lottery, he buys insurance that transfers 2 units of wealth from state 1 to state 2. This yields* $U_I(2)$ *in case of*

[9] The insurance pays a benefit of 2 in case state 2 occurs and requires a premium payment of 1.

losing and $U_I(5)$ in case of winning. In expectation,

$$\begin{aligned} U^*(3) &= \frac{1}{3}U_I(5) + \frac{2}{3}U_I(2) \\ &= \frac{1}{3}\left[0.5 * u_1(6) + 0.5 * u_2(4)\right] + \frac{2}{3}\left[0.5 * u_1(0) + 0.5 * u_2(4)\right] \qquad = \frac{1}{3}(3) + \frac{2}{3}(-1) \\ &= \frac{1}{3} > U_I(3). \end{aligned}$$

Hence, the maximum utility can be reached through a combination of insurance and gambling.[10]

The above example illustrates that an individual can be willing to both gamble and insure even if the initial endowment is certain. In addition, it shows that gambling and insurance are far from "opposite" behaviors but may sometimes even complement each other.

Financing an indivisible state-dependent consumption opportunity that costs c_i and occurs in a state s with probability p_s at a wealth level $w < c_i$ is feasible if and only if $w \geq p_s c_i$. Basically, insurance is feasible if and only if the actuarially fair premium is affordable.

Desirability of insurance for financing exclusive opportunities

Insurance is desirable in terms of financing state-exclusive consumption opportunities, i.e., that $U_I(w) > U(w)$, if a transfer x into state s is worthwhile, i.e.,

$$\sum_{s' \neq s} p_{s'} U_{s'}(w - x) + p_s U_s(w + (1 - p_s)x/p_s) > \sum_s p_s U_s(w) = U(w). \tag{9}$$

The equation shows why insurance is particularly valuable if state s is rather improbable. If p_s is low, then it requires only a small transfer x from states $s' \neq s$ to allow a large increase in wealth in state s. This helps to finance consumption opportunities exclusive to state s that have large cost, but also large gains, associated with them.

Proposition 2. *If at wealth level w, there exists an indivisible consumption opportunity $i(s)$ in a state $s \in S$ such that $a_i^*(s) = 0$, $d_i > 1$, and $p_s c_{i(s)} \leq \min_{s \in S} w_d^*(s)$, then $U_I(w) > U(w)$.*

Although Proposition 2 only states a sufficient condition for insurance to be desirable, it is worthwhile pointing out how weak this condition is. It requires that the decision-makers optimal

[10] This requires either that insurance activity can be conditioned on the outcome of gambling or that gambling activity can be conditioned on the resolution of states $s \in S$. In the latter case, $U^*(3) = \frac{1}{3}$ can be attained by transferring 1 unit of wealth from state 1 to state 2 through insurance, and, if state 1 occurs, using the remaining wealth for lottery tickets that pay a prize of 6. If a conditioning is not possible, $U^*(w) = \max\left\{U_I(w), \bar{U}(w)\right\}$.

allocation includes some divisible consumption in every state $s \in S$. Then, all that is required for insurance to be valuable is that there exists an indivisible consumption opportunity in some state s, that is not already consumed and of which the actuarially fair premium is affordable out of divisible consumption.

Desirability of insurance for risk reduction and of gambling for risk increase Suppose that initial wealth is uncertain. More specifically, suppose that wealth is a random variable W with expected value w: $\mathbb{E}W = w$ and the realizations of W are independent of the states $s \in S$. Insurance is desirable for the purpose of risk reduction if and only if there exists a non-degenerate random variable W such that

$$\mathbb{E}U(W) < U(w), \tag{10}$$

while gambling is desirable for the purpose of risk increase if and only if there exists a non-degenerate random variable W such that

$$\mathbb{E}U(W) > U(w). \tag{11}$$

Consider a DM with initial certain wealth w. It is clear that if $\bar{U}(w) > U(w)$, then some gambles are desirable. But what can be said about a particular gamble? Specifically, when is it desirable for the DM to undertake a mean-zero gamble with a prize x that is paid with probability p?[11] Define $w_j,\ j = 0, 1, 2, \ldots$ as the wealth levels $w_0 = 0 < w_1 < w_2 < \ldots < w_\gamma$ at which $U(w) = \bar{U}(w)$ with $w_\gamma = \min\left\{w : U(w) = \bar{U}(w), \forall w \geq w_\gamma\right\}$. For any wealth level $w : w_j < w < w_{j+1},\ 1 \leq j \leq \gamma - 1$, define the wealth levels $\underline{w} = \min\left\{w' \leq w : U(w) - U(w') = (w - w')\right\}$ and $\bar{w} = \sup\left\{w' \geq w : U(w') - U(w) = (w' - w)\right\}$. $(\underline{w}, \bar{w})$ are the discontinuity points of $U(w)$ closest to w.

Proposition 3. *Low-probability, large-stakes gambles*

Consider any wealth level $w : \underline{w} < w < w_\gamma$.

The DM rejects all mean-zero gambles with a gain $G < \bar{w} - w$ and a loss $L > w - \underline{w}$.

The DM is willing to take any mean-zero gamble with a gain $G \geq \bar{w} - w$ and a loss $L \leq w - \underline{w}$.

The Proposition implies that people are willing to take gambles if they offer a long-shot chance

[11] Note that for such a lottery to constitute a gamble, it must be true that the winning probability p is independent of $s \in S$.

at a large gain and are reluctant to take gambles if they imply a long-shot chance at a large loss. Such behavior - that shows an aversion to risks with a small probability of a large loss and an inclination to take risks with a low probability of a large gain - has so far been attributed to probability weighting (Kahneman and Tversky 1979). The model offers a rational underpinning for such long-shot risk attitudes.

The following example serves two purposes. First, it illustrates a commonality in the motives underlying gambling and insurance that results in the relationship between gambling and insurance to be more complex than just being direct opposites. Second, it is helpful to address a popular objection by Bailey et al. (1980) that local convexities in the utility function remain inconsequential if the decision-maker has access to borrowing and saving.

Example 3. *Consider two equiprobable states s_1, s_2. There is one indivisible consumption opportunity $v = 8, c = 4$ that is available in both states, and another indivisible consumption opportunity with $v = 8, c = 6$ that is unique to state s_2. Suppose there is also a utility loss of 10 associated with state 2. Figure 3 depicts the setup.*

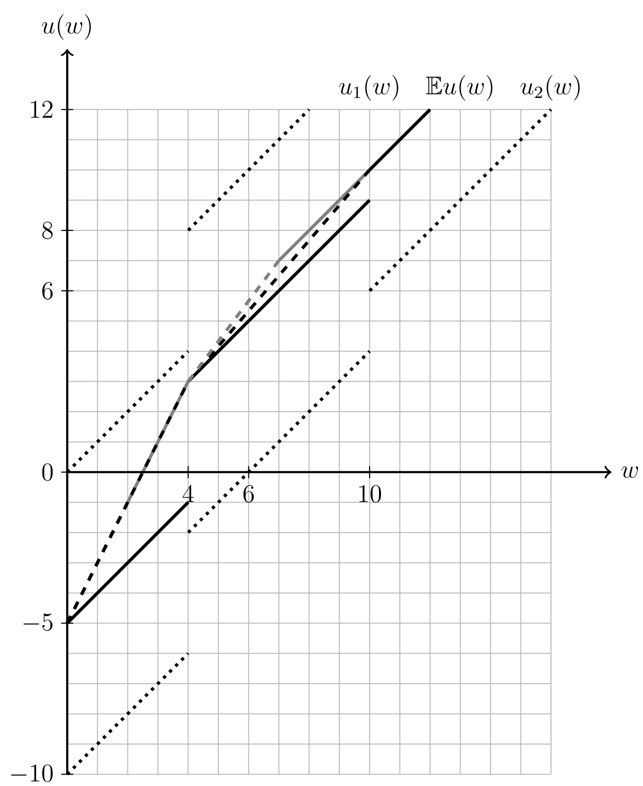

Figure 3: Example 3. The black dotted lines denote state-dependent utility $U_s(w)$. The black solid line denotes $U(w)$. The black dashed line denotes $\bar{U}(w)$. The gray solid line denotes $U_I(w)$ if it exceeds $U(w)$. The gray dashed line denotes $U^*(w)$.

At $w = 3$, $U^*(3) = U_I(3) = \bar{U}(3) > U(3)$. That is, gambling and insurance yield the same *expected utility. At $w = 8$, $U^*(8) = U_I(8) > \bar{U}(8) > U(8)$. That is, while gambling yields a utility gain, the larger utility gain is reached by insuring.*

Example 3 illustrates that gambling is desirable for financing *state-independent* indivisible consumption opportunities (Ng 1965; Jones 2008; Vasquez 2017). If it is possible to finance the same consumption opportunity through a directed wealth transfer ($w \geq 2$), then directed wealth transfers and undirected wealth transfers (gambling) are perfect substitutes. This is true because the consumption opportunity that is optimal to consume at $w = 4$ does not represent a conditional need, but an unconditional one. In a strict sense, a targeted wealth transfer across states is then no longer insurance, as it does not derive its value from addressing a conditional, but an unconditional need. In contrast, if there is a conditional need - a consumption opportunity exclusive to a state - then a directed wealth transfer through insurance is desirable, while gambling only serves as an imperfect substitute.

Notably, in neither case, gambling constitutes the opposite of insurance, but an (im)perfect substitute for it. This is due to a commonality in the motives underlying gambling and insurance. As suggested in Fels (2017b), both gambling and insurance can serve as means to finance indivisible consumption opportunities across states. Gambling is useful in financing state-independent consumption opportunities, while insurance is useful in financing consumption opportunities that are exclusive to particular states.

Example 3 is also useful to discuss a common critique of considering indivisibilities in consumption. As Bailey et al. (1980) point out, local convexities in the utility function as suggested by Friedman and Savage (1948) might be inconsequential if individuals have access to borrowing and saving opportunities at sufficiently low cost. The basic argument suggests that instead of gambling it is better to attain the wealth levels w_i^* by adjusting wealth temporally, e.g., by reducing consumption in one period to a lower level $w_i^* < w$ and increasing consumption to a higher wealth level $w > w_{i+1}^*$ in a later period. This is a valid criticism, but hinges on two implicit assumptions.

First, it assumes that the wealth levels w_i^*, w_{i+1}^* "neighboring" w are actually feasible trough saving and/or borrowing. This is not always the case. Consider the setup in Example 3 in a simple two-period model. Let a superscript $t = 1, 2$ denote the time period. The DM maximizes $U^1(w^1) + \beta U^2(w^2)$ where $\beta \in (0, 1)$ denotes the discount factor. Suppose that the individual has wealth level $w^t = 1$ in both periods associated with expected utility $-4(1 + \beta)$. By borrowing at interest rate r, the individual could reach a wealth level of at most $1 + \frac{1}{1+r} < 2$ in period 1 which results in expected utility $-4 + \frac{1}{1+r} - 5\beta$. This is superior to consuming 1 in each period if and only if $\beta < \frac{1}{1+r}$, but it yields a strictly smaller expected utility as compared to gambling in each period $\bar{U}(1)(1+\beta) = -3(1+\beta)$ for any $r > 0$. In contrast, by saving at interest rate r, the individual could reach a wealth level of at most $2 + r$ in period 2. As long as $r < 2$, this yields a utility of $\beta(-3 + r)$ which is strictly lower than $(1+\beta)\bar{U}(1)$. In short, as long as the interest rate does not reach 200%, an individual with wealth level $w = 1$ is strictly better off by gambling as compared to saving or borrowing. This is due to the fact that saving/borrowing can only redistribute a given lifetime wealth, while gambling can expand it. Intuitively, while it may not be possible to make a million Dollar out of a Dime through saving in a given period of time, it is possible to make a million Dollar out of a Dime through gambling in a rather short period of time (even if the odds are not exactly high).

Second, the indivisibilities should be state-independent. This, also, is not always true, and the model suggests insurance to be the preferred behavior in response to state-dependent indivisibilities. Consider again the setup in Example 3 in a simple two-period model. Suppose that the individual has wealth level $w^t = 9$ in both periods associated with expected utility $U(9) + \beta U(9) = 8(1+\beta)$. Insuring (in both periods) yields $9(1+\beta)$. Note that the wealth levels $w^*_{1(2)} = 4, w^*_{2(2)} = 10$ are now feasible with saving/borrowing at reasonable interest rates. Suppose that the DM would borrow in order to achieve a wealth of 10 in period 1 at the expense of reducing w^2 by $(1+r)$. This yields $U^1 = U(10)$ and $U^2 = (8 - r)$. With r not being too large, this yields a utility of $10 + \beta(7 - r)$, that exceeds the utility from insuring if and only if $\beta \leq 1/(2+r)$. Hence, for borrowing to yield a better outcome, the discount rate must exceed 100%, a rather strong requirement in many settings.[12] Suppose, in contrast, the individual would save $1/(1+r)$ units to achieve the wealth level 10 in period 2. This yields a utility $8 - \frac{1}{1+r} + \beta 10$ which exceeds the utility from insuring if and only if $\beta \geq 1 + \frac{1}{1+r}$. Hence, for saving to be better than insurance would require a negative discount rate. In sum, for saving/borrowing to dominate insurance, an extreme difference between the interest and discount rate is required. Again, the intuition for the superiority of insurance is pretty simple. Saving and borrowing reallocate wealth across time. The optimal realization of *state-dependent* indivisible consumption opportunities, however, requires reallocation across states of nature. The wealth level of 10 is not of particular

[12]Even in the case in which it is satisfied, it would be better to insure in both periods, but borrow the money for the premium payment of period 1. Hence, the optimal allocation of wealth always includes insurance.

interest in all states, but only in state 2. If it is costly to transfer wealth across time or states, it is only desirable to transfer as little wealth as is necessary. Realizing the consumption opportunity at wealth 10 does not require the individual to have this level of wealth in all states, but only in state 2. Hence, saving (or borrowing) constitutes an excessive transfer of wealth if the goal is to realize state-dependent indivisibilities.

In sum, the incentives to gamble and/or insure in the face of state-dependent indivisible consumption opportunities are not overcome through access to saving and borrowing.

4 Decreasing Marginal Utility of Divisible Consumption and the Functions of Insurance

Suppose now that the utility from divisible consumption remains state-independent, but it features a decreasing marginal utility. Let $u_d(w_d)$ be twice continuously differentiable with $u_d' > 0$ and $u_d'' < 0$. Furthermore, assume that $u_d(0) > -\infty$ and normalize $u_d(0) = 0$. We first consider the optimal allocation within each state $s \in S$.

Again, the relaxed problem with $\bar{\alpha}_i(s) \in [0, 1]$ (treating the indivisible objects as perfectly divisible) is easier to handle given that the greedy algorithm delivers the optimal solution $\bar{\alpha}^*(s)$. Define $\hat{w}_i$ implicitly by $u_d'(\hat{w}_i) = d_i$. Intuitively, $\hat{w}_i$ is the amount of money that an individual would prefer to invest in indivisible consumption before starting to invest in item i. Then the wealth level as of which it is optimal to buy the first i items is given by $w_i^* = \hat{w}_i + \sum_{j=1}^{i} c_j$. Note that both $\hat{w}_{i+1} > \hat{w}_i$ and $w_{i+1}^* > w_i^*$ holds for all i.

Define $\iota = \max\{i | w \geq w_i^* - c_i\}$ if $w > \hat{w}_1$ and $\iota = 0$ otherwise. Intuitively, ι denotes the marginal indivisible consumption opportunity into which a DM with wealth w is willing to invest money. $\iota = 0$ if and only if it is best to invest all money in divisible consumption.

Proposition 4. *In any state $s \in S$, the optimal consumption plan for a wealth level w - given that consumption opportunities $i \in I(s)$ are perfectly divisible - is described by*

$$\bar{w}_d^*(s) = \begin{cases} w \text{ if } \iota = 0 \\ \max\left\{\hat{w}_\iota, w - \sum_{j=1}^{\iota} c_j\right\} \text{ if } \iota > 0 \end{cases}, \tag{12}$$

$$\bar{a}_i^*(s) = \begin{cases} 1 \text{ if } i < \iota \\ \min\left\{\frac{w - \hat{w}_\iota - \sum_{j=1}^{\iota-1} c_j}{c_\iota}, 1\right\} \text{ if } i = \iota \\ 0 \text{ if } i > \iota \end{cases}. \tag{13}$$

It is optimal to allocate an incremental unit of wealth into divisible consumption if the marginal utility of doing so exceeds the density d_i of any "indivisible" object with $\bar{a}_i < 1$. That is, if an amount w_d of wealth is allocated into divisible consumption it must be true that $\bar{a}_i = 1$ for all indivisibles with $d_i > u_d'(w_d)$.

The utility $\bar{U}_s(w)$ resulting from such an allocation can again be interpreted as the utility from gambling optimally given wealth w. It constitutes the concave envelope of $U_s(w)$ with $\bar{U}_s(w) \geq U_s(w)$. $\bar{U}_s$ is a concave function alternating between strictly concave and linear parts. In contrast, $U_s(w)$ exhibits several jump discontinuities in its first derivative.[13] At these discontinuities, the right derivative of $U_s(w)$ exceeds the left derivative. This is intuitive as the indivisibilities in consumption result in too much wealth being allocated into divisible consumption until a threshold wealth is realized that allows to optimally invest into the indivisible consumption opportunities. This is an important consequence of indivisibilities: the marginal utility of wealth $U_s'(w)$ is not monotonically decreasing even if the utility from divisible consumption is (compare Ng 1965).

Proposition 5. *The marginal utility of wealth differs across states if and only if there exist states s, s' such that $\sum_{i \in I(s)} a_i^*(w, s)c_i \neq \sum_{i \in I(s')} a_i^*(w, s')c_i$.*

The proposition shows how indivisibilities in consumption *cause* marginal utility of wealth to differ across states, resulting in incentives to reallocate wealth into states with unique indivisible consumption opportunities even at wealth levels at which these opportunities have already been realized.

The following example illustrates that differences in marginal utility across states at a wealth

[13] $U_s(w)$ can be discontinuous itself if there exist consumption opportunities with $v_i > u_d(c_i)$. See below.

level w are still a sufficient but no longer a necessary requirement for insurance to be desirable.

Example 4. *Consider two equiprobable states: s_1, s_2. Utility from divisible consumption is given by $u_d(w_d) = 100\sqrt{w_d}$ in both states. In addition, state 2 offers an indivisible consumption opportunity with $v_i = 50$ and $c_i = 0.5$. Finally, state 2 exhibits a utility shock of -100. Figure 4 depicts the utility functions.*

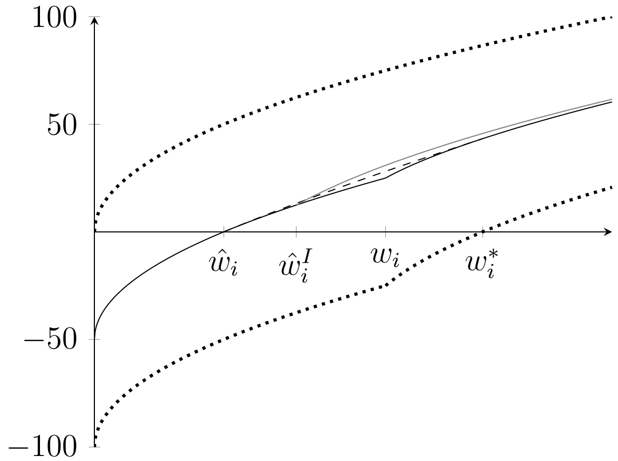

Figure 4: Example 4. The black dotted lines denote state-dependent utility $U_s(w)$. The black solid line denotes $U(w)$. The black dashed line denotes $\bar{U}(w)$. The gray solid line denotes $U_I(w)$ if it exceeds $U(w)$.

In the above example, a single state-dependent consumption opportunity results in marginal utility of wealth to differ across states for any wealth level above a threshold w_i. However, insurance is already valuable for wealth levels at which it is worthwhile to pay the actuarially fair premium: $w \geq w_i^I$. This illustrates how indivisibilities can (a) cause marginal utilities to differ across states, and (b) that, since they cause marginal utility to no longer be monotonically decreasing, there is an insurance value even for wealth levels at which marginal utilities do not differ (yet).

Note that if differences in marginal utility across states result from indivisibilities in consumption, there is still a consumption-smoothing motive despite difference in marginal utilities across states. It applies to the divisible part of consumption, not overall consumption. It is only if marginal utility from divisible consumption also differs across states, that there is no consumption-smoothing motive.

Define $w_{i(s)}$ implicitly by $u_d(w_{i(s)}) - u_d(w_{i(s)} - c_{i(s)}) = v_i$. $w_{i(s)}$ is the amount of wealth at which it becomes optimal to purchase opportunity i that occurs in state s rather than spend the amount $w_{i(s)}$ entirely on divisible consumption. By the strict concavity of u_d, it is optimal to equalize the impact of financing opportunity $i(s)$ across states:

$$p_s\left[u_d(w) - u_d(w - c_{i(s)})\right] > u_d(w) - u_d(w - p_s c_{i(s)}).$$

This again implies that there exists a wealth level $w^I_{i(s)} < w_{i(s)}$, implicitly defined by

$$w^I_{i(s)} = \min\left\{w \geq p_s c_s : u_d(w^I_{i(s)}) - u_d(w^I_{i(s)} - p_s c_{i(s)}) \leq p_s v_i\right\}.$$

$w^I_{i(s)}$ constitutes the wealth level from which it becomes optimal to finance indivisibility $i(s)$ across states.[14] Hence, for wealth levels $w \in \left[w^I_{i(s)}, w_{i(s)}\right)$, opportunity $i(s)$ is only consumed if insurance is feasible. Note that if $w^I_i > p_s c_s$, then

$$\frac{u_d(w^I_{i(s)}) - u_d(w^I_{i(s)} - p_s c_{i(s)})}{p_s c_{i(s)}} = d_i = \frac{u_d(w_{i(s)}) - u_d(w_{i(s)} - c_{i(s)})}{c_{i(s)}}.$$

This directly implies that $\lim_{p_s \to 0} w^I_{i(s)} = \hat{w}_i$ and $\lim_{p_s \to 1} w^I_{i(s)} = w_i$. Hence, $w^I_i \in (\hat{w}_i, w_i)$. The less likely the state s, in which the consumption opportunity opportunity arises, the more insurance is able to overcome the indivisibility problem associated with the consumption opportunity. This is intuitive. As $p_s \to 0$, the premium payment that is required to finance opportunity $i(s)$ becomes negligible. In the limit, all it needs is a marginal reduction in divisible consumption in all states. Such a marginal reduction is worthwhile as soon as $u'_d(w)$ reaches d_i, which - by definition - is at $\hat{w}_i$.

Example 4 also illustrates a crucial difference between indivisibilities in consumption and consumption commitments (Chetty and Szeidl 2007). Both produce a similar shape in the utility function with adjoining concave parts and local convexities at the points where the concave parts meet. However, the largest utility attainable with consumption commitments is the utility associated with optimal gambling over wealth $\bar{U}(w)$. In contrast, if these local convexities are the result of state-dependent indivisible consumption opportunities, insurance can yield a larger utility gain exactly in the region of local convexity. While the two models make similar predictions with respect to the shape of the utility function $U(w)$, they make opposite predictions about the optimal behavior as they attribute a different cause to the local convexities in the utility of wealth.[15]

[14] $w^I_i > p_s c_s$ holds whenever $v_i < u_d(c_i)$.

[15] Note that the behavioral predictions are identical if the kink in the utility function is produced by an indivisible consumption opportunity that is state-independent.

In contrast to insurance that addresses potential losses in wealth, insurance demand with the intention to realize conditional (indivisible) consumption opportunities exhibits non-monotonic wealth effects. Most notably, there is no insurance value and thus no insurance demand for any wealth below a threshold w_i^I. This is a simple consequence of the fact that realizing a particular consumption opportunity may not be desirable at low wealth levels. But even at wealth levels above w_i^I, the effect of wealth on insurance may not be monotonic.

Proposition 6. *Suppose that there exists a single state-dependent indivisible consumption opportunity i that is exclusive to state s. Then the value of insurance directed at financing the opportunity is*
(i) negative for any wealth level $w < w_i^I$,
(ii) strictly positive and strictly increasing in w for any $w \in (w_i^I, w_i)$, and
(iii) strictly positive and strictly increasing/decreasing in w for any $w > w_i$, if $u_d'(w)$ is a strictly concave/strictly convex function.

It is important to recognize the difference between the consumption-smoothing value of insurance that is derived from financing a consumption opportunity across states and the traditional consumption-smoothing value of insurance to address losses in wealth. First, the expense of c_i in state s is a deliberate *choice*, not an exogenous event. This is most obvious for wealth levels $w < w_i^I$. At these wealth levels, the DM is not willing to make the expense c_i in state s even if financed across states. At wealth levels $w_i^I < w < w_i$, the DM is willing to make the expense only if it financed across states, and, thus, only if consumption-smoothing through insurance is feasible. At wealth levels $w \geq w_i$, the DM is willing to make the expense c_i even by financing it within state s, but prefers to smooth the expense across states. Thus, the consumption-smoothing motive is based on the desire to optimally finance a conditional need, not to cover a loss.
Second, at wealth levels $w \in \left[w_{i(s)}^I, w_{i(s)}\right)$, opportunity $i(s)$ is only consumed if consumption-smoothing through insurance is available. Hence, the *purpose* of insurance is to increase consumption in the state s. This contradicts the notion that any increase in consumption of the insured asset/service is a sign of moral hazard that depresses the value of insurance. On the contrary, the increase in consumption is the basis of insurance value for some wealth levels. This idea is related, but not identical, to the access motive of Nyman (1999) further discussed below. It is similar to Nyman's idea that insurance might exactly be valuable for increasing consumption of the insured asset/service (beneficial moral hazard). The access motive posits that the insured asset is not

consumed without insurance because consumption is not feasible ($w < c_i$). Here, the insured asset $i(s)$ is not consumed because the detrimental impact on divisible consumption is too strong if the asset is only financed out of the divisible consumption of state s, but bearable if financed out of the divisible consumption of all states. That is, it is only through consumption-smoothing that the consumption opportunity is worthwhile at some wealth levels.

Beyond a consumption-smoothing motive, there can be an additional value in insuring if $u_d(c_i) < v_i$. In such a case, a decision-maker would prefer to spend his "first" units of wealth on indivisible consumption, but is not able to do so until $w \geq c_i$. Nyman (1999) has shown that insurance is valuable in allowing access to such unaffordable indivisible consumption opportunities as long as the premium is affordable. In that case, insurance transfers enough wealth into the respective state such that the valuable indivisibility is affordable.

Example 5. *Consider two equiprobable states: s_1, s_2. Utility from divisible consumption is given by $u_d(w_d) = 100\sqrt{w_d}$ in both states. In addition, state 2 offers an indivisible consumption opportunity with $v_i = 200$ and $c_i = 1$. Finally, state 2 exhibits a utility shock of -100. Figure 4 depicts the utility functions.*

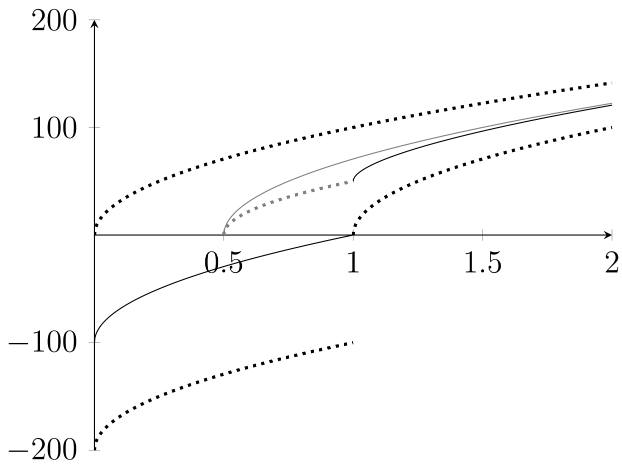

Figure 5: Example 5. The black dotted lines denote state-dependent utility $U_s(w)$. The black solid line denotes $U(w)$. The gray solid line denotes $U_I(w)$ if it exceeds $U(w)$. The gray dotted line denotes the part of $U_I(w)$ that constitutes the access value of insurance; the remaining part constitutes its consumption-smoothing value.

The example illustrates that insurance has an access value if there are indivisible consumption opportunities with $u_d(c_i) < v_i$. The indivisible consumption opportunity is only affordable with wealth $w \geq c_i$. For wealth levels $w_i^I \leq w < c_i$, insurance is valuable[16] as it allows to transfer enough wealth into state 2, such that c_i is affordable *and* the disutility of paying c_i is smoothed

[16] With $u_d(c_i) < v_i$ and $lim_{w_d \to 0} u_d' \to \infty$, there exists a unique probability $\hat{p}_s \in (0,1)$ such that $w_i^I = p_s c_s,\ \forall p_s \geq \hat{p}_s$ and $w_i^I > p_s c_s\ \forall p_s < \hat{p}_s$.

across states. The value of insurance is thus a composite of two values: the access value (AV) and the consumption-smoothing value (CSV).[17]

For all wealth levels $w > w_i^I$, there exists a consumption-smoothing value (CSV) as the cost of paying c_i is minimized by equalizing the payment across states.

$$CSV = \begin{cases} u_d(w - p_s c_i) - (1-p_s)u_d\left(\frac{1}{1-p_s}(w - p_s c_i)\right) & \text{if } w < c_i, \\ u_d(w - p_s c_i) - p_s u_d(w - c_i) & \text{if } w \geq c_i \end{cases} \tag{14}$$

For all wealth levels, $w_i^I \leq w < c_i$, there exists an additional access value as insurance allows the consumption of opportunity i at wealth levels at which paying c_i is not feasible:

$$AV = p_s v_i - p_s u_d(w) - (1-p_s)\left[u_d(w) - u_d(w-x)\right], \tag{15}$$

where $x = \frac{p_s}{1-p_s}(c-w)$ is the necessary wealth transfer from states $s' \neq s$ into state s that allows to pay c_i in state s. Note that the access value is not necessarily positive.[18] To date, most economic analysis of insurance and policy recommendations with respect to its efficient design focus on the consumption-smoothing value of insurance. The following proposition indicates that this focus might neglect the major part of insurance value.

Proposition 7. *(i) Given $w < c_i$, the access value of insurance exceeds its consumption-smoothing value if and only if*

$$p_s\left[v_i - u_d(c_i)\right] > \left[u_d(w) - (1-p_s)u_d(w-x) - p_s u_d(c_i)\right] + \left[u_d(w - p_s c_i) - (1-p_s)u_d(w-x)\right] \tag{16}$$

with $x = \frac{p_s}{1-p_s}(c-w)$ being the transfer from states $s_j \neq s_i$ that is necessary to finance c_i.
(ii) For $p_s \to 0$, the access value exceeds the consumption smoothing value if and only if

$$v_i - u_d(c_i) > \left[u_d(w) + u_d'(w)(c-w) - u_d(c)\right] + \left[u_d(w) - u_d'(w)w\right]. \tag{17}$$

(iii) Suppose $\lim_{w\to 0} u_d'(w) \to \infty$. Then there exists a unique wealth level $w_a \in (0, \frac{c}{2})$ such that

[17] Nyman (2003, p. 67 ff.) does not distinguish the two values and considers only the entire value of insurance in case it provides access.

[18] To see this, simply assume that $p_s v_i < u_d(p_s c_i)$. Then at $w = p_s c_i$, $AV < 0$.

the access motive dominates the consumption smoothing value for any $w \in (w_a, c)$ *if* $v_i - u_d(c_i) > u_d(c) - u_d'(c)c$. *The consumption-smoothing value always dominates the access value if* $v_i < 2u_d(\frac{c}{2})$.

The access value exceeds the consumption-smoothing value if the expected consumer surplus from buying the good at wealth $w = c_i$ exceeds both the loss of redistributing w across states such that c_i is feasible and the gain from equalizing the disutility of paying c_i across states. Note that the former is a first-order effect, although an uncertain one, while the other two are certain second-order effects that depend on the curvature of u_d. This suggests that insurance directed at financing high-cost (c_i), but also high-value expenditures (v_i) derives its major value from providing access. This corroborates the suggestion by Nyman (1999) that the major value of health insurance may lie in providing access instead of its consumption-smoothing role. It extends this hypothesis to any insurance that covers assets of particularly high value.

5 Conclusion

The existence of state-dependent indivisible consumption opportunities has a decisive impact on people's risk attitudes.

First, it produces a state-dependence of preferences over wealth that rule out the classic trichotomy of risk aversion, risk neutrality, and risk love. In general, risk aversion is confined to be an extremely special case.[19] In general, people have more nuanced attitudes regarding some risks as desirable and others - with similar distributional characteristics - as undesirable. This requires a redefinition of what constitutes insurance. Insurance must be viewed as a means for directed wealth transfers instead of a means for risk reduction. The novel definition of insurance questions the standard view of insurance and gambling as "opposite" activities. Instead, the two activities have a far more nuanced relationship depending on the situation, ranging from being complements to being imperfect substitutes. Second, state-dependent indivisibilities are a possible root cause of variations in the marginal utility of wealth across states, suggesting a link between the literature on indivisible consumption and state-dependent preferences. Finally, indivisibilities in consumption result in insurance to be desirable for two purposes: consumption-smoothing and access. The

[19]More precisely, diffidence - the dislike of any mean-zero lottery - is confined to a limited number of wealth levels. This rules out risk aversion which requires diffidence at all wealth levels (Gollier 2001). Risk aversion is then the special case that describes the non-existence of any state-dependent indivisible consumption opportunities, nor any other source of state-dependence of utility.

consumption-smoothing motive does not result from the intent to mitigate a loss, but is based on a desire to mitigate the impact of financing a state-dependent indivisible consumption opportunity. Beyond the motive to equalize marginal utility of wealth, insurance may create an access value (Nyman 1999). I show that the access motive presents the major value of insurance if insurance serves the purpose to finance a major state-dependent indivisible consumption opportunity.

These findings have important implications for policy and future research. First, optimal insurance design should no longer be based on the assumption that people are risk averse. Instead, it should focus on the degree to which insurance covers the conditional needs that it is supposed to address.[20] Second, empirical studies seeking to recover risk attitudes from insurance choices cannot be generalized to settings outside the domain in which they were conducted. There is no longer a theoretical reason why risk preferences should be stable across contexts. Third, laboratory studies that seek to elicit risk preferences might have little explanatory power with regard to actual insurance choices. Frequent findings of risk averse choices may say more about the general desirability of the typical risks offered in laboratory experiments than about a general risk attitude.

[20]Fels (2016) argues that the criticism of Medicaid to deliver only a minor consumption-smoothing value is misdirected if the program's major purpose is to guarantee access to basic long-term care services. Fels (ming) shows how the access value of insurance is critically undermined by imposing cost-sharing. Fels (2017a) proposes bonuses and rebates as an alternative form of incentivization that protects access.

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Appendix

Proof of Proposition 1

Given that $u_d'(w_d) > 0$, the entire wealth must be consumed in the solution to the optimal allocation problem. Thus, $w = w_d^* + \sum_{i \in I} \bar{a}_i^* c_i$. It is known that the greedy algorithm provides the optimal solution to the knapsack problem with $\bar{a}_i \in [0,1]$. Thus, if an amount $w_o = w - w_d$ is allocated to consuming "indivisible" consumption opportunities $i \in I$, the optimal allocation $\bar{a}(w_o)$ will realize opportunities according to their densities d_i. Let $\hat{i}(w_o)$ be the marginal indivisible consumption opportunity given the amount $w_o \leq w$ is allotted to indivisible consumption. That is, $\hat{i} = i | \sum_{j, j<i} c_j < w_o \leq \sum_{j, j \leq i} c_j$. Then the marginal utility of an increase in w_o is given by $d_{\hat{i}}$. The marginal utility of an increase in w_d is fixed at 1.

It follows that it is optimal to increase w_o until either $w_o = w$ or $\hat{i} = \iota$ and $\bar{a}_\iota = 1$. In the latter case, $w_o = \sum_{i=1}^{\iota} c_i$. In contrast, if $\hat{i} > \iota$, then it is optimal to decrease w_o until $\hat{i} = \iota$. Hence, $w_o^* = \min \{w, \sum_{i=1}^{\iota} c_i\}$ and $w_d^* = w - w_o^*$. $\bar{a}_i^*$ is the solution to the greedy algorithm with a budget w_o^*.

Proof of Proposition 2

If $p_s c_{i(s)} \leq \min_{s \in S} w_d^*(s)$, then is possible to finance consumption opportunity $i(s)$ across state by reducing only divisible consumption by an amount $p_s c_{i(s)}$ in each state $s' \in S$. That means a reduction in divisible consumption by $p_s c_{i(s)}$ in each state, freeing up enough resources to pay for $i(s)$ which yields an expected utility of $p_s v_{i(s)}$. Such a redistribution of wealth across states yields an expected utility

$$U(w) + p_s(v_{i(s)} - c_{i(s)}) > U(w) \tag{18}$$

since $v_{i(s)} - c_{i(s)} > 0$ as $d_{i(s)} > 1$. Given that this describes only one possible redistribution of wealth w across states $s' \in S$, it must be true that

$$U_I(w) \geq U(w) + p_s(v_{i(s)} - c_{i(s)}), \tag{19}$$

from which follows that

$$U_I(w) > U(w). \tag{20}$$

Proof of Proposition 3

Consider a wealth level w and its associated "boundary levels" $(\underline{w}, \bar{w})$. For any wealth levels w' with $w' > w$, it holds that $U(w') - U(w) \geq w' - w$ with strict inequality if and only if $w' \geq \bar{w}$. Similarly, for any $w'' < w$ it holds that $U(w) - U(w'') \geq w - w''$ with strict inequality if and only if $w'' < \underline{w}$.

A mean-zero gamble that offers a gain $G < \bar{w} - w$ with probability p and loss $L > w - \underline{w}$ with probability $(1-p)$ such that $pG - (1-p)L = 0$, then yields an expected utility

$$pU(w+G) + (1-p)U(w-L) < p(U(w)+G) + (1-p)(U(w)-L) = U(w),$$

where the inequality stems from $w - L < \underline{w}$ and $w + G < \bar{w}$.

Similarly, a mean-zero gamble that offers a gain $G \geq \bar{w} - w$ with probability p and loss $L \leq w - \underline{w}$ with probability $(1-p)$ such that $pG - (1-p)L = 0$, then yields an expected utility

$$pU(w+G) + (1-p)U(w-L) > p(U(w)+G) + (1-p)(U(w)-L) = U(w),$$

where the inequality stems from $w - L \geq \underline{w}$ and $w + G \geq \bar{w}$.

Proof of Proposition 4

If $\iota = 0$, then $w \leq \hat{w}_1$ and $u_d(w)' \geq d_i$ for all $i \in I(s)$. It is then optimal to invest the entire wealth into divisible consumption: $w_d^* = w$. If $\iota > 0$, then $w \geq w_\iota^* - c_\iota = \hat{w}_\iota + \sum_{j=1}^{\iota-1} c_j$ and $w < w_{\iota+1}^* - c_{\iota+1} = \hat{w}_{\iota+1} + \sum_{j=1}^{\iota} c_j$. That is, it is possible to both invest at least $\hat{w}_\iota$ into divisible consumption and $\sum_{j=1}^{\iota-1} c_j$ into indivisible consumption, thereby realizing any consumption opportunities, divisible or indivisible, with larger marginal utility/density than ι. This implies that $w_d^* \geq \hat{w}_\iota$.

Now, suppose that $w < \hat{w}_\iota + \sum_{j=1}^{\iota} c_j$, i.e., that - after paying for all indivisibilities $i < \iota$ - it is impossible to fully pay for ι and set $w_d = \hat{w}_\iota$. Then, $w_d > \hat{w}_\iota$ means $u_d'(w_d) < d_\iota$ while $\bar{a}_\iota < 1$.

This cannot be optimal as marginally reducing w_d in order to finance a marginal increase in $\bar{a}_\iota$ increases utility. Hence, $w_d = \hat{w}_\iota$ must hold.

If, in contrast, $w > \hat{w}_\iota + \sum_{j=1}^{\iota} c_j$, then it is possible to fully finance all indivisibilities $i \leq \iota$ and invest at least $\hat{w}_\iota$ into divisible consumption. Note, however, that by the definition of ι, $w < \hat{w}_{\iota+1} + \sum_{j=1}^{\iota} c_j$. That is $w - \sum_{j=1}^{\iota} c_j < \hat{w}_{\iota+1}$, i.e., $u_d'(w - \sum_{j=1}^{\iota} c_j) > d_{\iota+1}$. Hence, it is not optimal to invest in any indivisibilities $i > \iota$. Given that $u_d'(w) > 0$, $\forall w$, it is thus optimal to invest $\sum_{j=1}^{\iota} c_j$ into indivisible consumption - financing the first ι indivisible consumption opportunities - and to invest the remaining amount $w - \sum_{j=1}^{\iota} c_j$ into divisible consumption: $w_d^*(s) = w - \sum_{j=1}^{\iota} c_j$. Again, the greedy algorithm delivers the optimal investment into indivisibilities $\bar{a}_i^*(s)$ given that $w_o = w - w_d^*$ is allocated to buying indivisibilities.

Proof of Proposition 5

$a^*(s)$ denotes the optimal investment in indivisible consumption in state s. Given that all wealth is consumed in the optimal allocation, it follows that the optimal level of wealth allocated to divisible consumption in state s is given by $w_d^*(s) = w - \sum_{i \in I(s)} a_i^*(s) c_i$. Given that the marginal utility of wealth $U_s'(w)$ in a state - if defined - is identical to $u_d'(w_d^*(s))$ it follows that the marginal utility of wealth is identical across states if and only if the total amount of wealth invested in indivisible consumption, $\sum_{i \in I(s)} a_i^*(s) c_i$, is identical across states.

Proof of Proposition 6

By definition of w_i^I, the value of paying for i does not suffice to compensate for the disutility of paying $p_s c_i$ at a wealth level $w < w_i^I$ in all states. Hence, the insurance value is negative.

For any $w \in (w_i^I, w_i)$, the insurance value is given by

$$IV = p_s v_i - [u_d(w) - u_d(w - p_s c_i)]. \quad (21)$$

Taking the derivative w.r.t. w yields

$$\frac{d\, IV}{d\, w} = u_d'(w - p_s c_i) - u_d'(w). \quad (22)$$

The derivative is strictly positive as $u_d'' < 0$.

For any $w > w_i$, the insurance value is given by

$$IV = p_s \left[u_d(w) - u_d(w-c)\right] - \left[u_d(w) - u_d(w - p_s c_i)\right]. \tag{23}$$

Taking the derivative w.r.t. w yields

$$\frac{d\, IV}{d\, w} = u_d'(w - p_s c_i) - p_s u_d'(w - c_i) - (1-p_s)u_d'(w). \tag{24}$$

Given that $p_s(w-c) + (1-p_s)w = w - p_s c_i$, then this derivative is strictly positive/negative, if u_d' is a strictly concave (convex) function of w.

Proof of Proposition 7

Given $w < c_i$, such that there is an access value (AC), it exceeds the consumption-smoothing value (CSV) of insurance if and only if

$$p_s v_i - p_s u_d(w) - (1-p_s)\left[u_d(w) - u_d(w-x)\right] > u_d(w - p_s c_i) - (1-p_s)u_d(w-x) \tag{25}$$

$$\Leftrightarrow p_s(v_i - u_d(c_i)) > \left[u_d(w) - (1-p_s)u_d(w-x) - p_s u_d(c)\right] + \left[u_d(w - p_s c_i) - (1-p_s)u_d(w-x)\right]. \tag{26}$$

Define

$$\phi(p_s) := \left[u_d(w) - (1-p_s)u_d(w-x) - p_s u_d(c)\right], \tag{27}$$

$$\eta(p_s) := \left[u_d(w - p_s c_i) - (1-p_s)u_d(w-x)\right] \tag{28}$$

to rewrite above inequality as

$$v_i - u_d(c_i) > \frac{\phi + \eta}{p_s} \tag{29}$$

and note that $\phi(p_s)$ and $\eta(p_s)$ have the property $\lim_{p_s \to 0} \phi = \lim_{p_s \to 0} \eta = 0$. We can thus employ l'Hospital's rule to determine

$$\lim_{p_s \to 0} \frac{\phi + \eta}{p_s} = \frac{\partial \phi}{\partial p_s}(0) + \frac{\partial \eta}{\partial p_s}(0). \tag{30}$$

Given that $x = \frac{p_s}{1-p_s}(c-w)$, the two derivatives are

$$\frac{\partial \phi}{\partial p_s} = u_d(w-x) - u_d(c) + \frac{1}{1-p_s} u_d'(w-x)(c-w), \tag{31}$$

$$\frac{\partial \eta}{\partial p_s} = u_d(w-x) - u_d'(w-p_s c)c + \frac{1}{1-p_s} u_d'(w-x)(c-w) \tag{32}$$

Evaluating both at $p_s = 0$ yields

$$\frac{\partial \phi}{\partial p_s}(0) = u_d(w) + u_d'(w)(c-w) - u_d(c), \tag{33}$$

$$\frac{\partial \eta}{\partial p_s}(0) = u_d(w) - u_d'(w)w. \tag{34}$$

Thus, for $p_s \to 0$, the AC dominates the CSV if and only if

$$v_i - u_d(c_i) > [u_d(w) + u_d'(w)(c-w) - u_d(c)] + [u_d(w) - u_d'(w)w]\,. \tag{35}$$

Define the RHS as $z(w) := \frac{\partial \phi}{\partial p_s}(0) + \frac{\partial \eta}{\partial p_s}(0)$ and note that

$$\frac{\partial z}{\partial w} = u_d''(w)(c-2w) \tag{36}$$

which is negative for $w < c/2$ and positive for $w > c/2$ since $u_d'' < 0$. Hence, the function $z(w)$ has a minimum at $w = c/2$. $z(c) = u_d(c) - u_d'(c)c$ and $\lim_{w \to 0} z(w) \to \infty$ if $\lim_{w \to 0} u_d'(w) \to \infty$. Given that, it follows that if $v_i - u_d(c_i) > z(c)$ there must exist a unique wealth level w_a with $0 < w_a < c/2$ such that $v_i - u_d(c_i) > z(w)$ for all $w \in (w_a, c)$.

In addition, if $v - u_d(c_i) < z(\frac{c}{2}) = \min_{w \in [0,c]} z(w) = 2u_d(\frac{c}{2}) - u_d(c)$, then the CSV dominates the AV for all $w \in (0, c)$.