Loss Aversion, Probability Weighting, and the Demand for Insurance

Abstract

We examine the demand for insurance within the framework of prospect theory preferences to determine whether such preferences can explain the choice of low deductibles observed in the market. Prospect theory implies individuals make decisions by evaluating gains and losses relative to a reference point, where utility is concave over gains and convex over losses; furthermore, losses are weighed more heavily than gains in this setting. We incorporate such preferences in the utility function for an individual and investigate various reference points for an individual making insurance purchasing decisions. We find that insurance purchase decisions made within the context of prospect theory can explain several phenomena observed in insurance markets: the preference for low deductibles for mandatory insurance, the lack of demand for non-mandatory insurance like catastrophe insurance, and the over-demand to insure small losses as seen with the purchasing of warranties.

1 Introduction

The preference for low deductibles is well established in the insurance literature (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993). Yet, this preference cannot be explained by risk aversion alone. It has been suggested that this preference could be caused by prospect theory type preferences (Koszegi Rabin 2006, 2007, 2009), and in this paper, we formally examine whether preferences in alignment with prospect theory can explain this phenomena.[1] We do so by considering how preferences described by prospect theory can impact the demand for insurance including both the decision to insure and the deductible level chosen. We then compare this deductible level to that chosen by an individual without prospect theory type preferences to show how components of prospect theory (loss aversion and probability distortions) can explain the preference for low deductibles.

Cumulative prospect theory, developed by Kahneman and Tversky (1979, 1992) implies individuals make decisions by evaluating gains and losses relative to a reference point rather than evaluating expected final wealth. Prospect theory shows people process these gains/losses using a value function that is concave for gains and convex for losses. This S-shaped value function captures individuals' risk-aversion over gains and risk-seeking behavior over losses. Furthermore, people with prospect theory preferences are willing to take on additional risk in order to avoid feeling a loss. This feature implies individuals weigh losses more heavily than gains, and this aspect of prospect theory that has been termed "loss aversion." Finally, prospect theory preferences use a weighting function that overweights small probabilities since individuals have been shown to be more sensitive to small gains/losses relative to larger ones.

In this paper, we investigate the impact of these components from prospect theory (loss aversion and probability distortions) on the demand for insurance to see if it can explain the deductible levels observed, especially the preference for low deductibles. Preferences consistent with prospect theory induce individuals to take actions to avoid losses and maximize gains. Our intuition is that people will therefore make insurance decisions in order to minimize the domain where a loss is experienced and maximize the domain where a gain is experienced. In this vein, individuals will choose their insurance coverage so as to minimize the experience of a loss should one occur. That is, preferences described by prospect theory may cause individuals to have a preference for full insurance (or low deductibles).

Previous work that has suggested prospect theory might lead to low deductibles has mostly considered the Koszegi Rabin (2006, 2007, 2009) (KR) framework for prospect theory. The KR framework allows for endogenous reference points; yet applications of this framework to an insurance setting require the elimination of diminishing marginal utility within the framework and assume a linear utility function instead. Braseghyan et al. (2011) implements the KR framework to empirically examine insurance decisions for moderate-stake risks and finds that probability distortions are an important factor in the preference for low deductibles; here probability distortions are with regard to the overweighting of claims probabilities. In the estimation the authors are unable to distinguish between loss aversion and probability weighting, and they do not examine the decision of whether to insure or not, but instead focus on the deductible chosen assuming

[1] We will examine several cases, one of which conforms to the conditions of cumulative prospect theory as described by Kahneman and Tversky (1979, 1992). While our other cases do not precisely conform to the axioms set forth under cumulative prospect theory, they are consistent with loss aversion and probability distortions.

insurance will be purchased. The authors also assume the loss is always greater than the deductible chosen; therefore they do not necessarily capture how prospect theory preferences may cause the choice for low deductibles initially. Using deductible choices from homeowners' data, Sydnor (2010) finds high risk aversion over modest stakes and alludes that prospect theory could potentially be the cause of this in the discussion; again, the assumption that the loss experienced is always greater than deductible chosen is made. Furthermore, both Braseghyan et al. (2011) and Sydnor (2010) make assumptions about the loss distribution.

In what follows, we follow the KT approach to model prospect theory which allows for diminishing marginal utility. Eliminating diminishing marginal utility for a framework that examines insurance decisions seems counter-intuitive. Schmidt (2011) utilizes the KT method for prospect theory to investigate insurance demand, but implements a two state model (either there is a loss or no loss) and assumes individuals benchmark to full insurance; furthermore, the decision analyzed is whether to insure or not insure, not how much insurance to purchase. With this framework, Schmidt (2011), finds prospect theory can explain some of the observed phenomena. Our model accommodates any continuous loss distribution though and makes no assumption about the size of the loss relative to the deductible chosen. We consider several benchmarks from which individuals define gains and losses which allows us to distinguish how prospect theory type preferences impact deductible choices when it is assumed insurance will be purchased and when this assumption is not made. That is, we investigate what deductible is chosen when it is known insurance will be purchased, and we also examine situations where individuals decide whether or not to buy insurance altogether. We find that benchmarks that seem appropriate for insurance that is mandatory will cause those with prospect theory-like preferences to choose low deductibles. Preferences consistent with prospect theory with reference points that seem relevant for non-mandatory insurance can explain the lack of demand for insurance against small probability, high loss events (catastrophe insurance) while also explaining the over-demand to insure small losses (warranties). Initially we examine the loss aversion component of prospect theory and then we incorporate the probability weighting function so as to see how each aspect of prospect theory impacts our results.

In the next section we discuss the previous literature and the two methods for modeling prospect theory (Koszegi-Rabin and Kahneman-Tversky). In section 3, we examine how prospect theory impacts insurance decisions using the Kahneman Tversky framework and investigate how various benchmarks from which individuals can evaluate gains and losses impact insurance decisions. Finally in Section 4, we conclude.

2 Prospect Theory Preferences and Previous Literature

Prospect theory was first documented by Kahneman and Tversky (1979) and later examined and quantified further by Tversky and Kahneman (1981, 1992). It implies individuals make decisions by evaluating gains and losses relative to a reference point. The value function to evaluate gains and losses is concave over gains and convex over losses. Furthermore, losses are weighed more heavily than gains. Also probabilities are weighted unevenly with low probabilities being overweighted and moderate/high probabilities being underweighted. Incorporating prospect theory preferences in a model involves the following:

1. Defining a reference point from which to evaluate gains and losses;

2. Invoking a value function that is S-shaped to capture concavity for gains and convexity over losses and that small gains/losses are weighed more heavily than large gains/losses; and

3. Invoking a probability weighting function to capture the overweighting of low probabilities and underweighting of moderate and high probabilities.

Tversky and Kahneman (1992) experimentally tested their idea of cumulative prospect theory and proposed the following functional form for the value function with which to evaluate gains and losses:

$$u(w) = \left\{ \begin{array}{c} w^a \text{ if } w \geq 0 \\ -\lambda \left(-w\right)^{\beta} \text{ if } w < 0 \end{array} \right\}. \tag{1}$$

where w is an outcome, either positive or negative. The authors found that $a = \beta = 0.88$ and $\lambda = 2.25$. Furthermore, they suggest the following weighting functions to capture the overweighting of small probabilities and underweighting of moderate and high probabilities:

$$\begin{aligned} w^{+}(p) &= \frac{p^{\theta}}{\left(p^{\theta} + (1-p)^{\theta}\right)^{1/\theta}} \\ w^{-}(p) &= \frac{p^{\delta}}{\left(p^{\delta} + (1-p)^{\delta}\right)^{1/\delta}} \end{aligned} \tag{2}$$

where p is probability of the outcome and w^{+} and w^{-} are the decision weights on positive outcomes and negative outcomes respectively. Tversky and Kahneman (1992) find that $\theta = 0.61$ and $\delta = 0.69$. The value function given in (1) and weighting function shown in (2) describe the KT framework for prospect theory. It is this framework which we will utilize to consider insurance demand. That is, we will use the utility function and weighting functions as given in (1) and (2) which capture loss aversion and probability distortions. In what follows, we denote individuals who make decisions using this utility function and weighting functions as "prospect theory" individuals.

More recently both theoretical and experimental work has been done with prospect theory. Using experimental data and a mixture model, Harrison and Rutstrom (2009) investigate how both expected utility theory (EUT) and prospect theory (PT) influence the observed choices made by subjects. They find support for both theories and are able to address the demographic domains where one theory is more dominant than the other. Furthermore they derive additional estimates of KT's parameters for both the value function and probability weighting functions; their work supports the equivalency of weights on gains and losses, i.e. that $\theta = \delta$. Wu and Markle (2008) investigate the separability of gains and losses within the prospect theory framework and find that individuals are less sensitive to probability differences for choices involving mixed gambles versus those gambles that involve only gains or only losses.

Theoretically, several extensions of KT's cumulative prospect theory have been made. Bleichrodt et al. (2009) show how the KT framework can accommodate having different reference points for various attributes within a decision, and Schmidt et al. (2008) show how the KT framework can be adapted for state dependent reference points. This latter work can explain the disparity between an individual's willingness to pay and willingness to accept for a lottery. Also, Schmidt et al. (2009) develop an axiomatic foundation for a linear form of cumulative prospect theory. Schmidt and Zank (2008) specify the conditions needed for strong risk aversion under cumulative prospect theory, and they find that concave utility is not necessary. They also develop an index of probabilistic loss aversion which equals the ratio of gain to loss decision weights.

Previous work in the finance area to examine the effect of prospect theory preferences on financial markets has relied on the framework initially derived by Tversky and Kahneman (1981, 1992) by utilizing the KT framework described here. With this approach, prospect theory has been shown to explain the disposition effect (Barberis and Xiong, 2009; Henderson, 2012), the high mean, excess volatility, and predictability of stock returns (Barberis et al., 2001), and the pricing of a security's own skewness (Barberis and Huang, 2008). Furthermore it has been implemented in the pricing of financial derivatives (Pena et al., 2010; Polkovnichenko and Zhao, 2009; Versluis et al., 2010).

Recently, Koszegi and Rabin (2006) have incorporated the concept of prospect theory preferences into a general model of dependent preferences although they do not use the KT framework shown here. Their model evaluates absolute consumption and then has a second attribute which evaluates gains/losses which allows for an endogenous reference point equal to one's recent rational expectations about outcomes. They investigate how such preferences impact willingness to pay for a good (2006), preferences over monetary risk (2007), and intertemporal consumption decisions (2009). The Koszegi-Rabin framework has been implemented by Braseghyan et al. (2011) to examine risk preferences calculated from auto and home insurance deductible choices. Sydnor (2010) uses deductible choices from home insurance to discuss the over-insurance of modest risks and discusses how his work could be adapted to the KR framework.

The KR framework does allow for intertemporal consumption and endogenous reference points. The application of the KR framework to an insurance setting necessitates the loss of curvature of the utility function and therefore eliminates diminishing marginal utility (Sydnor, 2010). Furthermore, implementation of the KR framework for insurance decisions (Braseghyan et al., 2011; Sydnor, 2010) assumes the loss realized is always greater than deductible chosen. This somewhat eliminates the ability to capture the possibility that prospect theory is causing the choice for a low deductible since the loss incurred is always greater than the deductible. These papers calculate the impact of prospect theory from data on deductible choices and assume specific loss distributions.

Furthermore, Barseghyan et al. (2011) do not examine the decision of whether to insure or not. They assume insurance is bought and examine how prospect theory influences the deductible level chosen after the initial purchasing decision is made. In this way, the authors' model does not apply to all types of insurance. It is not clear that the calculations would be similar if it included catastrophe insurance and/or warranties which are not mandatory types of insurance. Their model is applicable for the data they used for calibration which includes moderate stake risks. Implementation of the KR framework in Barseghyan et al. (2011) also assumes a loss if felt whenever a claim is made and gain is felt whenever one does not make a claim; this method may not truly depict the way individuals evaluate and/or view gains and losses. Even if a claim is not made, if a loss is realized individuals might feel a loss. If individuals benchmark gains and losses relative to their wealth then Barseghyan et al. (2011) does not accurately capture the individual's decision making process.

In their explanation for under-annuitization, Brown et al. (2008) find evidence that individuals tend to view insurance as an investment and simply evaluate the expected gains and/or losses associated when deciding whether to buy a policy. In this manner they do not evaluate absolute consumption and then have a second attribute to evaluate the gains and/or losses relative to a reference point, which is what the KR framework assumes. A model that only evaluates gains/losses as the KT framework does might be more appropriate for an insurance setting then.

In what follows, we implement the KT framework shown above, which has been utilized in the finance area, to examine the impact of prospect theory on insurance decisions. This model maintains diminishing marginal utility in the value function and is general enough to accommodate any continuous loss distribution. Also we do not make an assumption about the loss size relative to the deductible chosen which allows us to better examine the impact of prospect theory preferences on the deducible choice. Furthermore we utilize a reference point which is relevant for when it is not assumed insurance will be purchased. In this way we are able to see how prospect theory influences insurance demand for smaller types of insurance, such as warranties, and also larger types of insurance, such as catastrophe insurance. The manner in which we define gains/losses represents more "status quo loss aversion" rather than the definition for gains and losses used by Barseghyan et al. (2011). We implement several benchmarks to define gains and losses but define them relative to what an individual's previous "status quo" is; this definition is more aligned with that of KT.

A growing literature examines how differences in preferences might affect insurance decisions. Most previous work which considers how psychology impacts insurance decisions focuses on regret and/or disappointment. Braun and Muermann (2004) show how regret impacts the demand for insurance, Muermann et al. (2006) examine how regret impacts portfolio choice for defined contribution pension plans, and Huang et al. (2008) analyze how regret can impact an equilibrium insurance setting. Similarly, Gollier and Muermann (2010) consider a decision-making model where individuals have beliefs which include ex-ante optimism and ex-post disappointment to explain the preference for low deductibles. Shapira and Venezia (2008) show experimentally that individuals are subject to an anchoring effect and anchor their preferences on the prices associated with full insurance policies. As a result, they undervalue partial insurance policies (policies with deductibles) causing them to prefer full insurance and low deductibles. Wakker et al. (2007) find experimentally that individuals' willingness to take insurance is consistent with prospect theory preferences which further motivates the model put forth in this paper. We show how prospect theory type preferences can lead to low deductibles for insurance that is seen as mandatory; on the other hand it can also explain the lack of demand to insure small probability, high loss events and the over-demand to insure small losses.

3 Impact of Prospect Theory on the Demand for Insurance

Suppose an individual is endowed with initial wealth $w_0 \geq 0$ and faces a monetary loss L which is described by cumulative distribution function $F(L)$ with $F(0) = 0$. We denote $f(L)$ as the probability density function for the loss distribution. An insurance company offers indemnity contracts with premiums equal to the expected indemnity plus a proportional loading factor, $\gamma \geq 0$. This assumption is consistent with a riskneutral insurer in a perfectly competitive insurance market with transactional costs but no entry costs. Our setting does not include any information asymmetries that would lead to moral hazard or adverse selection problems.

An insurer offers a set of deductible contracts with deductible levels $D \in [0, w_0]$. The indemnity schedule is therefore

$$I(L) = \max(L - D, 0) = (L - D)^+$$

and for a given deductible, the premium is given by

$$\begin{aligned} P(D) &= (1+\gamma) E[I(L)] \\ &= (1+\gamma) E\left[(L-D)^+\right]. \end{aligned}$$

The individual chooses a deductible level, D, to maximize expected utility of final wealth. We assume individual's preferences can be represented by the KT value function given in (1). For a given benchmark from which to define gains and losses, people choose the level of insurance that maximizes their expected utility; that is they maximize the overall value of gains and losses. We initially focus on how the value function associated with prospect theory impacts insurance decisions. We will then incorporate the probability weighting function described by KT to determine how that feature further effects insurance demand.

The probability weighting functions proposed by KT and given in (2) can be used for discrete probabilities. To utilize these weightings for a continuous probability distribution, a modification is needed as shown in Barberis and Huang (2008). Since we assume a continuous loss distribution to analyze the optimal demand for insurance, we implement these modified weighting functions. That is, the goal utility function for an individual is given by

$$v(W) = u\left(W^-\right) + u\left(W^+\right)$$

where W^- are losses and W^+ are gains and $u()$ is given in (1). As shown in Barberis and Huang (2008), maximizing $v(W)$ with cumulative prospect theory preferences as described in (1) and (2) is equivalent to

$$v(W) = \int u\left(W^-\right) dw^-(F(L)) - \int u\left(W^+\right) dw^+(1 - F(L)) \tag{3}$$

where $w^-(F(L))$ is the weighting function for losses from KT, $w^+(1 - F(L))$ is the weighting function for gains from KT, and $F(L)$ is the cumulative distribution function of L. This equation merely allows for continuous probability distributions. Yet, solving such an equation is a challenge. Therefore with several, reasonable, assumptions a simplification can be made. Barberis and Huang (2008) show that if one uses the KT value function and probability weighting functions as given in (1) and (2), if the expected loss and variance of the loss are less than infinity, the probability weighting functions for both losses and gains are the same (making $\theta = \delta$ in equation (2) and therefore $w^- = w^+ = w$) , and $a < 2\delta$, then (3) is equivalent to the following:

$$v(W) = -\int w(F(L))\, du\left(W^-\right) + \int w(1 - F(L))\, du\left(W^+\right). \tag{4}$$

The last assumption that $a < 2\delta$ is not needed if the loss distribution is assumed to be Normal or Lognormal. Therefore, for a given benchmark from which to define gains and losses, when the probability weighting component of prospect theory is incorporated into the model, individuals will choose the level of insurance which maximizes their expected utility as given in (4).[2]

In what follows, we first note the optimal demand for insurance for an individual not subject to prospect theory preferences. Then we determine the optimal demand for insurance for individuals with preferences consistent with prospect theory as described above using three different benchmarks. The first benchmark captures both the decision to buy insurance and how much insurance to buy. Here, individuals define gains and losses relative to initial wealth. This benchmark is consistent with the axioms that define prospect theory. This benchmark allows for an examination of prospect theory as described by Kahneman and Tversky in the context of insurance purchases. The second benchmark captures an insurance setting where individuals know they will buy insurance, but have to determine how much insurance to purchase. For this setting, individuals define gains and losses relative to initial wealth minus the premium as they assume the premium will be paid and do not factor that into any feeling of a loss. Though this benchmark does not precisely fit the axioms set forth by prospect theory, it is consistent with decision making with preferences consistent with prospect theory. Finally the third benchmark is an extension of the second benchmark but is state dependent. It is possible that people determine ex-post feelings of losses and gains depending on the outcome that occurred. That is, they implement different benchmarks for different states of the world. For each benchmark we determine the optimal deductible level when individuals maximize expected utility of gains and losses. We denote D^* as the optimal deductible for an individual with prospect theory preferences.

Expected Utility Theory Individual In addition to understanding how prospect theory influences insurance demand we would like to compare our results with deductibles optimally chosen by an individual not subject to prospect theory, but rather makes choices based on Expected Utility Theory (EUT). An individual that does not have prospect theory preferences does not evaluate gains and losses. They choose the optimal deductible by maximizing expected utility of final wealth where final wealth is given by

$$\begin{aligned} W(D) &= w_0 - P(D) - L + (L - D)^+ \\ &= w_0 - P(D) - \min(L, D). \end{aligned}$$

We denote D_0^* as the optimal deductible for an individual that has EUT preferences. For an increasing, concave utility function, as shown in Mossin (1968), a fully rational, risk-averse individual will buy full insurance ($D_0^* = 0$) if the contract is fairly priced. Partial insurance will be chosen if there is a loading factor ($\gamma > 0$), moral hazard (Holmstrom, 1979), or adverse selection (Rothchild and Stiglitz, 1976).

In order to make the utility for an EUT maximizing individual more comparable to that of a prospect theory individual, we assume they have utility that is increasing and concave and specify it by the following function:

$$u(w) = w^a. \tag{5}$$

[2]Previous work supports the assumption that probability weighting functions for gains and losses are equal ($\theta = \delta$); see, for instance, Harrison and Rutstrom (2009).

The parameter, a, is the same as that from the KT value function. Note that this formula is similar to that for CRRA utility. Additionally, in order for the utility function specified above to be concave, it is necessary that $a < 1$. This condition is still consistent with KT as they find $a = 0.88$ (Tversky and Kahneman, 1992). Using the utility given in (5) we find that EUT individuals will buy full insurance ($D_0^* = 0$) if the contract is fairly priced ($\gamma = 0$) and will demand partial insurance ($D_0^* > 0$) if there is a positive loading factor ($\gamma > 0$). Please see Appendix A.1 for details.

3.1 Benchmark to Initial Wealth

The first benchmark we consider utilizes a reference point equal to initial wealth.[3] This benchmark would be applicable for non-mandatory insurance where the individual would potentially not want to buy the insurance altogether and needs to evaluate whether paying the premium is worthwhile. With this reference point, the gain experienced would need to be enough so that insurance "pays off"; that is, the gain felt needs to offset the premium paid and the loss or deductible, should a loss occur. By benchmarking to initial wealth, the individual needs to decide whether it is "worth it" to purchase insurance initially; if so, then he needs to decide how much insurance to buy.

In this setting, gains and losses are defined as follows. If no loss occurs, final wealth equals initial wealth minus the premium $(w_0 - P(D))$ and the individual feels a loss equal to the premium paid since the insurance did not "pay off." If a loss occurs but it is lower than the deductible level, then the individual feels a loss of both the premium and the loss that occurred (final wealth would be $w_0 - P(D) - L$ in this case). If the loss incurred is greater than the deductible then final wealth is given by: $w_0 - P(D) - D$. If the amount of the loss above the deductible is less than the premium paid ($L - D < P(D)$), then the individual will still feel the insurance didn't "pay off" and will feel a loss equal to the premium plus deductible minus the loss (i.e. loss of $P(D) + D - L$). To understand this situation better consider the following example. Suppose the premium paid is \$500, the deductible is \$1000 and the loss incurred is \$1100. On net, the individual paid \$1500 (premium plus deductible). If the individual had not bought insurance, he would have incurred the \$1100 loss. In this way, the loss felt is the difference between being insured and not (-\$1500 vs. -\$1100); the loss felt is \$400 which is equal to the premium plus deductible minus the loss (\$1000 + \$500 - \$1100 = \$400). If the loss is greater than the deductible chosen and the amount of the loss above the deductible is greater than the premium paid ($L - D > P(D)$) then the individual feels a gain equal to how much the loss offset the costs of buying insurance; that is a gain of $L - (P(D) + D)$ is felt.[4]

Anticipating these gains and losses, an individual chooses the deductible to maximize his expected utility

[3]Here, initial wealth does not depend on our choice variable, and is indeed exogenously given. In this way, this benchmark conforms to prospect theory axioms.

[4]*We note that the insured does not actually obtain a monetary gain. Rather, the insured feels as if the insurance "paid off" when the loss offsets both the deductible and premium.*

of gains and losses. The maximization problem is as follows:

$$\max_{D\in[0,w_0]}\left[\begin{array}{c}\int_0^D u(-P(D)-L)dF(L)+\int_D^{D+P(D)} u\left(-P\left(D\right)-D+L\right)dF\left(L\right)\\ +\int_{D+P(D)}^{\infty} u\left(L-P\left(D\right)-D\right)dF\left(L\right)\end{array}\right]$$

which substituting in the utility from KT as given in (1) we can write as

$$\max_{D\in[0,w_0]}\left[\begin{array}{c}\int_0^D -\lambda(P(D)+L)^{\beta}dF(L)+\int_D^{D+P(D)} -\lambda\left(P\left(D\right)+D-L\right)^{\beta}dF\left(L\right)\\ +\int_{D+P(D)}^{\infty} \left(L-P\left(D\right)-D\right)^{a}dF\left(L\right)\end{array}\right].$$

In the next proposition we show that for actuarially fair premiums, individuals with prospect theory like preferences using initial wealth as a benchmark demand full insurance. When premiums are actuarially unfair, partial insurance is demanded. However, for loss distributions that are weighted heavily toward higher losses, individuals with prospect theory type preferences will choose a lower deductible than EUT individuals. For loss distributions that weighted toward lower losses, prospect theory individuals choose a higher deductible.

Proposition 1 *If a prospect theory individual uses initial wealth as his reference point, he will demand full insurance if premiums are actuarially fair and partial insurance if premiums are actuarially unfair. That is, $D^*=0$ if $\gamma=0$ and $D^*>0$ if $\gamma>0$. Furthermore, if the following holds and $\lambda>1$, then prospect theory individuals will demand less insurance than an EUT individual ($D^*>D_0^*$):*

$$\begin{aligned}&\int_0^{D_0^*}\beta(P(D_0^*)+L)^{\beta-1}f\left(L\right)dL \qquad (6)\\ >&\int_{D_0^*}^{D_0^*+P\left(D_0^*\right)}\beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1}f\left(L\right)dL+\int_{D_0^*+P\left(D_0^*\right)}^{\infty} a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1}f\left(L\right)dL.\end{aligned}$$

Proof. See Appendix A.2. ■

When the reference point is given as initial wealth, insurance is seen more as an investment. A gain is only felt if the loss is greater than both the premium and the deductible. In this scenario, the result is not as straightforward as will be seen in later benchmarks. Overall, individuals with prospect theory preferences will demand full insurance if prices are actuarially fair, just as EUT individuals do. Yet it's possible there are situations where a prospect theory individual will demand more or less insurance than an EUT individual. Prospect theory individuals by definition have $\lambda > 1$ and condition (6) holds when the probability distribution for losses is skewed left. Therefore, for small losses that occur with a high probability prospect theory individuals will be less likely than EUT individuals to buy insurance and will optimally choose a higher deductible. In this instance, the chance that a gain will be felt is small and hence less insurance is purchased. Catastrophic events have small probabilities associated with high loss sizes and therefore may fall into this instance. In this way, prospect theory may explain the lack of demand for catastrophe insurance. For loss distributions that are not skewed left though, there is a greater chance a gain will be felt, and therefore prospect theory can lead to a preference for more insurance and lower deductibles. Individuals may feel that warranties are more likely to "pay off" and therefore are more willing to buy this type of insurance (at an unfair rate).

We also consider variations in the gain/loss definition by considering only monetary gains/losses. That is, when benchmarking to initial wealth, and the loss is less than the deductible, individuals feel a loss equal to the actual loss and the premium paid. When the loss realized is above the deductible, they feel a loss of the deductible and premium. Again, a loss is always realized but there is trade-off between choosing a lower deductible (which will decrease the first part of any loss felt) and the increased premium. Analysis for this definition of gains/losses relative to initial wealth are in Appendix A.5, and results are consistent with the gain/loss definition shown above for this benchmark.

3.2 Benchmark to Initial Wealth Minus Premium

The second benchmark we consider is a reference point equal to initial wealth minus the premium.[5] That is, the decision to buy insurance is already made and individuals assume the premium will be paid but have to decide the optimal deductible level. That is, individuals do not view the premium as a loss and instead think of their starting wealth as the level of wealth which already accounts for paying a premium. One can think of this setting as that associated with insurance that is mandatory, such as insurance demanded by a lien-holder (e.g. a mortgage, or an automobile loan). The decision to insure has already been made, but people need to decide "how much" insurance to purchase. This decision is influenced by evaluating the expected gains and or losses associated with buying insurance. That is, if they experience a monetary loss which is lower than the deductible chosen, individuals will *feel* a loss. When the monetary loss experienced is greater than the deductible chosen, individuals will *feel* a gain. [6] Our intuition is that individuals will consequently choose lower deductibles in order to minimize the situations in which they feel a loss and maximize situation in which they feel a gain.

The manner in which we define gains and losses for this reference point is as follows: suppose there is no loss. Final wealth equals initial wealth minus the premium $(w_0 - P(D))$ which implies that final wealth is equal to the individual's benchmark. Therefore the individual does not feel either a gain or a loss. When a loss occurs, the feeling of a gain or loss is dependent on whether the loss incurred is less than or

[5]We note here that if the premium is a function of the deductible chosen, then the benchmark is not exogenously given. That is, the choice of deductible can affect the premium. In this sense, the model presented below is not precisely prospect theory. This problem is easily solved by considering an "expected premium" paid by the insured that does not depend on the deductible given. Using this more technically correct reference point would yield the exact same result presented below.

[6]We note again that the insured does not actually obtain a monetary gain. Rather, the insured feels as if the insurance "paid off."

greater than the deductible chosen. If the loss is less than the deductible chosen, then final wealth equals $(w_0 - P(D) - L)$. If individuals benchmark to initial wealth minus the premium, they feel a loss equal to the loss incurred, i.e. $-L$. If the loss incurred is greater than the deductible chosen final wealth equals initial wealth minus the premium and deductible ($w_0 - P(D) - D$). In this case, the individual gains the difference between the loss incurred and the deductible paid; that is, a gain of $(L - D)$ is experienced.

Anticipating these gains and losses, an individual chooses the deductible to maximize his expected utility of gain and losses. The maximization problem is as follows:

$$\max_{D \in [0, w_0]} \left[\int_0^D u(-L)\, dF(L) + \int_D^\infty u(L - D)\, dF(L) \right]$$

which can be reduced to

$$\max_{D \in [0, w_0]} \left[\int_0^D -\lambda (L)^\beta\, dF(L) + \int_D^\infty (L - D)^a\, dF(L) \right].$$

In the next proposition we show that when individuals do not factor the premium paid into their evaluation of gains and losses, they will always choose full insurance (zero deductible).

Proposition 2 *If a prospect theory individual uses initial wealth minus the premium as his reference point, he will demand full insurance. That is, $D^* = 0$ for all γ.*

Proof. See Appendix A.3. ■

When an individual with prospect theory preferences uses initial wealth minus the premium as his reference point, full insurance is always optimal, even at unfair prices ($\gamma > 0$). If full insurance is not offered, then within the contracts offered by the insurer, the individual will choose the policy that provides the most coverage. When insurance is mandatory it is possible that people might "write off" the premium knowing that it will have to be paid. Therefore, gains and losses will be evaluated relative to initial wealth minus the premium paid. If a loss occurs and the loss size is less than the deductible, the individual will feel a loss. However, if the loss size is greater than the deductible chosen a gain will be felt. In order to maximize the number of situations in which a gain is experienced, the individual will choose the lowest deductible available. In the next proposition, we show that an individual with prospect theory preferences will optimally always demand more insurance than an EUT individual.

Proposition 3 *If a prospect theory individual uses initial wealth minus the premium as his reference point, he will demand a higher level of insurance than an EUT individual (i.e. $D^* < D_0^*$) for all γ.*

Proof. See Appendix A.4. ■

Without prospect theory preferences, individuals demand full insurance for actuarially fair premiums ($\gamma = 0$) and partial insurance for unfair premiums ($\gamma > 0$) (Mossin, 1968). With prospect theory preferences, if individuals use initial wealth minus the premium as a benchmark to evaluate gains and losses, an insured only experiences a gain when the loss incurred is greater than the deductible chosen. Consequently a prospect theory individual will demand full insurance ($D^* = 0$) even for unfair premiums to maximize feeling a gain. Both results above demonstrate that individuals with prospect theory type preferences optimally select the lowest deductible available when using initial wealth minus the premium as their reference point. This benchmark seems applicable to mandatory insurance as insureds know the premium will need to paid. Therefore, the results here can help explain the preference for low deductibles that has been documented in the literature both empirically for auto insurance and also through experiments (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993).

Note that in practice zero deductibles are not usually offered by insurers. The analysis above does not include transactions costs which would potentially cause the optimal deductible to be greater than zero. Yet, relative to each other, a prospect theory individual will still choose a lower deductible than that chosen by an EUT individual, even if there were transaction costs.

Furthermore, in the manner in which we have defined the benchmark above, the gain is somewhat of a "mental" gain rather than a monetary one. If individuals benchmark to initial wealth minus the premium but evaluate the monetary value of an outcome relative to this benchmark, then they will feel a loss of L if the loss is less than the deductible and a loss of D if the loss is greater than the deductible chosen. In this case, a loss is always felt. In order to maximize utility by minimizing losses, the lowest deductible will be chosen. That is, if individuals only evaluate the monetary outcome relative to the reference point of initial wealth minus the premium, they will demand full insurance for all loading factors.

In a similar way, individuals may experience both the monetary and mental loss/gain relative to the reference point. More specifically, if the loss is less than the deductible, the individual will feel a loss of L, and if the loss is greater than the deductible they feel both a loss of the deductible, D, but also a gain equal to the amount of the loss that is greater than the deductible ($L - D$). In this instance, the individual feels a loss unless the loss is greater than the deductible, at which point he feels a gain. We find that if individuals define gains/losses relative to the benchmark of initial wealth minus the premium in this manner, our results from before do not change: they will choose a zero deductible for all loading factors. In order to minimize losses and maximize gains they choose the lowest deductible possible. For further details on these additional ways to define gains/losses with a benchmark of initial wealth minus the premium, please see Appendix A.5.

3.3 State-Dependent Benchmark

The last benchmark is an extension of the prior benchmark where we consider a benchmark that depends on the state that occurs. In this scenario, we again assume the decision to buy insurance is already made and only the deductible level needs to be chosen. Individuals will assume the premium will be paid and do not consider this in their evaluation of gains and losses. They will benchmark their view of gains and losses relative to initial wealth minus the premium for small losses (losses less than the deductible). For large losses that exceed the deductible, however, it is possible insureds feel satisfied that they do not have to incur the loss. However, they did have to pay the deductible so they might not feel a gain either. That is, they feel neither a loss of the deductible nor a gain of how much the loss exceeded the deductible. In this setting if no loss occurs, final wealth equals initial wealth minus the premium and therefore neither a gain nor a loss is felt. If a loss occurs that is less than the deductible chosen, a loss equal to the size of the loss incurred will be felt. If the loss incurred is greater than the deductible, then the individuals feel neither a gain nor a loss since they feel the insurance was worth it.

For this setting, the individual chooses the deducible to maximize expected utility of gains and losses as given by:

$$\max_{D\in[0,w_0]}\left[\int_0^D u\left(-L\right)dF\left(L\right)\right]$$

which can be reduced to

$$\max_{D\in[0,w_0]}\left[\int_0^D -\lambda\left(L\right)^{\beta}dF\left(L\right)\right].$$

In the following proposition we show it is optimal for the prospect theory individual to choose full insurance.

Proposition 4 *If a prospect theory individual has a state-dependent reference point, he will demand full insurance. That is, $D^* = 0$ for all γ. This solution is a local maximum.*

Proof. See Appendix A.6. ■

When using initial wealth minus the premium as the benchmark for small losses only, a loss is felt as long as the loss incurred is less than the deductible. Once a loss is greater than the deductible, the individual feels the insurance was worth it and does not feel a loss. At the same time, if the loss exceeds the deductible, the individual does have to pay the deductible and so a gain is not felt either. In order to minimize the feeling of a loss, the individual chooses the lowest deductible available to maximize the chance that the loss will be greater than the deductible. In this way, prospect theory supports the preference for low deductibles that has been observed (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993).

3.4 Probability Weighting

Thus far our results have not included the decision weights as given by KT which capture people's increased sensitivity to low probability gains/losses relative to medium or large gains/losses. KT suggest probability weighting functions which capture the overweighting of small probabilities and underweighting of moderate and high probabilities which are given in equation (2). In this section we incorporate this feature of the model into our analysis to see how it further impacts our results.

As shown in Barberis and Huang (2008), maximizing utility with cumulative prospect theory preferences as described in (1) and (2) is equivalent to (3) which allows for continuous probability distributions. Barberis and Huang (2008) show that if one uses the KT value function and probability weighting functions as given in (1) and (2), if the expected loss and variance of the loss are less than infinity, the probability weighting functions for both losses and gains are the same (making $\theta = \delta$ in equation (2) and therefore $w^- = w^+ = w$) , and $a < 2\delta$, then (4) can be used to represent expected utility for an individual. The last assumption that $a < 2\delta$ is not needed if the loss distribution is assumed to be Normal or Lognormal. To use Barberis and Huang's method for implementing the probability weighting for continuous distributions, one needs to assume a specific loss distribution though. Loss distributions vary greatly by the type of loss being considered. Therefore, we implement discrete probabilities in our model rather than specifying a specific distribution to analyze how probability weighting impacts the demand for insurance when individuals have prospect theory preferences. In this section we consider the three benchmarks looked at previously.[7]

3.4.1 Benchmark to Initial Wealth

Suppose individuals do not assume insurance will be bought and do not ignore the premium when evaluating gains and losses. Instead, they benchmark the payoff to initial wealth. In order for a gain to be felt, the loss would have to be large enough to offset both the premium and the deductible. In this case a mental gain of $(L - P(D) - D)$ is felt. If the loss is less than the deductible a loss of $-L$ is felt. If the loss is greater than the deductible but not large enough to offset both the premium and the deductible a loss of $(P(D) + D - L)$ is felt. Let $p_1 = p(0 \leq L \leq D)$, $p_2 = p(D < L \leq D + P(D))$, and $p_3 = p(L > D + P(D))$. An individual's maximization problem is

$$\max_{D \in [0, w_0]} \left[\begin{array}{c} w^-(p_1)\left(-\lambda(P(D)+L)^\beta\right) + w^-(p_2)\left(-\lambda(P(D)+D-L)^\beta\right) \\ +w^+(p_3)(L-P(D)-D)^a \end{array} \right].$$

As we show in the next proposition, prospect theory individuals will choose partial insurance, even at actuarially fair prices.

Proposition 5 *If a prospect theory individual uses initial wealth as his reference point and preferences include KT's probability weights, he will demand partial insurance ($D^* > 0$ for all γ) if $\frac{\partial w^-(p_2)}{\partial D} < 0$ and $\frac{\partial w^+(p_3)}{\partial D} > 0$.*

Proof. See Appendix A.7. ■

This result is somewhat consistent with the results found previously when we did not incorporate probability weights. Previously we found that when individuals benchmark to their initial wealth, PT individuals will choose full insurance at actuarially fair prices and partial insurance at unfair prices. Yet for catastrophic losses, PT individuals will choose less insurance (higher deductibles) than EUT individuals. In what we find here, partial insurance is demanded, yet this result is dependent on how the probability distribution looks and also how premiums adjust with deductible levels. As deductible levels increases, if the probability of the loss is between the deductible level chosen and the deductible plus premium associated with that deductible level decreases, then $\frac{\partial w^-(p_2)}{\partial D} < 0$. This result is inconsistent with modest risks but would be appropriate for catastrophic risks. Similarly the requirement that $\frac{\partial w^+(p_3)}{\partial D} > 0$ is appropriate for catastrophic risks. In this way, we find that for catastrophic losses, partial insurance will be demanded, for all loading factors. An EUT individual would demand full insurance at actuarially fair prices, but a PT individual using initial

[7] *In this section we do not specify whether the weighting parameters, delta and gamma, are equal or not. Cumulative prospect theory allows these parameters to be different whereas original prospect theory assumes they are equal. All results below remain the same if one assumes delta equals gamma as in the original prospect theory framework.*

wealth as a benchmark would still demand partial insurance. It is less likely that the insurance will "pay off", offsetting both the premium and the deductible, and hence, less insurance is demanded.

3.4.2 Benchmark to Initial Wealth Minus Premium

From our second benchmark, individuals view insurance as something they know they will buy (or it is mandated) and they choose their level of coverage. For these types of insurance, individuals "write off" the premium and therefore benchmark their feelings of gains and losses without considering the premium they have already paid. That is, if the loss is less than the deductible, an insured will feel a loss equal to the loss that incurred, $-L$. If the loss is greater than the deductible chosen, the individual feels a "mental" gain of $(L - D)$. Let $p_{L<D} = prob\,(0 \leq L \leq D)$ be the probability the loss is less than the deductible and $p_{L>D} = prob\,(L > D)$ be the probability the loss is greater than the deductible. The individual's maximization problem incorporating probability weights is as follows:

$$\max_{D\in[0,w_0]} \left[w^{-}\left(p_{L<D}\right) u\left(-L\right) + w^{+}\left(p_{L>D}\right) u\left(L-D\right) \right].$$

With KT's value function the previous equation can be reduced to

$$\max_{D\in[0,w_0]} \left[w^{-}\left(p_{L<D}\right) \left(-\lambda L^{\beta}\right) + w^{+}\left(p_{L>D}\right) \left(L-D\right)^{a} \right].$$

In the next proposition we show that when individuals do not factor the premium paid into the evaluation of gains and losses, they will always choose full insurance (zero deductible).

Proposition 6 *If a prospect theory individual with $a < 1$ uses initial wealth minus the premium as his reference point and preferences include KT"s probability weights, he will demand full insurance. That is, $D^{*} = 0$ for all γ. This solution is a local maximum with weighting parameters $\theta \in (.5, 1)$ and $\delta \in (.5, 1)$and is a global maximum if $\frac{\partial^2 w^{-}(p_{L<D})}{\partial D^2} > 0$ and $\frac{\partial^2 w^{+}(p_{L>D})}{\partial D^2} < 0$.*

Proof. See Appendix A.8. ■

Incorporating probability weighting yields the same results we found earlier. When an individual does not consider the premium when evaluating gains and losses, he will choose full insurance, even for actuarially unfair prices. If full insurance is not offered, then within the contracts offered by the insurer, the individual will choose the policy that provides the most coverage. This result holds for weighting parameters which are consistent with what has been found in previous work. In this setting, a loss is felt if the loss incurred is less than the deductible chosen, but a gain is felt if the loss incurred is greater than the deductible chosen. Therefore, in order to minimize losses and maximize gains, the insured will choose the lowest deductible possible. This result is consistent with the preference for low deductibles as seen with auto insurance.

3.4.3 State Dependent Benchmark

For the last benchmark, gains and losses felt depend on the state that occurs. Individuals assume the premium will be paid and do not consider it when evaluating gains and losses. If the loss that occurs is less than the deductible, the individual feels a loss of $-L$, but if the loss is greater than the deductible they feel neither a loss nor a gain. Although the loss is greater than the deductible and insureds are satisfied that they do not have to incur the entire loss, they do have pay the deductible and hence, they do not feel a gain either. Let $p_{L<D} = prob\,(0 \leq L \leq D)$ be the probability the loss is less than the deductible and $p_{L>D} = prob\,(L > D)$ be the probability the loss is greater than the deductible. The individual's maximization problem incorporating probability weights and KT's value function is as follows:

$$\max_{D \in [0, w_0]} \left[w^- \left(p_{L<D} \right) \left(-\lambda L^\beta \right) \right].$$

As stated in the next proposition, we find it is optimal for the prospect theory individual to choose full insurance when using this reference point.

Proposition 7 *If a prospect theory individual has a state-dependent reference point and preferences include KT's probability weights, he will demand full insurance. That is $D^* = 0$ for all γ. This solution is a global max if $\frac{\partial^2 w^-(p_{L<D})}{\partial D^2} > 0$ which holds locally for $\delta \in (.5, 1)$.*

Proof. See Appendix A.9. ∎

When the loss is small (less than the deductible), a loss is felt, and when a loss is large the insured feels the insurance was worth it and does not feel a loss (of the deductible). Yet, in the latter situation, a gain is not felt either since the deductible has to be paid. Here, only a loss can be felt when the loss incurred is less than the deductible. In order to minimize losses felt, the insured chooses the lowest deductible possible. This result is consistent with what we found earlier prior to including probability weights.

3.5 Impact of PT on Willingness to Pay: lllustrative Example

Using deductible choices from homeowners' insurance data, Sydnor (2010) finds individuals are willing to pay (WTP) \$95 to reduce their deductible from \$1000 to \$500. The average claim rate in the data was 4% so in expectation the value of this extra coverage was worth less than \$25. Why was an individual's WTP so high? The author gives a variety of explanations for the results, one of which being that prospect theory could potentially explain the high level of risk aversion over such a modest stake. In this section, we relate our model to Sydnor's to see if PT can explain this high WTP. Sydnor looks at claim data for homeowners insurance, which is viewed as mandatory by those who own it, so his work is most applicable to our benchmark for mandatory insurance: initial wealth minus the premium. Recall that for this benchmark if the loss is less than the deductible, the individual feels a loss of L. If the loss is greater than the deductible, the individual feels a gain of $(L - D)$. Assuming the probability of loss as 4% (to be consistent with Sydnor's work), we determine an individual's WTP (in utility terms) to have a deductible of \$500 relative to a \$1000 deductible by determining the following:

$$E\left[u\left(\cdot\right)_{D=1000}\right] = E\left[u\left(\cdot\right)_{D=500}\right] - WTP_{utility}.$$

Once we solve for $WTP_{utility}$, we back out the dollar value (WTP) associated with $WTP_{utility}$. We investigate the WTP with and without the probability weighting feature of KT. We also determine how WTP changes with parameter values as estimated by KT (1992) and Harrison and Rutstrom (2009).

First we investigate the WTP for a \$500 deductible without incorporating the probability weighting feature of KT's prospect theory preferences. Therefore we solve the following:

$$\begin{aligned} E\left[u\left(\cdot\right)_{D=1000}\right] &= E\left[u\left(\cdot\right)_{D=500}\right] - WTP_{utility} \\ \left[\begin{array}{c} p\left(L \leq 1000\right)\left(-\lambda L^{\beta}\right) \\ +p\left(L > 1000\right)\left(\left(L-1000\right)^{a}\right) \end{array}\right] &= \left[\begin{array}{c} p\left(L \leq 500\right)\left(-\lambda L^{\beta}\right) \\ +p\left(L > 500\right)\left(\left(L-500\right)^{a}\right) \end{array}\right] - WTP_{utility} \end{aligned}$$

Using KT parameter values of $\alpha = \beta = .88$ and $\lambda = 2.25$, we find that if the probability of a loss is 4%, an individual's WTP to have a \$500 deductible is \$95 if they believe the loss will be about \$1215. If they think the average loss is higher, their WTP is even greater. If they believe the loss is \$1000 (i.e. at the current deductible level of \$1000), their WTP to have a \$500 deductible is \$83. Using Harrison and Rutstrom (2009) parameter values for their conditional PT model of $\alpha = \beta = .7$, $\lambda = 1.38$, we find that if the probability of a loss is 4%, an individual is willing to pay \$95 for a \$500 deductible if they believe the loss is about \$5,570.

Now we investigate the WTP for a \$500 deductible with the probability weighting in KT's prospect theory preferences. That is, we solve:

$$\begin{aligned} E\left[u\left(\cdot\right)_{D=1000}\right] &= E\left[u\left(\cdot\right)_{D=500}\right] - WTP_{utility} \\ \left[\begin{array}{c} w^{-}\left(p\left(0 \leq L \leq 1000\right)\right)\left(-\lambda L^{\beta}\right) \\ +w^{+}\left(p\left(L > 1000\right)\right)\left(\left(L-1000\right)^{a}\right) \end{array}\right] &= \left[\begin{array}{c} w^{-}\left(p\left(0 \leq L \leq 500\right)\right)\left(-\lambda L^{\beta}\right) \\ +w^{+}\left(p\left(L > 500\right)\right)\left(\left(L-500\right)^{a}\right) \end{array}\right] - WTP_{utility} \end{aligned}$$

Using KT parameter values of $\alpha = \beta = .88$, $\delta = .61$, $\theta = .69$, and $\lambda = 2.25$, we find that if the probability of a loss is 4%, an individual's WTP to have a \$500 deductible is \$270 if they believe the loss will be about \$1000 (that is, the current deductible level). Using Harrison and Rutstrom (2009) parameter values for their conditional PT model of $\alpha = \beta = .7$, $\lambda = 1.38$, $\delta = \theta = .911$ we find that if the probability of a loss is 4%, an individual's WTP for a \$500 deductible will be \$95 if the loss is \$3670. Using HR mixture model parameter values of $\alpha = .614$, $\beta = .312$, $\lambda = 5.781$, $\delta = \theta = .681$ we find the loss needs to be \$42,000 for an individual to have a willingness to pay of \$95. Here, if the individual believes the loss will be \$1000, the willingness to pay for a \$500 deductible is approximately \$39.

From this illustrative example, it appears including the probability weighting matters when using prospect theory preferences to describe individual decision-making. Incorporating the over-weighting of low probability events and under-weighting of moderate/high probability events leads to estimates of WTP which are more consistent with what Sydnor (2010) finds in his work. This result also corresponds with Braseghyan et al.'s (2011) result that probability weighting matters when considering insurance claims data and with Barberis and Huang's (2008) work on the importance of probability weighting for asset pricing.

4 Conclusion and Future Work

We examine the effect of prospect theory type preferences on the demand for insurance to determine whether loss aversion and probability distortions can explain the preference for low deductibles as suggested by Sydnor (2010). Prospect theory implies individuals make decisions by evaluating gains and losses relative to a reference point, where utility is concave over gains and convex over losses; furthermore, losses are weighed more heavily than gains in this setting. We incorporate such preferences in the utility function for an individual and investigate various reference points for an individual making insurance purchasing decisions. We find that prospect theory which captures loss aversion and probability distortions can explain several documented phenomena about deductible choices: the preference for low deductibles for mandatory insurance, the lack of demand for non-mandatory insurance like catastrophe insurance, and the over-demand to insure small losses as seen with the purchasing of warranties. This work provides additional insight into how consumers behave which has implications for insurance companies on how to better induce individuals to buy coverage.

The current version of the paper does not account for the fact that the premium would increase when a smaller deductible is chosen. With a larger premium, then there is a more of a "loss" that the individual would feel. We plan to incorporate into the model a way to capture that the loss the individual feels with loss averse preferences would be greater when a lower deductible is chosen. We will incorporate a punishment function which will increase the loss the individual experiences when a higher premium is charged due to a lower deductible being chosen.

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A Appendix: Proofs and Derivations - For Online Publication

A.1 Proof of Optimal Demand for EUT Individual

The maximization problem for an EUT investor is given by

$$\max_{D_0\in[0,w_0]}\left[\int_0^{D_0} u(w_0-P(D_0)-L)dF(L)+\int_{D_0}^{\infty} u(w_0-P(D_0)-D_0)dF(L)\right]$$

where utility is given by

$$u(w)=w^a.$$

The maximization problem is therefore reduced to

$$\begin{aligned}
&\max_{D_0\in[0,w_0]}\left[\int_0^{D_0}(w_0-P(D_0)-L)^a dF(L)+\int_{D_0}^{\infty}(w_0-P(D_0)-D_0)^a dF(L)\right]\\
=\;&\max_{D_0\in[0,w_0]}\left[\int_0^{D_0}(w_0-P(D_0)-L)^a dF(L)+(w_0-P(D_0)-D_0)^a\left(1-F(D_0)\right)\right].
\end{aligned}$$

Using Liebnitz' Rule the first derivative is

$$\begin{aligned}
\frac{dEU_0}{dD_0} &= (w_0-P(D_0)-D_0)^a f(D_0)+\int_0^{D_0} a(w_0-P(D_0)-L)^{a-1}\left(-P'(D_0)\right)dF(L)\\
&\quad +(w_0-P(D_0)-D_0)^a\left(-f(D_0)\right)+\left(1-F(D_0)\right)a(w_0-P(D_0)-D_0)^{a-1}\left(-P'(D_0)-1\right)\\
&= -P'(D_0)\int_0^{D_0} a(w_0-P(D_0)-L)^{a-1}dF(L)-\left(P'(D_0)+1\right)a(w_0-P(D_0)-D_0)^{a-1}\left(1-F(D_0)\right).
\end{aligned}$$

Using Liebnitz' Rule again, the second derivative is

$$\begin{aligned}
\frac{d^2EU_0}{dD_0^2} &= -P'(D_0)\,a(w_0-P(D_0)-D_0)^{a-1}+\left(P'(D_0)\right)^2\int_0^{D_0} a(a-1)(w_0-P(D_0)-L)^{a-2}dF(L)\\
&\quad -P''(D_0)\int_0^{D_0} a(w_0-P(D_0)-L)^{a-1}dF(L)\\
&\quad -\left(P'(D_0)+1\right)\begin{bmatrix} -f(D_0)\,a(w_0-P(D_0)-D_0)^{a-1}\\ -\left(1-F(D)\right)a(a-1)(w_0-P(D_0)-D)^{a-2}P'(D_0)\end{bmatrix}\\
&\quad -a(w_0-P(D_0)-D_0)^{a-1}P''(D_0)\left(1-F(D)\right)
\end{aligned}$$

Mossin (1968) showed that the second derivative for the above problem was less than zero if utility is increasing and concave. Note that for this problem, $u'(w)=aw^{a-1}$ and $u''(w)=a(a-1)w^{a-2}$; therefore utility is increasing and concave for $a<1$ which we assume to be consistent with Kahneman and Tversky. Hence $\frac{d^2EU_0}{dD_0^2}<0$ which implies the solution to $\frac{dEU_0}{dD_0}=0$ is a global maximum.

Evaluating the first derivative at the full insurance point ($D_0=0$) we find

$$\begin{aligned}
\frac{dEU_0}{dD_0}\Big|_{D_0=0} &= -\left(P'(0)+1\right)a(w_0-P(0))^{a-1}\left(1-F(0)\right)\\
&= \gamma a(w_0-P(0))^{a-1}
\end{aligned}$$

Define D_0^* as the optimal deductible for an EUT individual which satisfies $\frac{dEU_0}{dD_0}|_{D_0^*} = 0$. If insurance is actuarially fair ($\gamma = 0$) then $\frac{dEU_0}{dD_0}|_{D_0=0} = 0$ and full insurance is optimal ($D_0^* = 0$). If insurance has a positive loading ($\gamma > 0$) then $\frac{dEU_0}{dD_0}|_{D_0=0} > 0$ and partial insurance is optimal ($D_0^* > 0$).

A.2 Proof of Proposition 1

Maximization problem is given by

$$\max_{D\in[0,w_0]} \left[\int_0^D -\lambda(P(D)+L)^\beta dF(L) + \int_D^{D+P(D)} -\lambda(P(D)+D-L)^\beta dF(L) + \int_{D+P(D)}^\infty (L-P(D)-D)^a dF(L) \right].$$

Since choice variable, D, is in both the limit and term being integrated for each part of the above equation we use Leibnitz' rule to find the FOC which is given by:

$$\begin{aligned}
&-\lambda(P(D)+D)^\beta f(D) + P'(D) \int_0^D -\lambda\beta(P(D)+L)^{\beta-1} dF(L) \\
&+\lambda(P(D))^\beta f(D) + (1+P'(D)) \int_D^{D+P(D)} -\lambda\beta(P(D)+D-L)^{\beta-1} dF(L) \\
&-(1+P'(D)) \int_{D+P(D)}^\infty a(L-P(D)-D)^{a-1} dF(L)
\end{aligned}$$

and reduces to

$$\begin{aligned}
\frac{dEU}{dD} &= -\lambda(P(D)+D)^\beta f(D) + \lambda(P(D))^\beta f(D) - \lambda\beta P'(D) \int_0^D (P(D)+L)^{\beta-1} dF(L) \\
&-\lambda\beta(1+P'(D)) \int_D^{D+P(D)} (P(D)+D-L)^{\beta-1} dF(L) \\
&-a(1+P'(D)) \int_{D+P(D)}^\infty (L-P(D)-D)^{a-1} dF(L)
\end{aligned}$$

Recall

$$\begin{aligned}
P(D) &= (1+\gamma) E\left[(L-D)^+\right] \\
&= (1+\gamma) \int_D^\infty (L-D) dF(L)
\end{aligned}$$

which implies

$$\begin{aligned}
P'(D) &= \frac{d}{dL}(1+\gamma) \int_D^\infty (L-D) dF(L) \\
&= -(1+\gamma)(1-F(D)).
\end{aligned}$$

Evaluating the FOC at $D = 0$ we find

$$\begin{aligned}
\frac{dEU}{dD} \mid_{D=0} &= -\lambda(P(0))^{\beta} f(0) + \lambda (P(0))^{\beta} f(0) - \lambda\beta (1 + P'(0)) \int_{0}^{P(0)} (P(0) - L)^{\beta-1} dF(L) \\
&\quad -a (1 + P'(0)) \int_{P(0)}^{\infty} (L - P(0))^{a-1} dF(L) \\
&= \lambda\beta\gamma \int_{0}^{P(0)} (P(0) - L)^{\beta-1} dF(L) + a\gamma \int_{P(0)}^{\infty} (L - P(0))^{a-1} dF(L) \\
&= \gamma \left(\lambda\beta \int_{0}^{P(0)} (P(0) - L)^{\beta-1} dF(L) + a \int_{P(0)}^{\infty} (L - P(0))^{a-1} dF(L) \right)
\end{aligned}$$

since

$$\begin{aligned}
P(0) &= (1 + \gamma) E[L] \\
P'(0) &= -(1 + \gamma).
\end{aligned}$$

For an actuarially fair premium ($\gamma = 0$) we find

$$\frac{dEU}{dD} \mid_{D=0} = 0$$

This result implies individuals will choose full insurance($D^* = 0$).

For a positive loading factor ($\gamma > 0$),

$$\frac{dEU}{dD} \mid_{D=0} = \gamma \left(\lambda \int_{0}^{P(0)} \beta (P(0) - L)^{\beta-1} dF(L) + \int_{P(0)}^{\infty} a (L - P(0))^{a-1} dF(L) \right) > 0$$

which implies that partial insurance is optimal ($D^* > 0$).

To compare the deductible chosen by a PT individual relative to that chosen by an EUT individual, evaluate the

FOC for a PT individual at the optimal deductible for an EUT individual as:

$$
\begin{aligned}
\frac{dEU}{dD}|_{D=D_0^*} &= -\lambda(P(D_0^*)+D_0^*)^{\beta} f\left(D_0^*\right)+\lambda\left(P\left(D_0^*\right)\right)^{\beta} f\left(D_0^*\right)-\left(\lambda\beta P'\left(D_0^*\right)\int_0^{D_0^*}(P(D_0^*)+L)^{\beta-1}dF\left(L\right)\right) \\
&\quad -\left(\lambda\beta\left(1+P'\left(D_0^*\right)\right)\int_{D_0^*}^{D_0^*+P(D_0^*)}\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1}dF\left(L\right)\right) \\
&\quad -\left(\left(1+P'\left(D_0^*\right)\right)\int_{D_0^*+P\left(D_0^*\right)}^{\infty}a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1}dF\left(L\right)\right) \\
&= \lambda f\left(D_0^*\right)\left[\left(P\left(D_0^*\right)\right)^{\beta}-(P(D_0^*)+D_0^*)^{\beta}\right] \\
&\quad -\lambda P'\left(D_0^*\right)\int_0^{D_0^*}\beta(P(D_0^*)+L)^{\beta-1}dF\left(L\right) \\
&\quad -\lambda\left(1+P'\left(D_0^*\right)\right)\int_{D_0^*}^{D_0^*+P(D_0^*)}\beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1}dF\left(L\right) \\
&\quad -\left(1+P'\left(D_0^*\right)\right)\int_{D_0^*+P\left(D_0^*\right)}^{\infty}a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1}dF\left(L\right).
\end{aligned}
$$

The integrals in the last 3 terms are all positive. The first term in the equation above is negative and for $\gamma > 0$, $D_0^* > 0$ which implies that $P'\left(D_0^*\right) < 0$. Therefore the 2nd term is positive. To determine the sign of the last three terms consider the FOC for the EUT individual which implies

$$
\begin{aligned}
0 &= -P'\left(D_0^*\right)\int_0^{D_0^*}a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right) \\
&\quad -\left(P'\left(D_0^*\right)+1\right)a(w_0-P(D_0^*)-D_0^*)^{a-1}\left(1-F\left(D_0^*\right)\right) \\
\left(1+P'\left(D_0^*\right)\right) &= -\frac{P'\left(D_0^*\right)\int_0^{D_0^*}a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)}{a(w_0-P(D_0^*)-D_0^*)^{a-1}\left(1-F\left(D_0^*\right)\right)}.
\end{aligned}
$$

We know $P'\left(D_0^*\right) < 0$, the integral in the term above will be positive as will the denominator which implies that $\left(1+P'\left(D_0^*\right)\right) > 0$. Going back to the FOC for a PT individual evaluated at D_0^* we can see that the last two terms will be negative. Therefore all terms except the 2nd term in $\frac{dEU}{dD}|_{D=D_0^*}$ are negative.

Therefore, $\frac{dEU}{dD}|_{D=D_0^*} < 0$ unless the 2nd term outweighs. That is unless the following condition holds:

$$\int_0^{D_0^*} \beta(P(D_0^*)+L)^{\beta-1}dF(L) > \left[\begin{array}{c} \frac{\left(1+P'(D_0^*)\right)}{-P'\left(D_0^*\right)} \int_{D_0^*}^{D_0^*+P(D_0^*)} \beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1} dF\left(L\right) \\ +\lambda\frac{\left(1+P'(D_0^*)\right)}{-P'\left(D_0^*\right)} \int_{D_0^*+P\left(D_0^*\right)}^{\infty} a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1} dF\left(L\right) \\ +\frac{f(D_0^*)\left[(P(D_0^*))^{\beta}-(P(D_0^*)+D_0^*)^{\beta}\right]}{P'\left(D_0^*\right)} \end{array} \right]$$

Reduce this term as follows:

$$\begin{aligned} \int_0^{D_0^*} \beta(P(D_0^*)+L)^{\beta-1}dF(L) &> \left[\begin{array}{c} \frac{\left(1+P'(D_0^*)\right)}{-P'\left(D_0^*\right)} \int_{D_0^*}^{D_0^*+P(D_0^*)} \beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1} dF\left(L\right) \\ +\lambda\frac{\left(1+P'(D_0^*)\right)}{-P'\left(D_0^*\right)} \int_{D_0^*+P\left(D_0^*\right)}^{\infty} a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1} dF\left(L\right) \\ +\frac{f(D_0^*)\left[(P(D_0^*))^{\beta}-(P(D_0^*)+D_0^*)^{\beta}\right]}{P'\left(D_0^*\right)} \end{array} \right] \\ &> \left[\begin{array}{c} \int_{D_0^*}^{D_0^*+P(D_0^*)} \beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1} dF\left(L\right) \\ + \int_{D_0^*+P\left(D_0^*\right)}^{\infty} a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1} dF\left(L\right) \end{array} \right] \end{aligned}$$

where the last condition holds because

$$\frac{\left(1+P'\left(D_0^*\right)\right)}{-P'\left(D_0^*\right)} > 1.$$

That is, through the FOC for EUT individual we know

$$\begin{aligned} \frac{\left(1+P'\left(D_0^*\right)\right)}{-P'\left(D_0^*\right)} &= \frac{\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF(L)}{a(w_0-P(D_0^*)-D_0^*)^{a-1}\left(1-F\left(D_0^*\right)\right)} \\ &> \frac{\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF(L)}{a(w_0-P(D_0^*)-D_0^*)^{a-1}} \\ &> 1. \end{aligned}$$

Therefore if

$$\int_{0}^{D_0^*} \beta(P(D_0^*)+L)^{\beta-1} f(L)\, dL$$

$$> \int_{D_0^*}^{D_0^*+P(D_0^*)} \beta\left(P\left(D_0^*\right)+D_0^*-L\right)^{\beta-1} f(L)\, dL + \int_{D_0^*+P\left(D_0^*\right)}^{\infty} a\left(L-P\left(D_0^*\right)-D_0^*\right)^{a-1} f(L)\, dL.$$

then $\frac{dEU}{dD}|_{D=D_0^*} > 0$ which implies $D^* > D_0^*$. This condition would hold if the pdf is weighted heavily toward losses lower than the deductible level. Therefore for loss distributions that are not skewed to the left, the above condition would not hold and we have $\frac{dEU}{dD}|_{D=D_0^*} < 0$ which implies $D^* < D_0^*$.

A.3 Proof of Proposition 2

Individual chooses deductible to maximize expected utility of gain/loss

$$\max_{D\in[0,w_0]} \left[\int_0^D u(-L)\, dF(L) + \int_D^{\infty} u(L-D)\, dF(L)\right]$$

which can be reduced as follows

$$\max_{D\in[0,w_0]} \left[\int_0^D -\lambda (L)^{\beta}\, dF(L) + \int_D^{\infty} (L-D)^{a}\, dF(L)\right]$$

$$\max_{D\in[0,w_0]} \left[-\lambda D^{\beta} F(D) - \int_0^D -\lambda\beta (L)^{\beta-1} F(L)\, dL + \int_D^{\infty} (L-D)^{a}\, dF(L)\right]$$

where the second line is obtained by integrating by parts. Using Liebnitz Rule the first derivative is

$$\begin{aligned}
\frac{dEU}{dD} &= -\lambda D^{\beta} f(D) - \lambda\beta D^{\beta-1} F(D) + \frac{\partial}{\partial D}\int_0^D \lambda\beta (L)^{\beta-1} F(L)\, dL - \int_D^{\infty} a (L-D)^{a-1}\, dF(L) \\
&= -\lambda D^{\beta} f(D) - \lambda\beta D^{\beta-1} F(D) + \lambda\beta D^{\beta-1} F(D) - \int_D^{\infty} a (L-D)^{a-1}\, dF(L) \\
&= -\lambda D^{\beta} f(D) - \int_D^{\infty} a (L-D)^{a-1}\, dF(L)
\end{aligned}$$

Evaluating the first derivative at zero we find:

$$\begin{aligned}
\frac{dEU}{dD} &\mid_{D=0} = -\int_0^{\infty} a (L)^{a-1}\, dF(L) \\
&< 0.
\end{aligned}$$

The above implies $D^* < 0$. As over-insurance isn't allowed, then full insurance ($D^* < 0$) is optimal for all loading factors. Using Liebnitz Rule again, the second derivative is given by

$$\frac{d^2EU}{dD^2} = -\lambda D^{\beta} f'(D) - \lambda \beta D^{\beta-1} f(D) + \int_{D}^{\infty} a(a-1)(L-D)^{a-2} \, dF(L).$$

Evaluate at zero as

$$\frac{d^2EU}{dD^2} \mid_{D=0} = \int_{0}^{\infty} a(a-1)(L)^{a-2} \, dF(L)$$

Since $a < 1$ then $\frac{d^2EU}{dD^2} \mid_{D=0} < 0$.

A.4 Proof of Proposition 3

Consider the first derivative for the prospect theory individual:

$$\frac{dEU}{dD} = -\lambda D^{\beta} f(D) - \int_{D}^{\infty} a(L-D)^{a-1} \, dF(L).$$

From the first order condition for the EUT individual we know

$$\frac{dEU_0}{dD_0} \mid_{D_0=D_0^*} = \begin{pmatrix} -P'(D_0^*) \int_{0}^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \\ -\left(P'(D_0^*) + 1\right) a(w_0 - P(D_0^*) - D_0^*)^{a-1} \left(1 - F(D_0^*)\right) \end{pmatrix} = 0$$

which implies

$$\begin{aligned}
-P'\left(D_0^*\right)\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right) &= \left(P'\left(D_0^*\right)+1\right)a(w_0-P(D_0^*)-D_0^*)^{a-1}\left(1-F\left(D_0^*\right)\right)\\
1-F\left(D_0^*\right) &= -\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\left(\frac{P'\left(D_0^*\right)}{P'\left(D_0^*\right)+1}\right)\\
&\quad \times\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\\
1-F\left(D_0^*\right) &= -\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\left(\frac{-\left(1+\gamma\right)\left(1-F\left(D_0^*\right)\right)}{-\left(1+\gamma\right)\left(1-F\left(D_0^*\right)\right)+1}\right)\\
&\quad \times\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\\
\left(1-F\left(D_0^*\right)\right)\left(\frac{-\left(1+\gamma\right)\left(1-F\left(D_0^*\right)\right)+1}{-\left(1+\gamma\right)\left(1-F\left(D_0^*\right)\right)}\right) &= -\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\\
\left(1-F\left(D_0^*\right)\right)-\frac{1}{1+\gamma} &= -\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\\
F\left(D_0^*\right) &= \frac{\gamma}{1+\gamma}+\left[\begin{array}{c}\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\\ \times\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\end{array}\right]\\
f\left(D_0^*\right) &= \frac{\partial}{\partial D_0^*}\left[\frac{\gamma}{1+\gamma}+\left(\begin{array}{c}\frac{1}{a}(w_0-P(D_0^*)-D_0^*)^{a-2}\\ \times\int_0^{D_0^*} a(w_0-P(D_0^*)-L)^{a-1}dF\left(L\right)\end{array}\right)\right]
\end{aligned}$$

Evaluate at the first derivative for the prospect theory individual at the optimal deductible for an EUT individual as

$$\begin{aligned}
\frac{dEU}{dD} \quad | \quad {}_{D=D_0^*} &= -\lambda\left(D_0^*\right)^{\beta}f\left(D_0^*\right)-\int_{D_0^*}^{\infty} a\left(L-D_0^*\right)^{a-1}dF\left(L\right)\\
&< \quad 0.
\end{aligned}$$

Since $\frac{dEU}{dD}\mid_{D=D_0^*}<0$ that implies that $D^*<D_0^*$ for all γ.

A.5 Variations of Gain/Loss Definition

In the first variation, no gain is felt. Individual's maximization problem is given by

$$\max_{D\in[0,w_0]}\left[\int_0^D-\lambda(L)^{\beta}dF\left(L\right)+\int_D^{\infty}-\lambda\left(D\right)^{\beta}dF\left(L\right)\right].$$

FOC is given by

$$\frac{dEU}{dD}=-\lambda D^{\beta}f\left(D\right)-\lambda\beta D^{\beta-1}\left(1-F\left(D\right)\right).$$

Evaluating the FOC at $D=0$:

$$\begin{aligned}\frac{dEU}{dD} \quad | \quad {}_{D=0} &= -\lambda(0)^{\beta} f(0) - \lambda\beta(0)^{\beta-1}(1-F(0)) \\ &= 0\end{aligned}$$

Therefore, $D^* = 0$ for all γ. The SOC is:

$$\begin{aligned}\frac{d^2EU}{dD^2} &= -\lambda D^{\beta} f'(D) - \lambda\beta(\beta-1) D^{\beta-2} + \lambda\beta(\beta-1) D^{\beta-2} F(D) \\ &= -\lambda D^{\beta} f'(D) - \lambda\beta(\beta-1) D^{\beta-2}(1-F(D))\end{aligned}$$

which holds if $f'(D) > \beta(1-\beta) D^{-2}(1-F(D))$.

In the second variation, a loss is felt if $L < D$ and if $L > D$, both a loss and gain are felt. An individual's maximization is given by

$$\max_{D\in[0,w_0]}\left[\int_0^D -\lambda(L)^{\beta} dF(L) + \int_D^{\infty}\left(-\lambda(D)^{\beta} + (L-D)^{a}\right) dF(L)\right].$$

FOC is given by

$$\frac{dEU}{dD} = -\lambda D^{\beta} f(D) + \lambda(D)^{\beta} - \int_D^{\infty}\left(\lambda\beta D^{\beta-1} + a(L-D)^{a-1}\right) dF(L).$$

Evaluating the FOC at $D=0$:

$$\begin{aligned}\frac{dEU}{dD} \quad | \quad {}_{D=0} &= -\lambda(0)^{\beta} f(0) + \lambda(0)^{\beta} - \int_0^{\infty} a(L-D)^{a-1} dF(L) \\ &= -\int_0^{\infty} a(L-D)^{a-1} dF(L) \\ &< 0\end{aligned}$$

Therefore, $D^* = 0$ for all γ. The SOC is:

$$\begin{aligned}\frac{d^2EU}{dD^2} &= -\lambda D^{\beta} f'(D) - \lambda\beta D^{\beta-1} f(D) + 2\lambda\beta(D)^{\beta-1} \\ &\quad -\int_D^{\infty}\left(\lambda\beta(\beta-1) D^{\beta-2} - a(a-1)(L-D)^{a-2}\right) dF(L)\end{aligned}$$

and evaluating at zero we have

$$\begin{aligned}\frac{d^2EU}{dD^2} \quad | \quad {}_{D=0} &= -\lambda(0)^{\beta} f'(0) - \lambda\beta(0)^{\beta-1} f(0) + 2\lambda\beta(0)^{\beta-1} + \int_0^{\infty}\left(a(a-1)(L-D)^{a-2}\right) dF(L) \\ &= \int_0^{\infty}\left(a(a-1)(L-D)^{a-2}\right) dF(L)\end{aligned}$$

If $a < 1$ then $\frac{d^2EU}{dD^2} \mid_{D=0} < 0$.

For the original benchmark (initial wealth), consider the following:

1. (a) If no loss occurs: feel a loss of $P(D)$

2. If $L < D$: feel a loss of $L + P(D)$

3. If $D < L < D + P(D)$: feel a loss of $D + P(D)$

4. If $L > D + P(D)$: feel a loss of $D + P(D)$

In this setting there will always be a loss. The analysis is below and we find that for an actuarially fair premium $(\gamma = 0)$, $\frac{dEU}{DD}|_{D=0} = 0$ which implies that full insurance is optimal ($D^* = 0$). For a positive loading factor $(\gamma > 0)$ we find $\frac{dEU}{DD}|_{D=0} > 0$ which implies partial insurance is optimal ($D^* > 0$). This coincides with we found already.

For a positive deductible we compare how a PT individual's optimal deductible compares to the optimal deductible chosen by a risk averse individual. As found previously, it will depend on the type of loss being considered and we find a similar relationship as found previously. If the following condition holds:

$$\int_0^{D_0^*} \beta(P(D_0^*) + L)^{\beta-1} dF(L) < \int_{D_0^*}^{\infty} \beta\left(P\left(D_0^*\right) + D_0^*\right)^{\beta-1} dF(L)$$

then, $\frac{dEU}{dD}|_{D_0^*} < 0$ which implies a PT individual will choose a lower deductible ($D^* < D_0^*$). That is, if losses are weighted more towards those that will be greater than the deductible, more insurance is demanded. For loss distributions that are skewed left, a PT individual will demand a higher deductible (less insurance) relative to that chosen by an EUT individual. That is for loss distributions where high severity losses occur with a very small probability, less insurance is demanded.

The maximization problem is:

$$\max_{D \in [0, w_0]} \left[\int_0^D -\lambda(P(D) + L)^\beta dF(L) + \int_D^\infty -\lambda\left(P(D) + D\right)^\beta dF(L) \right]$$

Using Leibnitz' rule we find FOC is

$$\begin{aligned} \frac{dEU}{dD} &= -\lambda(P(D) + D)^\beta f(D) + P'(D) \int_0^D -\lambda\beta(P(D) + L)^{\beta-1} dF(L) \\ &\quad -(1 + P'(D)) \int_D^\infty \lambda\beta\left(P(D) + D\right)^{\beta-1} dF(L) \end{aligned}$$

$$\begin{aligned} \frac{dEU}{DD}|_{D=0} &= -\lambda(P(0))^\beta f(0) - (1 + P'(0)) \int_0^\infty \lambda\beta\left(P(0)\right)^{\beta-1} dF(L) \\ &= \gamma \int_0^\infty \lambda\beta\left(P(0)\right)^{\beta-1} dF(L) \end{aligned}$$

since $P'(0) = -(1+\gamma)$. For an actuarially fair premium $(\gamma = 0)$ then $\frac{dEU}{DD}|_{D=0} = 0$ which implies that full insurance is optimal. For a positive loading factor we'd find $\frac{dEU}{DD}|_{D=0} > 0$ which implies partial insurance is optimal. These results are the same as we found previously. In order to determine if the PT individual chooses more or less insurance than a non-PT individual, evaluate the FOC for a PT individual at the optimal deductible for a risk-averse individual:

$$\frac{dEU}{dD}|_{D_0^*} = -\lambda(P(D_0^*)+D_0^*)^{\beta} f(D_0^*) + P'(D_0^*)\int_0^{D_0^*} -\lambda\beta(P(D_0^*)+L)^{\beta-1}dF(L)$$
$$-(1+P'(D_0^*))\int_{D_0^*}^{\infty}\lambda\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)$$

The first term is negative. The last term is also negative since $(1+P'(D_0^*))>0$ (determine by FOC for EUT person and explained in proof of Proposition 3 in our paper). Since $P'(D_0^*)<0$ then the 2nd term can be positive. Therefore $\frac{dEU}{dD}|_{D_0^*}<0$ if

$$\int_0^{D_0^*}\beta(P(D_0^*)+L)^{\beta-1}dF(L) < \left[-\frac{(P(D_0^*)+D_0^*)^{\beta} f(D_0^*)}{P'(D_0^*)} - \frac{(1+P'(D_0^*))}{P'(D_0^*)}\int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)\right].$$

Note the following:

$$\left[\begin{array}{c} -\frac{(P(D_0^*)+D_0^*)^{\beta} f(D_0^*)}{P'\left(D_0^*\right)} \\ -\frac{\left(1+P'(D_0^*)\right)}{P'\left(D_0^*\right)}\int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)\end{array}\right] > -\frac{(1+P'(D_0^*))}{P'(D_0^*)}\int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)$$
$$> \int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)$$

since $-\frac{(P(D_0^*)+D_0^*)^{\beta} f(D_0^*)}{P'\left(D_0^*\right)}>0$ and $-\frac{\left(1+P'(D_0^*)\right)}{P'\left(D_0^*\right)}>1$. If

$$\int_0^{D_0^*}\beta(P(D_0^*)+L)^{\beta-1}dF(L) < \int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)$$

then

$$\int_0^{D_0^*}\beta(P(D_0^*)+L)^{\beta-1}dF(L) < \left[\begin{array}{c} -\frac{(P(D_0^*)+D_0^*)^{\beta} f(D_0^*)}{P'\left(D_0^*\right)} \\ -\frac{\left(1+P'(D_0^*)\right)}{P'\left(D_0^*\right)}\int_{D_0^*}^{\infty}\beta\left(P(D_0^*)+D_0^*\right)^{\beta-1}dF(L)\end{array}\right]$$

and therefore $\frac{dEU}{dD}|_{D_0^*}<0$ and $D^*<D_0^*$. This will hold if the pdf is weighted heavily toward losses above the deductible level (skewed right). If losses are skewed left the above condition does not hold then $D^*>D_0^*$. The intuition here is that there will still be a trade-off as we found in earlier definitions of gains/losses when the individual benchmarks to initial wealth. In this instance, however, it will depend on whether a PT individual is willing to take on a low deductible given it will lead to a higher premium (which will increase the loss they feel). Previously it depended on whether the loss could offset both the premium and the deductible (where they felt a gain previously). Here we no longer have a gain being felt but there will still be a trade-off of sorts. For small losses which occur with high probability the higher premium associated with the lower deducitible isn't that much more and therefore is "worth it" to opt for a lower deductible to only feel a loss of the premium. For large losses which occur with very small probabilities (associated with cat insurance), the increase in the premium associated with a lower deductible isn't "worth it" since the loss occurs so rarely. Therefore to minimize the feeling of the loss, the insured chooses a higher deductible to have a lower premium as he will feel a loss of the premium for sure.

A.6 Proof of Proposition 4

Maximization problem is given by

$$\max_{D\in[0,w_0]}\left[\int_0^D -\lambda(L)^\beta\,dF(L)\right]$$
$$\max_{D\in[0,w_0]}\left[-\lambda D^\beta F(D)-\int_0^D -\lambda\beta(L)^{\beta-1}F(L)\,dL\right]$$

which has a first order condition of

$$\begin{aligned}\frac{dEU}{dD} &= -\lambda D^\beta f(D)-\lambda\beta D^{\beta-1}F(D)+\frac{\partial}{\partial D}\int_0^D \lambda\beta(L)^{\beta-1}F(L)\,dL\\ &= -\lambda D^\beta f(D)\end{aligned}$$

and a SOC of

$$\frac{d^2EU}{dD^2}=-\lambda D^\beta f'(D)-\lambda\beta D^{\beta-1}f(D)$$

In order for $SOC<0$ we need $f'(D)>0$. Given that $F(0)=0$ then at zero the pdf will be increasing. Therefore, SOC <0 around zero. Evaluate FOC at $D=0$:

$$\begin{aligned}\frac{dEU}{dD}|_{D=0} &= -\lambda(0)^\beta f(0)\\ &= 0.\end{aligned}$$

Therefore, $D^*=0$ which is a local max.

A.7 Proof of Proposition 5

Maximization problem is given by:

$$\max_{D\in[0,w_0]}\left[\begin{array}{c} w^-(p_1)\left(-\lambda(P(D)+L)^\beta\right)+w^-(p_2)\left(-\lambda(P(D)+D-L)^\beta\right)\\ +w^+(p_3)(L-P(D)-D)^a\end{array}\right].$$

First derivative is:

$$\begin{aligned}\frac{dEU}{dD} = {} & w^-(p_1)\left(-\lambda\beta(P(D)+L)^{\beta-1}P'(D)\right)+\left(-\lambda(P(D)+L)^\beta\right)\frac{\partial w^-(p_1)}{\partial D}\\ & +w^-(p_2)\left(-\lambda\beta(P(D)+D-L)^{\beta-1}(P'(D)+1)\right)+\left(-\lambda(P(D)+D-L)^\beta\right)\frac{\partial w^-(p_2)}{\partial D}\\ & -w^+(p_3)a(L-P(D)-D)^{a-1}(P'(D)+1)+(L-P(D)-D)^a\frac{\partial w^+(p_3)}{\partial D}\end{aligned}$$

Recall that

$$w^-(p_1)=\frac{prob(0\le L\le D)^\delta}{\left(prob(0\le L\le D)^\delta+(1-prob(0\le L\le D))^\delta\right)^{1/\delta}}$$

and therefore

$$\begin{aligned}\frac{\partial w^{-}\left(p_{1}\right)}{\partial D} &= \left(p_{0\leq L\leq D}^{\delta}+\left(1-p_{0\leq L\leq D}\right)^{\delta}\right)^{1/\delta}\left(\delta p_{0\leq L\leq D}^{\delta-1}\frac{\partial p_{0\leq L\leq D}^{\delta}}{\partial D}\right)\\ &\quad -p_{0\leq L\leq D}^{\delta}\left[\begin{array}{c}\frac{1}{\delta}\left(p_{0\leq L\leq D}^{\delta}+\left(1-p_{0\leq L\leq D}\right)^{\delta}\right)^{(1/\delta)-1}\\ \times\left(\delta p_{0\leq L\leq D}^{\delta-1}\frac{\partial p_{0\leq L\leq D}^{\delta}}{\partial D}-\delta\left(1-p_{0\leq L\leq D}\right)^{\delta-1}\frac{\partial p_{0\leq L\leq D}^{\delta}}{\partial D}\right)\end{array}\right].\end{aligned}$$

Evaluating at zero and substituting in $P'(0)=-(1+\gamma)$ we find

$$\begin{aligned}\frac{dEU}{dD}|_{D=0} &= \gamma\left[\lambda\beta\left(P(0)-L\right)^{\beta-1}w^{-}\left(p_{2}\right)+a\left(L-P(0)\right)^{a-1}w^{+}\left(p_{3}\right)\right]\\ &\quad -\left[\lambda\left(P(0)-L\right)^{\beta}\frac{\partial w^{-}\left(p_{2}\right)}{\partial D}-\left(L-P(0)\right)^{a}\frac{\partial w^{+}\left(p_{3}\right)}{\partial D}\right]\end{aligned}$$

since

$$w^{-}\left(p_{1}\right)|_{D=0}=\frac{prob\left(0\leq L\leq 0\right)^{\delta}}{\left(prob\left(0\leq L\leq 0\right)^{\delta}+\left(1-prob\left(0\leq L\leq 0\right)\right)^{\delta}\right)^{1/\delta}}=0$$

and

$$\begin{aligned}\frac{\partial w^{-}\left(p_{1}\right)}{\partial D}|_{D=0} &= \left(p_{0\leq L\leq 0}^{\delta}+\left(1-p_{0\leq L\leq 0}\right)^{\delta}\right)^{1/\delta}\left(\delta p_{0\leq L\leq 0}^{\delta-1}\frac{\partial p_{0\leq L\leq 0}^{\delta}}{\partial D}\right)\\ &\quad -p_{0\leq L\leq 0}^{\delta}\left[\begin{array}{c}\frac{1}{\delta}\left(p_{0\leq L\leq 0}^{\delta}+\left(1-p_{0\leq L\leq 0}\right)^{\delta}\right)^{(1/\delta)-1}\\ \times\left(\delta p_{0\leq L\leq 0}^{\delta-1}\frac{\partial p_{0\leq L\leq 0}^{\delta}}{\partial D}-\delta\left(1-p_{0\leq L\leq 0}\right)^{\delta-1}\frac{\partial p_{0\leq L\leq 0}^{\delta}}{\partial D}\right)\end{array}\right]\\ &= 0.\end{aligned}$$

We find $\frac{dEU}{dD}|_{D=0}>0$ if $\frac{\partial w^{-}(p_{2})}{\partial D}<0$ and $\frac{\partial w^{+}(p_{3})}{\partial D}>0$.

A.8 Proof of Proposition 6

Individual chooses deductible to maximize expected utility of gain/loss:

$$\max_{D\in[0,w_{0}]}\left[w^{-}\left(p_{L<D}\right)\left(-\lambda L^{\beta}\right)+w^{+}\left(p_{L>D}\right)\left(L-D\right)^{a}\right]$$

The first derivative is

$$\frac{dEU}{dD}=-\lambda L^{\beta}\frac{\partial w^{-}\left(p_{L<D}\right)}{\partial D}-w^{+}\left(p_{L>D}\right)a\left(L-D\right)^{a-1}+\left(L-D\right)^{a}\frac{\partial w^{+}\left(p_{L>D}\right)}{\partial D}$$

Evaluating the first derivative at zero we find:

$$\begin{aligned}\frac{dEU}{dD}|_{D=0} &= -\lambda L^{\beta}\frac{\partial w^{-}\left(p_{L<0}\right)}{\partial D}-w^{+}\left(p_{L>0}\right)a\left(L\right)^{a-1}+\left(L\right)^{a}\frac{\partial w^{+}\left(p_{L>0}\right)}{\partial D}\\ &< 0.\end{aligned}$$

Therefore $D^{*}=0$ for all γ. Looking at the second derivative:

$$\frac{d^{2}EU}{dD^{2}}=-\lambda L^{\beta}\frac{\partial^{2}w^{-}\left(p_{L<D}\right)}{\partial D^{2}}+w^{+}\left(p_{L>D}\right)a\left(a-1\right)\left(L-D\right)^{a-2}+\left(L-D\right)^{a}\frac{\partial^{2}w^{+}\left(p_{L>D}\right)}{\partial D^{2}}$$

if $\frac{\partial^2 w^-(p_{L<D})}{\partial D^2} > 0$ and $\frac{\partial^2 w^+(p_{L>D})}{\partial D^2} < 0$ then $\frac{d^2 EU}{dD^2} < 0$ since $a < 1$. Evaluating at zero we find:

$$\frac{d^2 EU}{dD^2}|_{D=0} = -\lambda L^{\beta} \frac{\partial^2 w^-\left(p_{L<0}\right)}{\partial D^2} + w^+\left(p_{L>0}\right) a\left(a-1\right)\left(L\right)^{a-2} + \left(L\right)^a \frac{\partial^2 w^+\left(p_{L>0}\right)}{\partial D^2}$$

which is less than zero if $a < 1$.

A.9 Proof of Proposition 7

Individual chooses deductible to maximize expected utility of gain/loss:

$$\max_{D\in[0,w_0]} \left[w^-\left(p_{L<D}\right)\left(-\lambda L^{\beta}\right)\right]$$

The first derivative is

$$\frac{dEU}{dD} = -\lambda L^{\beta} \frac{\partial w^-\left(p_{L<D}\right)}{\partial D}.$$

As the deductible increases, $p_{L<D}$ increases, and hence $w^-\left(p_{L<D}\right)$ increases, making $\frac{\partial w^-(p_{L<D})}{\partial D} > 0$ and $\frac{dEU}{dD} < 0$. Evaluating the first derivative at zero we find:

$$\begin{aligned} \frac{dEU}{dD}|_{D=0} &= -\lambda L^{\beta} \frac{\partial w^-\left(p_{L<0}\right)}{\partial D} \\ &= 0 \end{aligned}$$

since $\frac{\partial w^-(p_{L<0})}{\partial D} = 0$ given that $p_{L<0} = 0$. Also one can see that

$$\begin{aligned} \frac{\partial w^-\left(p_{L<D}\right)}{\partial D} &= \left(p_{L<D}^{\delta} + \left(1-p_{L<D}\right)^{\delta}\right)^{1/\delta}\left(\delta p_{L<D}^{\delta-1}\frac{\partial p_{L<D}^{\delta}}{\partial D}\right) \\ &\quad -p_{L<D}^{\delta}\left[\frac{1}{\delta}\left(p_{L<D}^{\delta} + \left(1-p_{L<D}\right)^{\delta}\right)^{(1/\delta)-1}\begin{pmatrix} \delta p_{L<D}^{\delta-1}\frac{\partial p_{L<D}^{\delta}}{\partial D} \\ -\delta\left(1-p_{L<D}\right)^{\delta-1}\frac{\partial p_{L<D}^{\delta}}{\partial D} \end{pmatrix}\right] \end{aligned}$$

and therefore

$$\frac{\partial w^-\left(p_{L<D}\right)}{\partial D}|_{D=0} = 0.$$

Therefore $D^* = 0$ for all γ. Looking at the second derivative:

$$\frac{d^2 EU}{dD^2} = -\lambda L^{\beta} \frac{\partial^2 w^-\left(p_{L<D}\right)}{\partial D^2}$$

if $\frac{\partial^2 w^-(p_{L<D})}{\partial D^2} > 0$ then $\frac{d^2 EU}{dD^2} < 0$ and the solution to $\frac{dEU}{dD} = 0$ is global max. As shown earlier $\frac{\partial^2 w^-(p_{L<D})}{\partial D^2} > 0$ for some loss distributions but always holds near a deductible level of zero for $\delta \in (.5, 1)$.