On a New Paradigm of Optimal Insurance Demand: How to Theoretically Model Insurance Changes Post-Catastrophe

Abstract

Post-loss insurance demand is shown to be fundamentally different to that of pre-loss period due to revised risk perception with respect to loss event occurring or not. In particular, optimal insurance demand when there is an intertemporal consideration increases relative to the standard single period mode, whereas when there is not the intertemporal consideration; the optimal insurance demand is low.

A basic theoretical model that examines insurance demand post-loss based on a two-period intertemporal setting is presented and applied to show the performance of the intertemporal insurance model that examines insurance demand behaviour after a loss event experience.

1.0. Introduction

It is globally observed that a unique type of market adjustment effect occurs in the aftermath of a major disaster event (Froot & O'Connell 1999; Auffret, 2003; Born & Viscusi, 2006). This affects both demand and supply sides of the insurance market. For instance, Born and Viscusi (2006) explains how premiums for catastrophe risk insurance typically increase dramatically when insurance and reinsurance firms suffer significant loss claims after natural disasters. There is evidence that experience with loss events changes insurance demand behaviour. Browne and Hoyt (2000) study shows that flood insurance purchases are highly correlated with flood losses in the same region previously affected by floods. This insurance demand behaviour is further explain in Cameron & Shah (2012) that finds individuals that have recently experienced disaster loss report higher probabilities for catastrophic events in the subsequent year.

The focus of this paper is to examine the effects generated by post-loss experience from an insurance demand-side standpoint. The influence of post-loss experience on insurance demand decisions appears in particular on insurance markets for catastrophic risk. For example, in California, before the 1989 San Francisco earthquake, 34 percent of the individuals considered insurance against an earthquake as an unimportant undertaking. But after the earthquake, only 5 percent held this opinion. Likewise, the earthquake occurrence increased insurance demand with 11 percent of the previously non-insured individuals subscribing an insurance contract Kunreuther (1996).

This paper presents a basic theoretical model that examines insurance demand post-loss based on a two-period intertemporal setting. The paper gives an in-depth analysis and discussion of the performance of the intertemporal insurance model that examines insurance demand behaviour after a loss event experience.

The paper is organised as follows: Section 1.1 present a review of the link between past insurance demand models and present study. Section 1.2 gives the hypotheses of the study. Section 1.3 presents the methodology; sub-section 1.3.1 gives the theory of the intertemporal model; sub-section 1.3.2 presents the analytical framework and the underlying properties of the intertemporal model; and sub-section 1.3.3 presents the numerical illustration of the intertemporal model using hypothetical datasets. Section 1.4 gives a discussion of several findings that can be drawn from the simulation results and conclusions.

1.1. Link between Past Insurance Demand Models and Present Study

Previous theoretical analysis on insurance demand focuses on insurance in isolation within a single period time horizon. Using static models most previous research assumes that, there is only one area of uncertainty in the insurance demand analysis. Several pioneering studies (Arrow, 1963, 1965; Mossin, 1968; Smith, 1968; Raviv, 1979; Dreze, 1981) are credited for enormous contributions to the analysis of insurance demand in a static setting. With such models, the question of the choice of the level of insurance coverage is not a simple one. Mossin (1968) shows that it is not optimal to purchase full insurance when insurance policies are not actuarially fairly priced. These classical models of insurance demand as described in a number of studies (Arrow, 1965, 1974; Dreze, 1981; Mossin, 1968) have an important deficiency arising from their static features. One clear deficiency is that, in single period models, wealth and consumption are exactly the same variables. A second deficiency lies in the fact that these models do not consider the post-loss implication for insurance demand in a multi-period model with updated wealth.

Briys, Kahane, and Kroll (1988) and Mayers and Smith (1983) introduced multiple sources of uncertainty in the analysis of the demand for insurance. More specifically, Mayers and Smith (1983) examined the interrelationship between insurance holdings and other portfolio decisions and found that the combined analysis leads to different predictions about insurance demand.

Extending some of the results obtained by these studies and using the notion of prudence as first introduced in Kimball (1991), Eeckhoudt and Kimball (1992) documents a detailed impact of background risk on the optimal coverage.

Building on this new dynamic approach to insurance analyses, Gollier (2003) examined a simple consumption lifecycle model where the representative consumer faces a sequence of independent risks over his lifetime. The implicit assumption made in this study is that the policyholders must transform immediately the retained loss into a corresponding reduction in demand. This assumption implies that utility for wealth, and the attitude towards risk, are constant over time. But in the real world, people mostly compensate for losses to their wealth by reducing their saving or by borrowing money rather than just reducing their demand over several periods. In Gollier (2002) it is shown that a time-consistent cooperative multiplicity strategy provokes consumers to be much more risk susceptible than in the static version of the model. Because of time multiplicity, the attitude towards risk on wealth and towards risk on consumption is not the same, specifically the aversion to risk on wealth should be smaller than the aversion to risk on consumption. Meyer and Meyer (2004) suggested that a lower degree of risk aversion for wealth in a multi-period setting translates to depressed demand and welfare gains from insurance. Two reasons are given for these results. First, consumers are keen to consume immediately rather than to perfectly smooth their consumption over time, which implies that they do not adopt a perfect dynamic strategy. Second, they usually face cash constraints, and when funds are needed, policyholders cannot withdraw to compensate for the loss, hence they are obliged to absorb incurred losses immediately. Gollier (2003) concluded that the ability of consumers to self-insure by accumulating wealth induces them to significantly reduce their demand for insurance relative to what the classical model suggests. However, consumers have a positive insurance demand when they have been unlucky enough to incur a sequence of accidents in the recent past, which reduce their accumulated wealth.

The model proposed in this analysis is in close agreement with Cohen, Etner, and Jeleva, (2008) which examines the implications of a model for multi-period demand decisions on the insurance market. Cohen et al. (2008) looked at the optimal insurance demand strategy of a consumer for a three-year period when the consumer faces a risk of loss in each period. Assuming that the estimated probability of incurring a loss is known and those losses in successive periods are independent; they looked at a perfectly competitive insurance market proposing insurance contracts at actuarially fair premia corresponding to the estimated expected loss. In this case, insurance contracts are subscribed for one period. Therefore, the consumer has to choose an amount of coverage characterised by the indemnity and premium at each period. Their study revealed that an individual is optimistic in the initial period and modifies his risk perception with respect to damages occurring or not.

This analysis takes in to cognisant the existing different literature results and built on this to sets up a dynamic intertemporal model; however, this is not the first study to do so. Other studies have already considered insurance demand in a dynamic setting (Cooper & Hayes, 1987; Schlesinger 2000; Cohen et al., 2008; Volkman-Wise 2015). Most of these studies do not allow agents to transfer wealth between different periods and only consider situations where consumers only differ in their accident probability. When considering the proposed model, there are new contributions to the insurance dynamic aspects. First, the model incorporates a parameter on probability updating in an effort to explain why demand for coverage might increase after a loss event in the previous period. Secondly, since there is no consumption decision, the only effect of the first period decision on the second period decision is through wealth, and all of the standard parameters in second period like the probability of loss, premium price etc. However, if we allow the free choice of the amount of wealth the agents are allowed to keep and/or transfer, then savings and loans would be essentially the same as consumption. Finally, the model provides a unique way to study the effects of increases in risk aversion, increases in the premium, and increases in the perceived probability of loss to the demand for insurance in different periods. In the end, this paper presents analysis beyond the current dynamic framework which only observed that, risk-averse agents benefit not only from period-by-period events insurance, but also from insurance against a bad risk in a loss event and being reclassified into a higher-risk pool with a associated increased in premiums. This is an important extension to the literature of dynamic insurance models.

1.2. Hypotheses of the Study

Theory and empirical analysis from the insurance markets suggests that a riskier and more uncertain market would usually be associated with an increase in insurance demand. Ranger and Surminski (2013) find this would remain so at least until some local threshold are reached where the affordability of insurance is threatened.

The analysis in this paper theorise that regular experience to natural disasters increases the risk perception and consequently raising the insurance demand. This effect is anticipated to be stronger for more recent events due to heuristics and availability biases influence on the prediction of future events. The paper aims to demonstrate how agents can maintain or modify insurance demand after occurrence of loss or no-loss event in an intertemporal setting.

1.3. Methodology

1.3.1. The Theory of the Intertemporal Model

The underlying research question in this paper is how the demand for insurance changes post-catastrophe, and how to model it theoretically. To address this question, this paper proposes an intertemporal dynamic model which significantly departs from the popular models in the economic theory of insurance but utilises much of the existing literature findings (see, (Cooper & Hayes, 1987; Schlesinger 2000; Cohen et al., 2008; Volkman-Wise 2015)). The intertemporal dynamic model investigates the insurance consumption decision in an intertemporal setting where an agent is allowed to update their insurance demand, initial wealth and a host of other risk and consumption decision parameters in the subsequent periods.

In this intertemporal model, insurance can be sought in two sequential periods of time, and at the second period, it is known whether or not a loss event happened in period one. Thus, it is possible to model the demand for insurance in period two conditional upon the loss event happening or not in period one. Above all, the model incorporates a parameter on probability updating that is, when a loss event happens in period one, the consumer increases the probability belief for a loss event in period two, in an effort to explain why demand for coverage might increase after a loss event. The model assumes that for the first period and in successive periods the estimated probability of incurring losses are independent. In essence, then, the intertemporal model of insurance is restricted to only two periods, with an identical insurable risk in each period and identical insurance supply characteristics in each period.

In period one, a decision is made on insurance, and then the period one risk is allowed to play out. Any wealth that is not lost as uninsured losses or premium payments in period one is passed to period two. Then, a decision is made on insurance in period two. It should be noted that the period two decision is made with full information on both the amount of insurance contracted in period one and the outcome of the period one contract, that is, if a loss occurs or not. Since the inherited wealth in period two depends upon the insurance decision in period one, it happens that the insurance decision in period two is expressible as a function of the decision in period one.

Therefore, in this model, it is easy to consider the comparison between (i) period two insurance conditional upon a loss in period one and conditional upon no loss in period one, and (ii) period two insurance (either with or without loss event in period one) and the period one insurance demand.

The pure theoretical results are derived in the next section. It turns out that the model is an interesting extension to the standard model of demand for insurance, which is based on a single period. So, in the end, the model generates results that can be compared to and contrasted with, those of the standard environment. More innovatively, this model provides unique contributions to the theory of insurance demand. In particular, then, the first novelty is the fact that the model provides a natural theoretical way to establish how the demand for insurance is affected by the size of the loss suffered in the previous period, that is the post-loss insurance demand. Second, the model provides a unique way to study the effects of increases in risk aversion, increases in the premium, and increases in the perceived probability of loss to the demand for insurance. Therefore, an agent is able to make a decision on the model's variables using an intertemporal approach.

1.3.2. Analytical Framework

Take an intertemporal perspective, with two consecutive periods. This analysis studies the insurance demand strategy of an individual for the two consecutive periods of time. In each period, a loss can occur, and insurance coverage can be sought in both periods. Assume there exists an insurer willing to offer insurance contracts that provide positive expected profit[1]. Assume the same loss in each period and that the individual faces a risk of loss of amount L at each period. An insurance contract C_t, where $t = 1, 2.$ is proposed for one period. Thus, individuals have to make a decision on the choice of the amount of coverage at each period. The insurance contract offers indemnity I_t in return of premium Π_t. Assume further that for each decision (insurance) period the estimated probability of incurring a loss is p and that losses in the consecutive periods are independent such that:

$$\text{P}(\text{loss at perod } t/\text{loss in period } t-1) = p.$$

[1] If the contracts were actuarially fair, the insured would only purchase full coverage always. We need, therefore, that the contracts be actuarially unfair, or in other words, that they offer positive expected profit to the insurer.

The outcome of the first-period, a situation which is governed by chance, will impact upon the choice to be made in period two. In the same way, the choice made in period one will impact upon the choice made in period two. For instance, in a simple two-dimensional loss situation, in period one a level of insurance coverage C_1 is purchased against a loss amount L that happens with probability p. If $C_1 < L$, that is partial coverage was purchased, and if the loss happens, then in period two, wealth is lower than it would otherwise have been by the amount of uninsured losses. This will impact upon the decision made in period two.

Thus, in period two, the optimal insurance choice, C_2, will be a function of (i) the size of loss in period one[2], (ii) the amount of coverage in period one, and (iii) all the standard parameters in period two[3]. The size of period one´s loss and the level of period one´s coverage will impact upon the level of initial wealth in period two.

It is also assumed that w_1 denotes the amount of initial wealth in period one, w_2^1 is the level of initial period two wealth conditional upon a loss occurring in period one, and w_2^2 is the level of initial period two wealth conditional upon no loss occurring in period one. Insurance is priced linearly, such that an indemnity of C costs qC, where $p < q < 1$. The model also introduces an intertemporal preference parameter β, which is used to measure period two utility in period one utility units.

Now, the consumer's problem is to choose C_1 to maximise the function (Equation 1.1);

$$\begin{aligned} &p[u(w_1 - L + C_1(1-q)) + \beta(pu(w_2^1 - L + C_2^1(1-q)) + (1-p)u(w_2^1 - qC_2^1))] + \\ &(1-p)[u(w_1 - qC_1) + \beta(pu(w_2^2 - L + C_2^2(1-q)) + (1-p)u(w_2^2 - qC_2^2))] \end{aligned} \tag{1.1}$$

Where C_2^i maximises $pu(w_2^i - L + C_2^i(1-q)) + (1-p)u(w_2^i - qC_2^i)$ (1.2)

[2] At this point, either loss or no loss.

[3]Probability of loss, premium price, etc.

Since w_2^i will be a function of C_1 for $i = 1, 2,$ this is a problem that needs to be solved recursively using backward induction. Starting then with the two optimisation problems in period two, this gives optimal choice functions $C_2^i(C_1)$. If a loss in period one leads to greater insurance in period two than if no loss happened in period one, then $C_2^1(C_1) > C_2^2(C_1)$. Moreover, if a loss in period one leads to more insurance in period two than what was purchased in period one, then it would see that $C_2^1(C_1) > C_1$. Once the two functions $C_2^i(C_1)$ have been found, it is then possible to do the first period optimisation problem and find the optimal choice for C_1.

Observation 1. *If the consumer is only partially insured in period one, and a loss event does happen, then initial wealth in period two would be lower by the amount of uninsured loss. So, by under-insuring, the consumer causes a larger decrease in period two wealth when a loss happens in period one, but a higher period two wealth if a loss does not happen in period one (since the period one premium would be lower).*

Standard Result: Under decreasing absolute risk aversion (DARA), Observation 1 would also imply a greater demand for coverage in period two. But this is conditional on a loss happening in period one, and partial coverage.

As an example, use $u(x) = \frac{x^{1-R} - 1}{1-R},$ which is constant relative risk aversion (CRRA), and for which $u'(x) = x^{-R}$. Start with the period two choices; we need to choose C_2^i to maximise

$pu(w_2^i - L + C_2^i(1-q)) + (1-p)u(w_2^i - qC_2^i)$.

The first-order condition is expressed in Equation 1.3;

$$\frac{p(1-q)}{q(1-p)} = \frac{u'(w_2^i - qC_2^i)}{u'(w_2^i - L + C_2^i(1-q))} = \frac{(w_2^i - qC_2^i)^{-R}}{(w_2^i - L + C_2^i(1-q))^{-R}} = \left(\frac{(w_2^i - qC_2^i)}{(w_2^i - L + C_2^i(1-q))}\right)^{-R} \quad (1.3)$$

Thus, in this case, we get the expression presented in Equation 1.4;

$$\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} = \frac{(w_2^i - qC_2^i)}{(w_2^i - L + C_2^i(1-q))}$$

$$\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (w_2^i - L + C_2^i(1-q)) = (w_2^i - qC_2^i)$$

$$\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} C_2^i(1-q) + qC_2^i = w_2^i - \left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (w_2^i - L)$$

$$C_2^i\left[\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (1-q) + q\right] = w_2^i - \left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (w_2^i - L)$$

$$C_2^i = \frac{w_2^i - \left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (w_2^i - L)}{\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} (1-q) + q} \quad (1.4)$$

If we set $\left(\frac{p(1-q)}{q(1-p)}\right)^{-\frac{1}{R}} \equiv k,$ then the optimal insurance purchase in period two is expressed as shown in Equation 1.5;

$$C_2^i = \frac{w_2^i - k(w_2^i - L)}{k(1-q) + q} \quad (1.5)$$

It can be noted that the only difference between the two options is the size of initial wealth, w_2^i. Thus, we now have Equation 1.6;

$$\frac{\partial C_2^i}{\partial w_2^i} = \frac{1-k}{k(1-q)+q} \qquad (1.6)$$

Since the denominator of this is positive, the effect of larger initial wealth upon the optimal insurance purchase is positive if $k<1$, and negative if $k>1$. But $k>1$ if $p(1-q)<q(1-p)$, in other words that is, if $p<q$. This is the condition for positive insurer profits, so it should be assumed to be so, in which case C_2^i is greater the smaller is w_2^i. Assuming partial insurance in period one, it is expected that $w_2^1 < w_2^2$, that is smaller initial period two wealth if a loss is suffered in period one than if not. In this case, then, the period two insurance purchase is greater after a period one loss has happened than when a period one loss did not happen.

The indirect utility function for period two can thus be written as in Equation 1.7;

$$pu(w_2^i - L + C_2^i(1-q)) + (1-p)u(w_2^i - qC_2^i) = p\frac{(w_2^i - L + C_2^i(1-q))^{1-R} - 1}{1-R} + (1-p)\frac{(w_2^i - qC_2^i)^{1-R} - 1}{1-R}$$

$$= Eu(w_2^i) \qquad (1.7)$$

In order to continue, an assumption is made on how the period one outcome affects period two initial wealth.

Assumption 1: In both periods, wealth is simply the level of inherited wealth from the previous period plus an intertemporally constant wage of amount, w. That means, savings are rewarded with an interest rate of zero, and that there is no consumption outside of insurance in period one.

This assumption now gives Equations 1.8 and 1.9;

$$w_2^1 = w_1 + C_1(1-q) - L + w \qquad (1.8)$$

$$w_2^2 \qquad = w_1 + qC_1 + w \tag{1.9}$$

When the period one insurance choice is made, the consumer now maximises as per equation 1.10;

$$p\left[u(w_1 - L + C_1(1-q)) + \beta Eu(w_2^1)\right] + (1-p)[u(w_1 - qC_1) + \beta Eu(w_2^2)] \tag{1.10}$$

The first-order condition is can be simplified as given in Equation 1.11;

$$p\left(u'(w_1 - L + C_1(1-q))(1-q) + \beta \frac{\partial Eu(w_2^1)}{\partial w_2^1}\frac{\partial w_2^1}{dC_1}\right) + (1-p)\left(u'(w_1 - qC_1)(-q) + \beta \frac{\partial Eu(w_2^2)}{\partial w_2^2}\frac{\partial w_2^2}{dC_1}\right) = 0$$

Using the specific utility function, this is equal to

$$p\left(\frac{(w_1 - L + C_1(1-q))^{1-R} - 1}{1-R} + \beta\left(p\frac{(w_2^1 - L + C_2^1(1-q))^{1-R} - 1}{1-R} + (1-p)\frac{(w_2^1 - qC_1)^{1-R} - 1}{1-R}\right)\right)$$

$$+(1-p)\left(\frac{(w_1 - qC_1)^{1-R} - 1}{1-R} + \beta\left(p\frac{(w_2^2 - L + C_2^2(1-q))^{1-R} - 1}{1-R} + (1-p)\frac{(w_2^2 - qC_2^2)^{1-R} - 1}{1-R}\right)\right)$$

$$p\{(w_1 - L + C_1(1-q))^{-R}(1-q)$$

$$+\beta\left(p(w_2^1 - L + C_2^1(1-q))^{-R}\left(1 + (1-q)\frac{\partial C_2^1}{\partial w_2^1}\right) + (1-p)(w_2^1 - qC_2^1)^{-R}\left(1 - q\frac{\partial C_2^1}{\partial w_2^1}\right)\right)(1-q)\}$$

$$+(1-p)\{(w_1 - qC_1)^{-R}(-q)$$

$$+\beta\left(p(w_2^2 - L + C_2^2(1-q))^{-R}\left(1 + (1-q)\frac{\partial C_2^2}{\partial w_2^2}\right) + (1-p)(w_2^2 - qC_2^2)^{-R}\left(1 - q\frac{\partial C_2^2}{\partial w_2^2}\right)\right)(-q)\}$$

$$= 0$$

with:

$$C_2^i \qquad = \frac{w_2^i - k(w_2^i - L)}{k(1-q) + q}$$

$$C_2^1 = \frac{w_2^1 - k(w_2^1 - L)}{k(1-q)+q}$$

$$C_2^2 = \frac{w_2^2 - k(w_2^2 - L)}{k(1-q)+q}$$

and

$$k = \left((\frac{p(1-q)}{q(1-p)})^{-\frac{1}{R}} \right)$$

also

$$w_2^1 = w_1 + C_1(1-q) - L + w$$

$$w_2^2 = w_1 + qC_1 + w$$

On substitution, it can be shown that;

$$C_2^1 = \frac{(w_1 + C_1(1-q) - L + w) - \left((\frac{p(1-q)}{q(1-p)})^{1-R} \right)((w_1 + C_1(1-q) - L + w) - L)}{\left((\frac{p(1-q)}{q(1-p)})^{1-R} \right)(1-q) + q}$$

$$C_2^2 = \frac{w_1 - qC_1 + w - \left((\frac{p(1-q)}{q(1-p)})^{-\frac{1}{R}} \right)(w_1 - qC_1 + w - L)}{\left((\frac{p(1-q)}{q(1-p)})^{-\frac{1}{R}} \right)(1-q) + q}$$

(1.11)

Equations 1.8 - 1.11 are as a result of assumption one and its defining conditions (1.8) and (1.9). Equations 1.8 and 1.9 imply that there is no consumption. The wealth in the second period is simply the wealth of the first period minus loss plus income. However, there is still utility derived from this wealth, but the model doesn’t expound on what this utility is derived from. It cannot be consumption as that would reduce the transferred wealth, but if we allow the free choice of the amount of wealth the agents are allowed to keep and/or transfer, then savings and loans are essentially the same as consumption. However, since the wealth is evaluated through two utility functions, it is ambiguous how the results translate to known comparative statics results. In most cases, models of insurance in an intertemporal setting have to be solved both over insurance demand and consumption choices. As it stands, the model needs some improvement in the future more so on the effect of consumption decision and corresponding methodology to solve a bivariate optimization problem.

Next is an illustrative example of the model numerically. The properties of this model have been demonstrated both analytically then numerically that $C_2^1(C_1) > C_2^2(C_1)$ and how C_2^i relates to C_1; this is also supported by the assumption of a CRRA utility function. The two derivations are important because the numerical illustration alone can only be reduced to a single point in a graph.

1.3.3. Numerical Illustration of the Intertemporal Model

This section uses hypothetical values to demonstrate how insurance demand changes pre- and post-loss and how it compares with insurance demand based on empirical data from both the demand and the supply sides of the market. The insurance scenario described by the intertemporal framework model using hypothetical values is simulated and results are shown in the following scenario.

Given the parameters;

w_1	w	L	p	q	R	β
100	10	40	0.4	0.5	2	0.9

Then;

$$C_2^1 = \frac{(100 + C_1(1-0.5) - 40 + 10) - \left((\frac{0.4(1-0.5)}{0.5(1-0.4)})^{-\frac{1}{2}} \right)((100 + C_1(1-0.5) - 40 + 10) - 40)}{\left((\frac{0.4(1-0.5)}{0.5(1-0.4)})^{-\frac{1}{2}} \right)(1-0.5) + .05}$$

$$= 29.898 - 0.10102 C_1$$

$$C_2^2 = \frac{100-0.5C_1+10-\left((\frac{0.4(1-0.5)}{0.5(1-0.4)})^{-\frac{1}{2}}\right)(100-0.5C_1+10-40)}{\left((\frac{0.4(1-0.5)}{0.5(1-0.4)})^{-\frac{1}{2}}\right)(1-0.5)+0.5}$$

$$= 0.10102C_1 + 21.816$$

Candidate(s) for extrema value for period one coverage are:

$$\{-13.864, 1.807, 1.875, 1.900, 1.964-0.130i, 1.964+0.130i, 1.887-7.530*10^{-2}i, 1.887+7.530*10^{-2}i\}$$

At

$$[C=-109.23-9.9402i],[C=-109.23+9.9402i],[C=168.50]$$

$$[C=195.77-7.2971i],[C=195.77+7.2971i],[C=179.95]$$

$$[C=-1456.0],[C=25.070]$$

So the optimum is at 25.070.

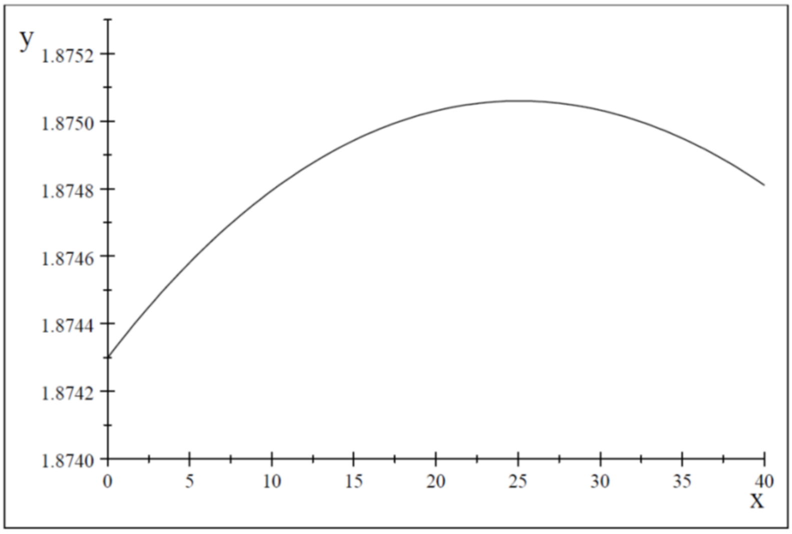

Figure 1: Optimal insurance coverage with intertemporal consideration

Comparing this to the solution for the case without intertemporal consideration,

$$0.4\left(\frac{(100-40+C(1-0.5))^{1-2}-1}{1-2}\right)+(1-0.4)\left(\frac{(100-0.5C)^{1-2}-1}{1-2}\right)$$

Candidate(s) for extrema: $\{0.98763, 0.99987\}$, at $\{[C=23.837],[C=-1543.8]\}$. Therefore optimum is at 23.837.

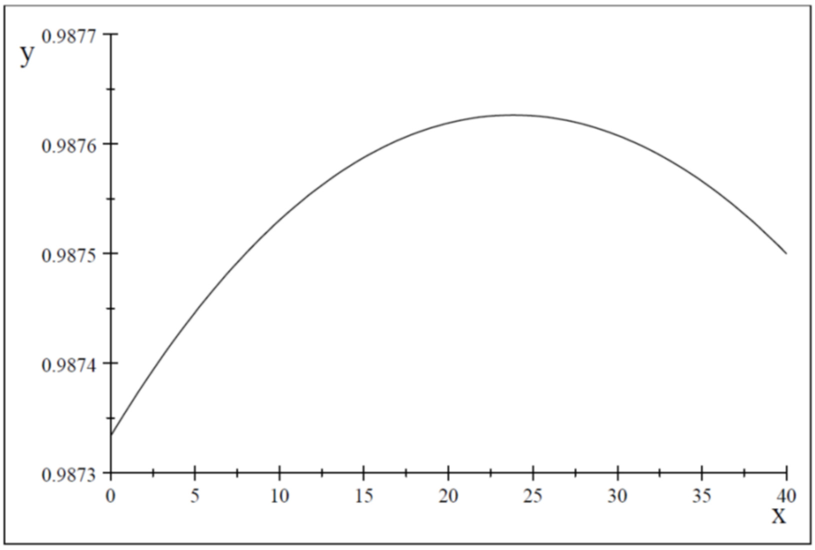

Figure 2: Optimal insurance coverage without intertemporal consideration

Results of the numerical simulation are plotted in Figure 1 and Figure 2. It can be clearly seen that the optimal insurance demand when there is an intertemporal consideration is at 25.070 whereas when there is not the intertemporal consideration; the optimal insurance demand is at 23.837.

So, for this example at least, taking into account the intertemporal nature increases insurance demand, finally;

$$C_2^1 = 29.898 - 0.10102 \times 25.070 = 27.365$$

$$C_2^2 = 0.10102 \times 25.070 + 21.816 = 24.349$$

So, insurance demand after loss increases by 9.2 percent (that is, from 25.070 to 27.365), and the demand if no loss falls by 2.9 percent (that is, from 25.070 to 24.349).

The principal findings in this analysis indicate that insurance demand increases immediately after the loss event. This is in line with the availability bias which corresponds to an overestimation of the probability of an event that recently occurred and implies an increase in insurance demand after a natural disaster, this demand being low after a long period without a catastrophe. However, the result found does not rely in any way on probability estimation bias or errors.

1.4. Discussion and Conclusions

This paper provides a theoretical explanation for the observation on insurance demand post-loss. Several findings can be drawn from the model's results. First, it is notable that using the theoretical intertemporal model proposed in this analysis; (i) period one demand for insurance increases relative to the standard single period model, when the period two is taken into consideration, (ii) period two insurance demand is higher post-loss, higher than both the period one demand, and the period two demand without a period one loss.

Second, when the intertemporal model results are compared to the findings from previous studies on the insurance demand, it can be inferred that insurers will tighten the underwriting criteria when the insurance demand increases post-loss. In essence, the tight underwriting criterion means that bad risks in a loss event are reclassified into a higher-risk category with higher premiums. Positive changes in the insurance demand will increase the cost of insurance cover in the short-run.

Lastly, this analysis is still incomplete in a number of important ways. The model has restricted the analytical analysis by requiring certain assumptions. The critical limitation of model analysis is the Assumption 1 and its defining conditions (1.8) and (1.9) implying no consumption. The wealth in the second period is simply the wealth of the first period minus loss plus income. However, there is still utility derived from this wealth in both period one and period two. It cannot be consumption as that would reduce the transferred wealth, but if we allow the free choice of the amount of wealth the agents are allowed to keep and/or transfer, then savings and loans would be essentially the same as consumption. However, since the wealth is evaluated through two utility functions, it is ambiguous how the results translate to known comparative statics results. In previous studies (Cooper and Hayes, 1987; Schlesinger 2000; Volkman-Wise 2015) models of insurance in an intertemporal setting have been solved both over insurance demand and consumption choices. Future work will need to relax this assumption then compare results presented in this analysis with evidence from the insurance market post-loss.

REFERENCES

Arrow, K. J. (1963). Uncertainty and the Welfare Economics of Medical Care. *The American Economic Review, 53*(5), 941-973.

Arrow, K. J. (1965). *Aspects of the theory of risk-bearing*. Helsinki: Yrjö Jahnssonin Säätiö, Helsinki.

Arrow, K. J. (1974). Optimal insurance and generalized deductibles. *Scandinavian Actuarial Journal, 1974*(1), 1-42.

Auffret, P. (2003). High consumption volatility: The impact of natural disasters? *World Bank Policy Research Working Paper*, No. 2962. http://dx.doi.org/10.1596/1813-9450-2962

Born, P., & Viscusi, W. K. (2006). The catastrophic effects of natural disasters on insurance markets. *Journal of Risk and Uncertainty*, *33*(1-2), 55-72.

Briys, E., Kahane, Y., and Kroll, Y. (1988). Voluntary insurance coverage, compulsory insurance, and risky-riskless portfolio opportunities. *Journal of Risk and Insurance*, *55*(4), 713-722.

Browne, M. J., and Hoyt, R. E. (2000). The Demand for Flood Insurance: Empirical Evidence. *Journal of Risk and Uncertainty, 20*(3), 291-306.

Cameron, L. and Shah, M. (2012) 'Risk-Taking Behavior in the Wake of Natural Disasters', IZA Discussion Papers, No. 6756, Institute for the Study of Labor (IZA).

Cohen, M., Etner, J., and Jeleva, M. (2008). Dynamic decision making when risk perception depends on past experience. *Theory and Decision*, *64*(2-3), 173-192.

Cooper, R., & Hayes, B. (1987). Multi-period insurance contracts. International Journal of Industrial Organization, 5(2), 221-231.

Dreze, J. H. (1981). Inferring risk tolerance from deductibles in insurance contracts. *Geneva Papers on Risk and Insurance*, 6(20), 48-52.

Eeckhoudt, L., and Kimball, M. (1992). Background risk, prudence, and the demand for insurance, in *Contributions to insurance economics* (Dionne G. [Ed]), Springer Netherlands, pp. 239-254.

Froot, K. A., and O'Connell, P. G. (1999). The pricing of US catastrophe reinsurance, in *The Financing of Catastrophe Risk* (Froot, K. A., [Ed]). University of Chicago Press, pp. 195-227

Gollier, C. (2002). Time diversification, liquidity constraints, and decreasing aversion to risk on wealth. *Journal of Monetary Economics*, *49*(7), 1439-1459.

Gollier, C. (2003). To Insure or Not to Insure?: An Insurance Puzzle. *GENEVA Papers on Risk and Insurance - Theory, 28*(1), 5-24.

Kimball, M. S. (1991). *Precautionary motives for holding assets*, NBER Working Paper No. 3586. National Bureau of Economic Research, Cambridge.

Kunreuther, H. (1996). Mitigating disaster losses through insurance. *Journal of risk and Uncertainty*, *12*(2-3), 171-187.

Mayers, D., and Smith Jr, C. W. (1983). The interdependence of individual portfolio decisions and the demand for insurance. *Journal of Political Economy, 91*(2), 304-311.

Meyer, D., and Meyer J. (2004). A More Reasonable Model of Insurance Demand, in *Assets, Beliefs, and Equilibria in Economic Dynamics* (Aliprantis, C. D, Arrow K. J., Hammond P., Kubler F., Wu H. M., and Yannelis N. C. [Eds.]), Essays in Honor of Mordecai Kurz, Studies in Economic Theory (vol. 18), Springer-Verlag, pp. 733-742.

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. *The Journal of Political Economy, 76*(4), 553-568.

Ranger, N., & Surminski, S. (2013). A preliminary assessment of the impact of climate change on non-life insurance demand in the BRICS economies. *International Journal of Disaster Risk Reduction*, *3*, 14-30.

Raviv, A. (1979). The design of an optimal insurance policy. *American Economic Review, 69*(1), 84-96.

Schlesinger, H. (2000). The theory of insurance demand. In Handbook of insurance (pp. 131-151). Springer Netherlands.

Smith, V. L. (1968). Optimal insurance coverage. *The Journal of Political Economy*, 68-77.

Volkman-Wise, J. (2015). Representativeness and managing catastrophe risk. Journal of Risk and Uncertainty, 51(3), 267-290.