On the Interaction between Saving and Risk Reduction

February 14, 2019

Abstract

This paper presents a model of the joint demand for saving and risk reduction. This is motivated by saving decisions in the presence of a future consumption risk, which is endogenous because the decision-maker anticipates to engage in risk reduction, for example by purchasing insurance. We show that the interaction between saving and insurance is driven by whether absolute risk aversion in the second period decreases or increases in wealth so that insurance is either a substitute or a complement for saving as long as relative risk aversion is bounded by unity. Furthermore, for decreasing absolute risk aversion saving is a substitute for insurance. These results carry over to more general forms of nth-degree risk reduction by formulating the associated conditions based nth-degree Ross instead of Arrow-Pratt risk aversion. We also show that risk reduction is a critical determinant of the intensity of the precautionary saving motive.

Keywords: insurance · risk · risk aversion · risk reduction · saving

JEL-Classification: D11 · D14 · D81 · D91 · G22

1 Introduction

It is common in the literature on optimal decision making under risk and uncertainty to focus on specific decision variables and to study them in isolation when modeling the agent's cost-benefit trade-off. For instance, the propensities to purchase insurance, to engage in precautionary saving or to perform prevention activities are in many cases dealt with separately. Although convenient analytically, this assumption is hardly descriptive because oftentimes individuals and households have more than one tool available to maximize expected consumption utility. Several contributions in the literature illustrate that different types of decision variables can interact in non-trivial ways, which is important both for normative as well as positive economics, and can inform empirical work. Ehrlich and Becker's (1972) classical contribution was the first analysis dedicated to the interaction between various instruments used to manage financial risks, insurance and self-protection on the one hand, and insurance and self-insurance on the other hand. This initiated a series of theoretical papers examining joint risk management decisions. Among these, Dionne and Eeckhoudt (1984) considered the relationship between saving and insurance decisions, Menegatti and Rebessi (2011), Hofmann and Peter (2016) and Peter (2017) examined simultaneous saving and self-protection efforts and Courbage et al. (2015) study the interplay of preventive activities targeting different and potentially interdependent sources of risk.[1]

In this paper, we study the demand for saving in the presence of an *endogenous* future consumption risk. This endogeneity arises because agents anticipate to engage in risk mitigation in the future (e.g., by purchasing insurance) so that our analysis can be viewed as a direct extension of the stream of literature that studies the interaction of optimal decisions under risk. Taken in isolation, the demand for precautionary saving has been shown to depend - in the expected utility model - on the sign of the third derivative of the utility function (see Leland, 1968; Sandmo, 1970; Drèze and Modigliani, 1972) while Kimball (1990) demonstrated that the intensity of this demand was measured by the ratio of minus the third derivative to the second derivative of the utility function. In these contributions, the future consumption

[1] Note that the analysis of joint actions undertaken to protect oneself against disease has been introduced into the health economics literature in recent years, see for instance the analysis of self-protection activities and disease treatment in Hennessy (2008), Menegatti (2014) or Brianti et al. (2017).

risk is exogenous because agents have no means of affecting it. Surprisingly, the role of endogenous risks and their effect on saving has not been discussed much.[2] This is despite the fact that individuals have a variety of instruments to address future consumption risks (insurance, self-protection, self-insurance, etc.). We thus consider in this paper that individuals can engage in risk reduction at that point in time when they will be exposed to the risk so that the demand for saving and the demand for risk reduction interact. We consider the case of insurance for its own sake before moving to the more general case. Therefore, the first part of our analysis can be seen as a complement to Dionne and Eeckhoudt (1984) who study Hicksian demand for insurance and saving in a set-up where both decisions are taken in the first period.[3] In our paper, we examine Walrasian demand instead to allow for wealth effects and focus on the case where the decision-maker anticipates that future consumption risks will be mitigated in the period they occur, when taking his saving decision.[4]

Our first result is that the interaction between saving and insurance depends on how the Arrow-Pratt measure of absolute risk aversion changes with wealth. Specifically, if relative risk aversion is bounded by unity an increase in the interest rate increases the demand for saving and decreases or increases the demand for insurance depending on whether absolute risk aversion is decreasing or increasing in wealth. So insurance can be a substitute or a complement for saving depending on the shape of the index of absolute risk aversion in the second period. Furthermore, as long as insurance is an ordinary good and absolute risk aversion is decreasing in wealth, an increase in the price of insurance increases the demand for saving so that saving is a substitute for insurance. We show that all of these results can be extended to general forms of nth-degree risk reduction by formulating the sufficient conditions in terms of how nth-degree Ross risk aversion depends on wealth (see Wang and

[2] Several papers study joint saving and labor supply decisions with exogenous wage rate risk, see, for example, Flodén (2006) and Nocetti and Smith (2011) for two-period models and Low (2005) and Marcet et al. (2007) for calibration results. In such a situation, the source of risk is exogenous although the effective exposure to it depends on the agent's labor supply.

[3] Hicksian demand is obtained when keeping expected utility constant in the comparative statics analysis by eliminating wealth effects. To the best of our knowledge, Dionne and Eeckhoudt (1984) is the only paper that studies the demand for insurance in the Hicksian sense.

[4] Besides that several authors have studied insurance and saving in continuous time. Briys (1986) demonstrates the separability between both decisions in case of actuarially fair premiums, Gollier (1994) shows that precautionary saving domintes insurance demand in the long run if the loading exceeds a critical level, and Moore and Young (2006) find that deductible insurance is optimal under certain assumptions.

Li, 2014). As it turns out, the substitutability between saving and risk reduction provides for a useful application of decreasing Ross risk aversion in our model. A third contribution of our analysis is to show that the endogeneity of risk influences the trade-off between consumption smoothing and precautionary saving. As is intuitive, introducing the possibility to mitigate risk lowers precautionary saving but increases savings to smooth consumption to compensate expenditures devoted to risk reduction. We present a numerical example that illustrates how precautionary saving as a fraction of the total demand for saving can vary between 0% and over 20% depending on the insurability of the consumption risk for one and the same decision-maker whose degree of relative prudence is at a fixed level.

The paper is organized as follows. Section 2 presents a simple model of saving and insurance and analyzes the comparative statics of the joint optimum with respect to the interest rate and the price of insurance. Section 3 extends the analysis to a more general model of nth-degree risk reduction and introduces sufficient conditions to generalize our results. The final section concludes.

2 A Simple Model of Saving and Insurance

2.1 Preliminaries

Consider a decision-maker (DM) who lives for two periods. His consumption stream $(c_1, \widetilde{c}_2)$ consists of certain consumption c_1 in the first period and risky consumption $\widetilde{c}_2$ in the second period, where the tilde indicates a random variable. The DM's discounted expected utility is given by $u(c_1) + \beta \mathbb{E} v(\widetilde{c}_2)$, where u denotes his first-period utility function, v his second-period utility function, and β the utility discount factor. We assume non-satiation and risk aversion in each period such that $u' > 0$, $u'' < 0$, $v' > 0$ and $v'' < 0$. The DM receives certain income of w_1 and w_2 in the first and second period, respectively.

The uncertainty in the second period results from a potential loss of size l that occurs with probability p. The DM's effective exposure to this risk is endogenous because we assume insurance to be available in the second period to protect against the financial consequences of a loss. We denote the level of coverage by $\alpha \in [0, 1]$ and the loading factor by $\lambda \geq 0$. Then, the per-unit price of insurance is given by $(1 + \lambda)$ and the premium π associated with a level of coverage of α is given by $(1+\lambda)\alpha pl$. Furthermore, the DM decides about his consumption allocation over time by specifying a level of saving s in the first period. Savings are deducted from first-period income and yield interest according to the non-random interest rate $r \geq 0$ in the second period. With these specifications, the DM's objective function is given by

$$\max_{\alpha,s} U(\alpha,s) = u(w_1 - s) + \beta\left[pv(w_2 + s(1+r) - \pi - (1-\alpha)l) + (1-p)v(w_2 + s(1+r) - \pi)\right].$$

Unlike previous literature, the saving decision in our model is undertaken in the presence of an *endogenous* additive consumption risk because the riskiness of the DM's second-period income depends on his insurance choice. To compress notation, we use subscripts '1', '2L' and '2N' to denote consumption in the first period, the second-period loss state and the second-period no-loss state, respectively. The first-order conditions for the DM's maximization problem are given by

$$\begin{aligned} U_\alpha &= \beta l\left[(1-(1+\lambda)p)pv'_{2L} - (1+\lambda)p(1-p)v'_{2N}\right] = 0, \\ U_s &= -u'_1 + \beta(1+r)\left[pv'_{2L} + (1-p)v'_{2N}\right] = 0. \end{aligned} \tag{1}$$

Optimal choices are indicated with an asterisk, that is α^* and s^*. The first equation determines the optimal level of insurance coverage such that the marginal rate of substitution between second-period consumption in the loss and the no-loss state is equal to the slope of the line of insurance. The second equation specifies the optimal level of saving such that expected marginal utility of consumption is equal at both points in time. We show in Appendix A.1 that the second-order conditions are satisfied.

Before we proceed, we inspect the cross-derivative of the DM's objective function with respect to the level of insurance and saving. Direct computation yields that

$$U_{\alpha s} = \beta(1+r)l\left[(1-(1+\lambda)p)pv''_{2L} - (1+\lambda)p(1-p)v''_{2N}\right],$$

which is sign ambiguous. If $A(w) = -v''(w)/v'(w)$ denotes the coefficient of Arrow-Pratt risk aversion of the utility function v in the second period,[5] we can use the first-order condition

[5] Bommier et al. (2012) show that only certain classes of utility functions over "certain $\times$ uncertain" con-

for optimal insurance demand to rewrite the cross-derivative as follows:

$$U_{\alpha s} = \beta(1+r)(1+\lambda)p(1-p)lv'_{2N}\left[A_{2N} - A_{2L}\right].$$

This informs us about the relationship between insurance and saving at an optimal choice, which we summarize in the following remark.

Remark 1. *Insurance and saving are Edgeworth-Pareto substitutes (complements) in the sense of Samuelson (1974) when second-period risk aversion is decreasing (increasing) in wealth.*

A higher level of saving increases the individual's certain level of consumption in the second period, which decreases the optimal demand for insurance when the utility function exhibits decreasing absolute risk aversion (DARA) in the second period (see Mossin, 1968). Likewise, a higher level of insurance coverage reduces expected wealth due to the loading, which stimulates more savings, but also decreases the riskiness of consumption in the second period, which stimulates less savings (see Kimball, 1990). Under DARA, prudence exceeds risk aversion so that the second effect preponderates and a lower level of saving will be optimal. Likewise, if the utility function in the second period exhibits increasing absolute risk aversion (IARA), the reverse intuition applies. This simple observation illustrates that there is a non-trivial interaction between the demand for saving and the demand for insurance as soon as Arrow-Pratt risk aversion of the utility function in the second period is not a constant function of wealth. This interaction will be important in the comparative statics analysis that follows.

2.2 Changes in the Interest Rate

In the sequel, we assume the optimal level of saving to be positive as in Eeckhoudt and Schlesinger (2008). In the presence of an exogenous income risk, it is well-known that optimal saving is increasing in the interest rate if relative risk aversion is less than unity (see Proposition 63 in Gollier, 2001). The DM trades off a substitution effect because a higher return on saving reinforces the incentive to save, against a wealth effect because a higher interest

sumption pairs are well ordered in terms of risk aversion. Our analysis is immune to this problem because we abstain from interpersonal comparisons in the sense that both u and v are fixed throughout the paper.

rate makes the DM wealthier in the second period, which attenuates the incentive to save.[6] When risk is endogenous via insurance, we also need to determine how the DM's demand for insurance is affected by a change in the interest rate. A higher interest rate increases the DM's wealth in the second period and therefore exerts a wealth effect on insurance demand. We summarize our findings in the following proposition.[7]

Proposition 1. *Assume relative risk aversion in the second period to be bounded by unity. An increase in the interest rate increases the demand for saving and decreases (increases) the demand for insurance if risk aversion in the second period is decreasing (increasing) in wealth.*

Said differently, insurance is a substitute (complement) for saving in the Walrasian sense when risk aversion in the second period is decreasing (increasing) in wealth. The intuition behind this result is the following. An increase in the interest rate has a direct effect on each, the demand for saving and the demand for insurance. For relative risk aversion below unity, the direct effect of a higher interest rate is to increase saving. The direct effect of a higher interest rate on insurance is a pure wealth effect, and is goverend by whether risk aversion in the second period is decreasing or increasing in wealth (see Mossin, 1968), in which case a higher interest rate decreases or increases insurance, respectively. In the DARA case, the increase in saving reinforces the decrease in insurance demand and vice versa (see Remark 1) so that indirect effects are aligned with direct ones. The demand for saving increases, whereas the demand for insurance decreases. Similarly, under IARA, the increase in saving reinforces the increase in insurance demand and vice versa (see Remark 1). Again, indirect effects are aligned with direct ones and both the demand for saving and the demand for insurance increase.

We point out the special case of constant absolute risk aversion (CARA) in the second period, which is obtained if v is of the negative exponential class. Then, interaction effects between saving and insurance are absent per Remark 1. As a consequence, the saving decision and the insurance decision are separable and the comparative statics analysis simplifies because the DM chooses the same level of insurance coverage for any interest rate.

[6] Notice that the wealth effect is positive when the optimal level of saving is negative because then a higher interest rate would impoverish the consumer.

[7] All proofs are gathered in the appendix.

Dionne and Eeckhoudt (1984) show that under decreasing temporal risk aversion[8], insurance is a substitute for saving in the Hicksian sense, so for compensated price changes that keep expected utility constant. Our Proposition 1 extends this result to the case where the insurance premium is paid in the second period and Walrasian instead of Hicksian demand is considered. We show further that insurance can be a complement for saving when risk aversion in the second period is increasing in wealth.

2.3 Changes in the Price of Insurance

As was first discussed by Hoy and Robson (1981), insurance can be a Giffen good in the standard model of insurance demand. The reason is that a price increase will induce a positive wealth effect on insurance demand under DARA. Hoy and Robson (1981), Briys et al. (1989) and Hau (2008) discuss necessary and/or sufficient conditions for insurance not to be Giffen. As it turns out, although theoretically possible, it is quite implausible for insurance to be a Giffen good. Therefore, and for lack of empirical evidence, we assume insurance to be an ordinary good that satisfies the law of demand. Then, the effect of price changes on the demand for saving and the demand for insurance are as follows.

Proposition 2. *Assume insurance to be an ordinary good and risk aversion in the second period to be decreasing in wealth. Then, an increase in the price of insurance decreases the demand for insurance and increases the demand for saving.*

This result says that saving is a substitute for insurance when risk aversion in the second period is decreasing in wealth. Intuitively, the direct effect of an increase in the price of insurance is to lower the demand for insurance and to increase the demand for saving. The first effect is per assumption, whereas the second one results from the DM's propensity to smooth consumption over the lifecycle. A higher price of insurance reduces expected consumption in the second period and therefore stimulates saving. In the DARA case, this increase in saving reinforces the decrease in insurance demand and vice versa per Remark 1. Indirect effects and direct effects are aligned and the comparative statics are clear.

[8] EXPLAIN THAT CONCEPT

Notice that unlike in the previous proposition, saving will not necessarily be a complement for insurance when risk aversion in the second period is increasing in wealth. Technically, the reason for this is that the direct effect of an increase in the price of insurance on saving is positive whenever the utility function in the second period is concave, no matter whether its risk aversion is increasing or decreasing in wealth. Intuitively, this is due to the DM's propensity to smooth consumption over the lifecycle. The direct and indirect effect of an increase in the price of insurance on saving are conflicting in such a situation and the net effect depends on their relative strength.

Dionne and Eeckhoudt (1984) show that saving is a substitute for insurance in the Hicksian sense under decreasing temporal risk aversion.

2.4 Consumption Smoothing and Precautionary Saving

Thus far we have shown that under reasonable conditions, insurance is a substitute for saving and vice versa. The interplay between insurance and saving also determines to what extent saving is utilized for precautionary purposes. To shed some light on this question, we analyze the effect of the insurability of the consumption risk at the margin. We start with the case of an exogenous risk and carve out the DM's saving response, both in terms of consumption smoothing and precautionary saving, when the risk becomes just insurable.

The case of an exogenous risk is a special case of our analysis for $\alpha^* = 0$. Let s_0 denote the optimal level of saving in the absence of insurance, which is determined by

$$u'(w_1 - s_0) = \beta(1+r)\left[pv'(w_2 + s_0(1+r) - l) + (1-p)v'(w_2 + s_0(1+r))\right].$$

These choices, $\alpha^* = 0$ and $s^* = s_0$, turn out to be the optimal decisions for any loading factor exceeding what we call the critical loading factor,

$$\lambda^{crit} = \frac{(1-p)\left[v'(w_{2L}^0) - v'(w_{2N}^0)\right]}{pv'(w_{2L}^0) + (1-p)v'(w_{2N}^0)}.$$

w_{2L}^0 and w_{2N}^0 are shorthand for the consumption levels $w_2 + s_0(1+r) - l$ and $w_2 + s_0(1+r)$. The DM pursues saving for two purposes, consumption smoothing and precaution. To disentangle the two, we denote by s_0^{cs} the DM's saving choice if second-period income was risk-free:

$$u'(w_1 - s_0^{cs}) = \beta(1+r)v'(w_2 + s_0^{cs}(1+r) - pl).$$

As first shown by Leland (1968) and Sandmo (1970) and more systematically by Kimball (1990), the DM engages in precautionary saving, $s_0 > s_0^{cs}$, if and only if he is prudent ($v''' > 0$). Our next result shows that the endogeneity of the second-period consumption risk can tilt the balance between consumption smoothing and precautionary saving towards the former.

Proposition 3. *Assume that second-period risk aversion is non-increasing in wealth. At the margin, the insurability of risk in the second period increases the demand for saving to smooth consumption and reduces the demand for precautionary saving.*

The intuition behind this result is similar to before. At the margin, the DM will start utilizing insurance as soon as the loading factor drops below the critical loading factor. The use of insurance results in a decrease in expected wealth as well as a decrease in risk in the second period. The first effect reinforces saving to smooth consumption whereas the second one reduces the DM's propensity to save for precautionary purposes. Due to the fact that the overall demand for saving decreases (see Proposition 2), the first effect reinforces the second one and the precautionary demand for saving is diminished. This finding is important because it highlights that the demand for saving and for precautionary saving depend critically on insurance market conditions via the endogeneity of risk. As a result, variations in these conditions across agents will induce variations in optimal saving choices even if the agents' underlying risk and time preferences are identical. Obviously, this source of heterogeneity bears significant empirical measurement ramifications.

2.5 A Numerical Example

The following example serves to illustrate our findings about the substitution between saving and insurance and about the effects of insurability on the trade-off between consumption smoothing and precautionary saving. We consider an individual with $u(w) = v(w) = \log(w)$ and $\beta = 0.99$, who earns risk-free income of $w_1 = w_2 = \$50,000$ in each period and is subject to a 10% chance of a $\$10,000$ loss in the second period. Table 1 summarizes the optimal

r	0%	1%	2%	3%	4%	5%
α^*	49.74%	49.49%	49.23%	48.98%	48.73%	48.47%
s^*	\$301.51	\$544.80	\$783.33	\$1,017.22	\$1,246.62	\$1,471.65

Table 1: Optimal demand for insurance and saving for different interst rates ($\lambda = 0.1$)

demand for insurance and saving as a function of the interest rate, when the loading factor is given by 10%.

Consistent with Proposition 1, we observe that insurance is a substitute for saving because as the demand for saving increases, the demand for insurance decreases. In this particular example, the effect of the interest rate on insurance demand turns out to be marginal, whereas the effect of the interest rate on optimal savings is quite sizeable. Table 2 illustrates Propositions 2 and 3. The interest rate is fixed at 1% and the loading varies in increments of 5%. The last two rows also state the portion of saving that arises from consumption smoothing (s_{cs}) and the difference between the total demand for saving and the level of saving to smooth consumption ($s^* - s_{cs}$), so in other words the precautionary demand for saving.

λ	0%	5%	10%	15%	20%	λ^{crit}
α^*	100%	73.68%	49.49%	27.15%	6.44%	0%
s^*	\$495.05	\$519.93	\$544.80	\$569.68	\$594.55	\$602.67
s_{cs}	\$495.05	\$513.38	\$519.67	\$515.31	\$501.46	\$495.05
$s^* - s_{cs}$	\$0	\$6.55	\$25.13	\$54.37	\$93.09	\$107.62

Table 2: Optimal demand for insurance and saving and the decomposition of saving into consumption smoothing and precautionary saving at different loading factors ($r = 0.01$)

Consistent with Proposition 2, we observe that saving is a substitute for insurance because as the demand for insurance decreases, the demand for saving increases. As shown in Proposition 3, when the risk of loss becomes just insurable (i.e., considering a marginal decrease of λ starting at λ^{crit}), saving to smooth consumption increases whereas the total demand for saving decreases so that the precautionary demand for saving decreases. Saving to smooth consumption is hump-shaped because the expected cost of the risk of loss in the second period (i.e., the insurance premium at the optimal insurance choice plus the expected loss cost) is hump-shaped, too. Notice that although the DM's relative prudence is given by 2 in all scenarios the intensity of precautionary saving varies with the price of insurance from 21.74% if the risk is exogenous ($\lambda = \lambda^{crit}$) to 0% if insurance is actuarially fair and full coverage is purchased.

3 A More General Model of Saving with Endogenous Risk

3.1 Preliminaries

Insurance is a costly activity that reduces the risk of second-period consumption in the sense of Rothschild and Stiglitz (1970). Indeed, for a given level of saving, we can rewrite second-period consumption levels in the loss and the no-loss state as follows:

$$\begin{aligned} w_{2L} &= w_2 + s(1+r) - \lambda\alpha pl - (1-\alpha(1-p))l, \text{ and} \\ w_{2N} &= w_2 + s(1+r) - \lambda\alpha pl - \alpha pl. \end{aligned}$$

Obviously, an increase in the level of coverage comes at a cost, which we can identify as $\lambda\alpha pl$, in the above equations. It is the portion of the premium that is in excess of the actuarially fair premium and this portion is increasing in the level of coverage. The remainder of the change is a mean-preserving contraction in the second-period wealth distribution because apparently $p \cdot (1-\alpha(1-p))l + (1-p)\cdot \alpha pl = pl$, which is the expected loss.

Inspired by this decomposition, we can analyze the interaction between saving and costly changes in risk more generally. To describe the effects of the risk-reducing activity performed in the second period, suppose that F and G are two cumulative distribution functions defined on the interval $[a, b]$. In what follows, $F_0(w)$ denotes the density function, $F_1(w)$ the cumulative distribution function, and more generally, $F_n(w) = \int_a^x F_{n-1}(z)\mathrm{d}z$ for $x \in [a, b]$ and $n \geq 1$. We recall the following definition based on Ekern (1980).

Definition 1. *The distribution F has more nth-degree risk than G, $G \succ_n F$, if*

(i) $G_k(b) = F_k(b)$ *for* $k = 1, 2, \ldots, n$,

(ii) $G_n(w) \leq F_n(w)$ *for all* $w \in [a, b]$ *with strong inequality for some* w.

The first condition is necessary and sufficient for the $(n-1)$ first moments of G and F to coincide whereas the second condition is sufficient (but not necessary) for the nth moment of F sign adjusted by $(-1)^n$ to exceed the nth moment of G sign adjusted by $(-1)^n$. In the expected utility model, preferences over nth-degree changes in risk in the sense of Ekern (1980) are identified by the signs of subsequent derivatives of the utility function, which motivates the following definition.

Definition 2. *An agent is nth-degree risk-averse if* $\operatorname{sgn} v^{(n)}(w) = (-1)^{n+1}$.

The link between nth-degree risk changes and nth-degree risk aversion is formulated below.

Theorem 1. *The following two statements are equivalent:*

(i) $G \succ_n F$,

(ii) $\int_a^b v(w)\mathrm{d}G(w) > \int_a^b v(w)\mathrm{d}F(w)$, *for all functions v such that* $\operatorname{sgn} v^{(n)}(w) = (-1)^{n+1}$.

It follows from this theorem that an nth-degree risk averter has a positive willingness to pay for a reduction of nth-degree risk. We use the approach in Jindapon and Neilson (2007) to model the DM's opportunity to reduce the level of nth-degree risk of his second-period wealth distribution through a costly activity, whose intensity is denoted by t. Formally, a given level of the risk-reducing activity induces the wealth distribution $H_1(w, t) = (1-t)F_1(w) + tG_1(w)$ with $G_1 \succ_n F_1$, and the unit cost of the activity is denoted by $c > 0$. Our discussion at the beginning of this subsection makes it clear that insurance is a special case of such an activity.

Under these assumptions, the DM's objective function is given by

$$\max_{t,s} \left\{ U(t,s) = u(w_1 - s) + \beta \int_a^b v(w + s(1+r) - ct)\mathrm{d}H_1(w,t) \right\}.$$

To compress notation, let $w + s(1+r) - ct$ be denoted by $z(w)$. The first-order conditions related to the DM's problem are:

$$\begin{aligned} U_t &= -\beta c \int_a^b v'(z(w))\mathrm{d}H_0(w,t) + \beta \int_a^b v(z(w))\mathrm{d}\left[G_1(w) - F_1(w)\right] = 0, \\ U_s &= -u'(w_1 - s) + \beta(1+r) \int_a^b v'(z(w))\mathrm{d}H_1(w,t) = 0. \end{aligned}$$

The first equation determines the optimal level of the risk-reducing activity in terms of marginal benefit and marginal cost. The second equation specifies the optimal level of saving by equating expected marginal utility over the lifecycle. As in Jindapon and Neilson (2007), we assume the objective function to be concave in the choice variables such that the system of first-order conditions uniquely identifies a maximum.

Via integration by parts, the first-order condition $U_t = 0$ can be rewritten as

$$U_t = -\beta c \int_a^b v'(z(w)) \mathrm{d}H_0(w,t) - \beta(-1)^n \int_a^b v^{(n)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w = 0$$

or equivalently as

$$-\frac{(-1)^n \int_a^b v^{(n)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w}{\int_a^b v'(z(w)) \mathrm{d}H_0(w,t)} = c. \tag{2}$$

As demonstrated by Jindapon and Neilson (2007), the choice of t is not ranked according to Arrow-Pratt risk aversion but according to Ross risk aversion (see their Theorem 2). As such it is not surprising that the interaction between risk reduction and saving is governed by the effect of changes in wealth on Ross risk aversion. We provide the following definition, which is obtained by the characterization in Proposition 2.5 in Wang and Li (2014) and interpreted in the strict sense.

Definition 3. *The utility function v displays decreasing nth-degree Ross risk aversion ($n \geq 2$) if there exists a scalar λ_n such that for any x and y,*

$$-\frac{v^{(n+1)}(x)}{v^{(n)}(x)} > \lambda_n > -\frac{v''(y)}{v'(y)}. \tag{3}$$

If the inequalities are reversed, we speak of increasing nth-degree Ross risk aversion and if the left hand side and the right hand side coincide for any x, y we speak of constant nth-degree Ross risk aversion. The cross-derivative of the DM's objective function with respect to the intensity of the risk-reducing activity and saving is given by

$$U_{ts} = -\beta(1+r)c \int_a^b v''(z(w)) \mathrm{d}H_0(w,t) - \beta(1+r)(-1)^n \int_a^b v^{(n+1)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w. \tag{4}$$

To understand when there is a substitution effect between both activities, we rewrite $U_{ts} < 0$

with the help of equation (2) as follows:

$$\frac{(-1)^n \int_a^b v^{(n+1)}(z(w))\,[F_n(w) - G_n(w)]\,\mathrm{d}w}{(-1)^n \int_a^b v^{(n)}(z(w))\,[F_n(w) - G_n(w)]\,\mathrm{d}w} < \frac{\int_a^b v''(z(w))\mathrm{d}H_0(w,t)}{\int_a^b v'(z(w))\mathrm{d}H_0(w,t)}. \tag{5}$$

If v exhibits decreasing nth-degree Ross risk aversion, we obtain from equation (3) that

$$-(-1)^n v^{(n+1)}(x)v'(y) < -(-1)^n v^{(n)}(x)v''(y) \quad \forall x, y,$$

which implies that inequality (5) is satisfied. On the contrary, if utility function v displays increasing nth-degree Ross risk aversion, the reverse inequality holds such that $U_{ts} > 0$. We summarize this in the following remark.

Remark 2. *nth-degree risk reduction and saving are Edgeworth-Pareto substitutes (complements) in the sense of Samuelson (1974) when second period nth-degree Ross risk aversion is decreasing (increasing) in wealth.*

At a higher level of saving, nth-degree risk reduction is less desirable to the agent whenever nth-degree Ross risk aversion is decreasing in wealth. Similarly, a higher level of nth-degree risk reduction reduces expected wealth, which incentivizes the agent to increase savings, but also reduces nth-degree risk which incentivizes the agent to reduces savings (see Eeckhoudt and Schlesinger, 2008). Inspection of Equation (3) reveals that decreasing nth-degree Ross risk aversion can be interpreted as nth-degree Ross risk aversion exceeding second-degree Ross risk aversion such that the latter effect dominates. As a result, higher levels of nth-degree risk reduction are accompanied by lower levels of saving. In the case of increasing nth-degree Ross risk aversion, the relationships are reversed.

3.2 Changes in the Interest Rate

We first wonder how the joint demand for saving and risk reduction is affected by changes in the interest rate. As it turns out, we can recover a version of Proposition 1 for the general case of costly nth-degree risk reduction activities. Our result is summarized in the following proposition.

Proposition 4. *Assume relative risk aversion of second-period utility to be bounded by unity. An increase in the interest rate increases the demand for saving and decreases (increases) the demand for nth-degree risk reduction if second-period nth-degree Ross risk aversion is decreasing (increasing) in wealth.*

This result shows that nth-degree risk reduction is a substitute (complement) for saving in the sense of Walrasian demand functions when second-period nth-degree Ross risk aversion is decreasing (increasing) in wealth. Assume that the interest rate increases. The direct effect on saving is positive as long as relative risk aversion is bounded by unity. Furthermore, this increase in the interest rate increases the individual's wealth in the second period, which reduces his propensity to invest in nth-degree risk reduction due to decreasing nth-degree Ross risk aversion. So the direct effect on the demand for risk reduction is negative. The indirect effects are governed by the substitution effect between saving and risk reduction, see Remark 2, and under decreasing nth-degree Ross risk aversion these indirect effects are aligned with the direct ones.

3.3 Changes in the Price of Risk Reduction

Similarly, we can conduct comparative statics with respect to c, the per-unit price of nth-degree risk reduction. As in the case of insurance, the demand for risk reduction can be Giffen, which is a possibility that we exclude from our analysis.[9] Under these presuppositions, we obtain the following result.

Proposition 5. *Assume nth-degree risk reduction not to be Giffen and nth-degree Ross risk aversion to be non-increasing in wealth. Then, an increase in the per-unit price of risk reduction decreases the demand for risk reduction and increases the demand for saving.*

If the per-unit price of risk reduction increases, this exerts a negative direct effect on the demand for risk reduction per assumption. Also, wealth in the second period is reduced due to the higher cost of risk reduction which exerts a positive effect on the demand for saving.

[9] A sufficient condition for nth-degree risk reduction not to be Giffen is that nth-degree relative risk aversion be bounded by n and that total spending on risk reduction as a fraction of wealth be bounded by $1/(n+1)$, see the Appendix.

This effect results from the concavity of utility. To achieve consistency between the direct and the indirect effects, nth-degree Ross risk aversion needs to be decreasing in wealth or at least not increasing in wealth so that the direct effect that less risk reduction exerts on saving is positive and the direct effect that higher savings exert on risk reduction is negative.

3.4 Consumption Smoothing and Precautionary Saving

We will finally use our results to understand the effect of risk reduction on saving and especially on the decomposotion of saving into consumption smoothing and precautionary saving. The analysis is similar to before and we start out with the case in which risk reduction is prohibitively expensive. When marginally lowering the per-unit cost of risk reduction, we are able to isolate the effect of the risk exposure's endogeneity at the margin.

The DM abstains from nth-degree risk reduction whenever $t^* = 0$ is optimal, in which case $H(w, 0) = F(w)$. Let s_0 denote the optimal level of saving when the DM does not engage in risk reduction. It is the solution to

$$u'(w_1 - s^0) = \beta(1+r)\int_a^b v'(w + s^0(1+r))\mathrm{d}F(w).$$

The choices $t^* = 0$ and $s^* = s_0$ are optimal as soon as the per-unit cost of risk reduction exceeds a certain critical level,

$$c^{crit} = \frac{\int_a^b v(w + s^0(1+r))\mathrm{d}\left[G(w) - F(w)\right]}{\int_a^b v'(w + s^0(1+r))\mathrm{d}F(w)}.$$

In microeconomic jargon, this is sometimes referred to as the "choke price" because it is the price that just chokes off demand. The agent engages in saving for two purposes, consumption smoothing and precaution. To disentangle them, we denote by s_0^{cs} the agent's saving choice if second-period income was risk-free,

$$u'(w_1 - s_0^{cs}) = \beta(1+r)v'\left(\mathbb{E}_F\widetilde{w} + s_0^{cs}(1+r)\right),$$

where $\mathbb{E}_F$ denotes the expectation operator with respect to the distribution function F. It is clear that $\mathbb{E}_F\widetilde{w} \succ_2 \widetilde{w}$, where the latter is distributed according to F, so that it follows directly from Kimball (1990) that the DM engages in precautionary saving if and only if he is prudent ($v''' > 0$). In such a case, $s_0 > s_0^{cs}$, and the difference between the two is the amount of precautionary saving. Risk reduction tilts the balance between consumption smoothing and precautionary saving as summarized in the following proposition.

Proposition 6. *Assume nth-degree Ross risk aversion to be non-increasing in wealth. At the margin, nth-degree risk reduction increases the demand for saving to smoogh consumption and reduces the demand for precautionary saving.*

4 Conclusion

Many papers have been written on the analysis of one specific decision under risk and uncertainty. This is the case for precautionary saving which is defined as the extra saving due to risky future income. In the literature, the risk in question is usually considered as a background risk since it is assumed that it cannot be modified through preventive actions, diversified or insured against. These paper thus deal with the way the introduction of an exogenous risk affects savings. Less attention has been dedicated to the interaction between saving and other economic decisions that could be used to deal with a future risk, which would then be endogenous. This is the question we address in this paper since we analyze the relationship between the demand for saving and insurance or between saving and risk reduction more generally.

In the case of insurance, we find that the way the Arrow-Pratt coefficient of absolute risk aversion changes in wealth determines the interaction between the demand for saving and the demand for insurance. As a result, in those cases where saving reacts positively to an increase in the interest rate, insurance is a substitute of complement for saving whenever absolute risk aversion is decreasing or increasing in wealth, respectively. Similarly, if we exclude insurance to be Giffen, saving is a substitute for insurance whenever absolute risk aversion is non-increasing in wealth. As a consequence, in the more plausible case of decreasing absolute risk aversion, insurance and saving are substitutes for each other. We also show that this result extends to more forms of costly nth-degree risk reduction by drawing on the notion of decreasing nth-degree Ross risk aversion instead of Arrow-Pratt risk aversion.

We believe that these findings are interesting on their own behalf. However, as we illustrate for the case of saving and insurance, they have very direct empirical measurement implications and need to be taken into account when assessing the strength of the precautionary saving motive or inferring the intensity of certain preferences conditions from it. We illustrate that the insurability of risk or, more generally, the market conditions for risk-reducing activities are directly reflected in the extent to which decision-makers engage in precautionary saving. As a consequence, prudent decision-makers may not engage in precautionary saving when they anticipate sufficient risk reduction activities in the future, or they may engage in substantial precautionary saving when they anticipate the opposite. It becomes clear that their *beliefs* about future risk reduction opportunities and their associated costs will critically modulate the amount of precautionary savings. Our paper formalizes the exact mechanism behind this interaction and provides guidance for more informed identification and estimation.

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Appendix

A.1 Second-order conditions for system (1)

Direct computation shows that

$$\begin{aligned} U_{\alpha\alpha} &= \beta l^2 \left[(1-(1+\lambda)p)^2 p v''_{2L} + (1+\lambda)^2 p^2 (1-p) v''_{2N}\right] < 0, \quad \text{and} \\ U_{ss} &= u''_1 + \beta(1+r)^2 \left[p v''_{2L} + (1-p) v''_{2N}\right] < 0. \end{aligned}$$

After some simplifications, the determinant of the Hessian matrix of $U(\alpha, s)$ can be shown to take the following form:

$$D = U_{\alpha\alpha}U_{ss} - U_{\alpha s}^2 = u''_1 U_{\alpha\alpha} + \beta^2(1+r)^2 l^2 p(1-p)(1-2(1+\lambda)p)^2 v''_{2L} v''_{2N} > 0.$$

As a result, the DM's objective function is globally concave in (α, s).

A.2 Proof of Proposition 1

We utilize the two-dimensional Implicit Function Theorem and obtain that

$$\frac{\mathrm{d}\alpha}{\mathrm{d}r} = \frac{1}{D}\left(-U_{ss}U_{\alpha r} + U_{\alpha s}U_{sr}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}r} = \frac{1}{D}\left(-U_{\alpha\alpha}U_{sr} + U_{\alpha s}U_{\alpha r}\right).$$

To sign these two expressions, we need to determine the two remaining cross-derivatives $U_{\alpha r}$ and U_{sr}. Direct computation shows that

$$U_{\alpha r} = \beta l s^* \left[(1-(1+\lambda)p) p v'_{2L} - (1+\lambda)p(1-p) v''_{2L}\right] = \beta l s^* (1+\lambda) p(1-p) v'_{2N} \left[A_{2N} - A_{2L}\right],$$

where the second equality is obtained by using $U_\alpha = 0$. As a result, $U_{\alpha r}$ is non-positive (non-negative) whenever risk aversion in the second period is decreasing (increasing) in wealth. Per Remark 1 this shows that $U_{\alpha r}$ and $U_{\alpha s}$ coincide in sign. Furthermore,

$$\begin{aligned} U_{sr} &= \beta\left[p v'_{2L} + (1-p) v'_{2N}\right] + \beta(1+r)s^*\left[p v''_{2L} + (1-p) v''_{2N}\right] \\ &= \beta\left\{p v'_{2L}\left[1 + (1+r)s^* \frac{v''_{2L}}{v'_{2L}}\right] + (1-p) v'_{2N}\left[1 + (1+r)s^* \frac{v''_{2N}}{v'_{2N}}\right]\right\}, \end{aligned}$$

where the square brackets compare partial risk aversion in the second-period loss state and the second-period no-loss state with unity. According to Lemma 2 in Chiu et al. (2012), partial risk aversion is uniformly less than unity if and only if relative risk aversion is, in which case U_{sr} is non-negative. This completes the proof.

A.3 Proof of Proposition 2

Another application of the Implicit Function Theorem shows that

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\lambda} = \frac{1}{D}\left(-U_{ss}U_{\alpha\lambda} + U_{\alpha s}U_{s\lambda}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}\lambda} = \frac{1}{D}\left(-U_{\alpha\alpha}U_{s\lambda} + U_{\alpha s}U_{\alpha\lambda}\right).$$

To sign these expressions, the two missing cross-derivatives $U_{\alpha\lambda}$ and $U_{s\lambda}$ will be determined in the sequel. We obtain that

$$U_{\alpha\lambda} = -\beta l\left[p^2 v'_{2L} + (1-p)pv'_{2N}\right] - \beta\alpha pl^2\left[(1-(1+\lambda)p)pv''_{2L} - (1+\lambda)p(1-p)v''_{2N}\right].$$

Its sign is a priori ambiguous but we exclude insurance demand to be Giffen so that $U_{\alpha\lambda} \leq 0$.[10] The cross-derivative of the objective function with respect to saving and the price of insurance is

$$U_{s\lambda} = -\alpha pl\beta(1+r)\left[pv''_{2L} + (1-p)v''_{2N}\right],$$

which is positive due to risk aversion in the second period. This concludes the proof.

A.4 Proof of Proposition 3

For $\lambda = \lambda^{crit}$, we know that $\alpha^* = 0$ is optimal. Therefore, $U_{s\lambda} = 0$ when evaluated at λ^{crit} so that the effect of insurability on the total demand for saving reduces to

$$\left.\frac{\mathrm{d}s_0}{\mathrm{d}\lambda}\right|_{\lambda=\lambda^{crit}} = \frac{1}{D}U_{\alpha s}U_{\alpha\lambda}.$$

[10] Exploiting the first-order condition for optimal insurance demand, a sufficient condition for $U_{\alpha\lambda} \leq 0$ is that

$$A_{2L} - A_{2N} \leq \frac{1}{\alpha pl(1-p)(1+\lambda)}.$$

Loosely speaking, this is more likely to be satisfied if second-period risk aversion does not decrease too quickly in wealth and/or if the loading factor is not excessive.

Furthermore, at $\lambda = \lambda^{crit}$ the effect of the price of insurance on insurance demand reduces to $U_{\alpha\lambda}|_{\lambda=\lambda^{crit}} = -\beta l \left[p^2 v'_{2L} + (1-p)pv'_{2N}\right] < 0$. To investigate the demand for saving to smooth consumption, we start with an arbitrary loading factor, $\lambda \in [0, \lambda^{crit})$, and associated insurance demand of $\alpha = \alpha^*$. Expected wealth in the second period is given by

$$\overline{w}_2 = w_2 + s(1+r) - (1+\lambda)\alpha^* pl - (1-\alpha^*)pl$$

and the demand for saving to smooth consumption over time is implicitly defined via

$$-u'(w_1 - s_0^{cs}) + \beta(1+r)v'(\overline{w}_2) = 0.$$

The effect of a change in the price of insurance is obtained from the Implicit Function Rule:

$$-\beta(1+r)v''(\overline{w}_2)\alpha^* pl - \beta(1+r)v''(\overline{w}_2)\frac{\mathrm{d}\alpha^*}{\mathrm{d}\lambda}\lambda pl.$$

The first term is zero when evaluated at $\lambda = \lambda^{crit}$ due to $\alpha^* = 0$, and as a result

$$\left.\frac{\mathrm{d}s_0^{cs}}{\mathrm{d}\lambda}\right|_{\lambda=\lambda^{crit}} < 0.$$

It follows that a marginal decrease of λ starting at λ^{crit} strictly increases the level of saving to smooth consumption whereas the total level of saving remains constant or decreases, depending on whether second-period risk aversion is constant or decreasing. As a consequence, the demand for precautionary saving decreases.

A.5 Proof of Proposition 4

We utilize the two-dimensional Implicit Function Theorem and obtain that

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \frac{1}{D}\left(-U_{ss}U_{tr} + U_{ts}U_{sr}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}r} = \frac{1}{D}\left(-U_{tt}U_{sr} + U_{ts}U_{tr}\right).$$

where D denotes the determinant of the Hessian of U. It holds that

$$U_{ss} = u''(w_1 - s) + \beta(1+r)^2 \int_a^b v''(z(w))\mathrm{d}H_1(w,t) < 0$$

due to concavity of u and v. For the determinant D of U to be positive for maximality, it follows that $U_{tt} < U_{ts}^2/U_{ss} < 0$. Direct computation shows that

$$U_{tr} = -\beta cs \int_a^b v''(z(w))\mathrm{d}H_0(w,t) - \beta s(-1)^n \int_a^b v^{(n+1)}(z(w))\left[F_n(w) - G_n(w)\right]\mathrm{d}w,$$

and recalling Equation (4), we find that $(1+r)U_{tr} = sU_{ts}$. As a consequence, U_{tr} and U_{ts} have the same sign[11] and we can directly apply Remark 2 to sign U_{tr}: U_{tr} is negative (positive) if nth-degree Ross risk aversion is decreasing (increasing) in wealth. Finally, we obtain that

$$\begin{aligned} U_{sr} &= \beta \int_a^b v'(z(w))\mathrm{d}H_1(w,t) + \beta(1+r)s\int_a^b v''(z(w))\mathrm{d}H_1(w,t) \\ &= \beta \int_a^b v'(z(w))\left[1 + (1+r)s\frac{v''(z(w))}{v'(z(w))}\right]\mathrm{d}H_1(w,t), \end{aligned}$$

where the square bracket is non-negative if partial risk aversion is less than unity, which is the case if relative risk aversion is uniformly less than unity (Chiu et al., 2012). Combining the conditions for the various signs accordingly completes the proof.

A.6 Footnote 9

The effect of a change in the per-unit cost of nth-degree risk reduction on its demand is governed by the following cross-derivative:

$$U_{tc} = -\beta \int_a^b v'(z(w))\mathrm{d}H_0(w,t) + \beta ct \int_a^b v''(z(w))\mathrm{d}H_0(w,t) - \beta t \int_a^b v'(z(w))\mathrm{d}\left[G(w) - F(w)\right].$$

The first two terms are negative but the third one is sign ambiguous. The reason is that an increase in the per-unit cost of nth-degree risk reduction reduces the DM's consumption and without further restrictions it is not clear how lower consumption influences the marginal benefit of nth-degree risk reduction. If we assume the agent to exhibit mixed risk aversion, then $(-v'(w))$ is nth-degree risk-averse such that the last term is positive.

We can utilize the first-order condition for the optimal level of risk reduction ($U_t = 0$) and

[11] Under the assumption that individuals save rather than dissave at the optimum, see also Eeckhoudt and Schlesinger (2008).

combine the first and third term in U_{tc}:

$$\begin{aligned} & -\beta\int_a^b v'(z(w))\mathrm{d}H_0(w,t)-\beta t\int_a^b v'(z(w))\mathrm{d}\left[G(w)-F(w)\right] \\ = & -\frac{\beta}{c}\int_a^b v(z(w))\mathrm{d}\left[G(w)-F(w)\right]-\beta t\int_a^b v'(z(w))\mathrm{d}\left[G(w)-F(w)\right] \\ = & -\frac{\beta}{c}\int_a^b \big(v(z(w))+ctv'(z(w))\big)\,\mathrm{d}\left[G(w)-F(w)\right]. \end{aligned}$$

Whenever the integrand as a function of w is nth-degree risk-averse of nth-degree risk-neutral, the entire integral is non-negative rendering the expression non-positive. This would be sufficient for $U_{tc}<0$. Straightforward calculations show that this is the case if

$$-z(w)\frac{v^{(n+1)}(z(w))}{v^{(n)}(z(w))}\leq\frac{z(w)}{ct},$$

which holds whenever nth-degree relative risk aversion is bounded by n and total expenditures on risk reduction as a fraction of consumption $(ct/(w+s(1+r)))$ are bounded by $1/(n+1)$.

A.7 Proof of Proposition 5

Another application of the Implicit Function Theorem shows that

$$\frac{\mathrm{d}t}{\mathrm{d}c}=\frac{1}{D}\left(-U_{ss}U_{tc}+U_{ts}U_{sc}\right)\quad\text{and}\quad\frac{\mathrm{d}s}{\mathrm{d}c}=\frac{1}{D}\left(-U_{tt}U_{sc}+U_{ts}U_{tc}\right).$$

The only missing cross-derivative to sign these expressions is U_{sc}, which is given by

$$U_{sc}=-t\beta(1+r)\int_a^b v''(z(w))\mathrm{d}H(w,t),$$

which is positive due to risk aversion. Combining all signs accordingly proves the proposition.

A.8 Proof of Proposition 6

When $c = c^{crit}$, then $t^* = 0$ is optimal by definition. In that case, $U_{sc} = 0$ and the effect of introducing risk reduction on optimal saving is given by:

$$\left.\frac{\mathrm{d}s_0}{\mathrm{d}c}\right|_{c=c^{crit}} = \frac{1}{D} U_{ts} U_{tc}.$$

Also, as soon as $c = c^{crit}$ and $t^* = 0$, the effect of a change in the per-unit cost of risk reduction on optimal demand for risk reduction is unambiguous, because

$$U_{tc}|_{c=c^{crit}} = -\beta \int_a^b v'(w + s^0(1+r))\mathrm{d}F(w) < 0.$$

At $c = c^{crit}$ not further assumptions are required for the demand for risk reduction not to be Giffen. Hence, whenever nth-degree Ross risk aversion is non-increasing in wealth, $U_{ts} \geq 0$ such that $\left.\frac{\mathrm{d}s_0}{\mathrm{d}c}\right|_{c=c^{crit}} \geq 0$.

To determine the effect of risk reduction on consumption smoothing, we determine the optimal level of saving in the absence of risk in the second period. For $c \in (0, c^{crit})$ and optimal demand for risk reduction of $t = t^*$, this renders

$$-u'(w_1 - s_0^{cs}) + \beta(1+r)v'\left(\mathbb{E}_{H(w,t^*)}\widetilde{w} + s_0^{cs}(1+r) - ct^*\right) = 0.$$

A change of the per-unit price of risk reduction affects consumption smoothing, and the direction of this effect can be determined with the help of the Implicit Function Rule as follows:

$$-\beta(1+r)v''\left(\mathbb{E}_{H(w,t^*)}\widetilde{w} + s_0^{cs}(1+r) - ct^*\right)\left[t^* + c\frac{\mathrm{d}t^*}{\mathrm{d}c}\right].$$

The sign is determined by the sign of the square bracket and when $c = c^{crit}$, it follows that $t^* = 0$ and that $U_{tc}|_{c=c^{crit}} < 0$ such that

$$\left.\frac{\mathrm{d}s_0^{cs}}{\mathrm{d}c}\right|_{c=c^{crit}} < 0.$$

Hence, a marginal reduction of c starting at c^{crit} strictly increases saving to smooth consumption, whereas the total demand for saving remains constant or decreases, depending on whether nth-degree Ross risk aversion is decreasing or constant in wealth. Therefore, the demand for precautionary saving decreases.