

## Life Insurance Settlement and Information Asymmetry

Abstract: We examine the effect of life insurance settlement on insurance market with two-dimensional asymmetric information on mortality and liquidity risks. We first show that under Rothschild and Stiglitz equilibrium conditions (1976), semi-pooling equilibrium may exist without settlement. When settlement is allowed, the equilibrium is pooling contract and the utilities of all insureds decrease. Second, under Wilson conjecture (1977), pooling equilibrium at which the utility of high mortality risks are maximized is possible without settlement. When settlement is allowed, pooling equilibrium is possible and the utilities of all insureds decrease.

Keyword: asymmetric information, life settlement, pooling equilibrium

## Life Insurance Settlement and Information Asymmetry

### I. Introduction

Life insurance settlement (life settlement) is a transaction that policyholders can sell their life insurance contract to a third-party investors. Life insurance market is primary market in which the insurance policies are issued while settlement market is secondary market in which the policies are traded.

Life insurance contract consists of death benefit and surrender value. Specifically, policyholders can choose to surrender their insurance contract rather than keeping insurance when they lose bequest motive or face liquidity needs. However, surrender value is a small portion of the death benefit in general. According to Fang and Wu (2017), cash surrender value is only 3-5% of death benefit. Thus, the settlement market provides the opportunity to get higher value of to policyholders as long as settlement price is higher than surrender value.

The effect of settlement on consumer welfare and efficiency is controversial. Some studies point out that the introduction of settlement enhances consumer welfare while others show that settlement deteriorates consumer welfare and only increases insurance premium.

On the proponent side, there are the researches represented by Doherty and Singer (2003, DS hereafter), Hong and Seog (2018, hereafter HS), Fang and Wu (2017, hereafter FW) and Gottlieb and Smetters (2014, hereafter GS).

At first, DS demonstrated that a competitive secondary market for life insurance policies improves the welfare of policyholders. According to DS, policyholders have no choice but to surrender when they face liquidity needs. Thus, insurers have monopsony power when policyholders surrender. If settlement is introduced, then policyholders can sell the insurance policies to others and monopsony power decreases as the competition increases among the buyers of insurance.

HS suppose that policyholders have different liquidity risks and the monopolistic insurer cannot discriminate the policyholders along with the liquidity risks. In general, it is forbidden to discriminate against the risks others than mortality risk in life insurance. They show that the introduction of settlement may increase insurance demand even if insurance premium increases. They states that the condition to improve consumer welfare.

Unlike the previous two studies that assume fully rational policyholders, FW and GS assume that policyholders are overconfident either of their bequest motive or mortality risks. Then the policyholders can correct their belief to both risks through the settlement market and welfare is improved.

On the other hand, the research like Daily, Hendel and Lizzery (2008, hereafter DHL), Fand and Kung (2010, hereafter FK) are in the opponent side of settlement. Both studies are based on the study of Hendel and Lizzery (2003, hereafter HL). HL shows that in a two period model, optimal insurance premium is front-loaded and the premium for each period is the same. Due to the flat premium, insureds hedge the reclassification risk that consumer health risk may change over time. Using this model, DHL and FK show that policyholders cannot hedge the reclassification risk when settlement is introduced since the premium reflects the second period health risk. As a result, consumer welfare decreases.

In this study, we suppose that insureds have different liquidity risk and mortality risk and the information of both risks is private. We investigate the equilibrium contract under Rothschild and Stiglitz equilibrium condition (1976) and Wilson condition (1977) when settlement is not allowed. Next, we investigate the equilibrium contract when settlement is allowed and compare the utilities of insureds. As a result, when settlement is not allowed, then semi-pooling equilibrium at which the same mortality risk s but different liquidity risks

are pooled constitutes RS equilibrium if it exists. On the other hand, when settlement is allowed, then pooling contract at which all insureds are pooled is RS equilibrium. The utilities of insureds decrease. Under Wilson conditions, we show that the pooling contract at which the utility of insureds with high liquidity risks are maximized when settlement is not allowed. When settlement is allowed, the utilities of insureds decrease as well.

The remainder of the paper is organized as follows. In section II, we consider a model without information asymmetry as a benchmark model. In section III and IV, we investigate the model with information asymmetry under Rothschild and Stiglitz and Wilson condition respectively. In section V, we conclude.

## II. The Basic Model

We consider a competitive insurance market. Insureds have two sources of risk: mortality risk and liquidity risk. There are four risk types of insureds HH, HL, LH, and LL along with liquidity risk  $i, i = H, L$  and mortality risk  $j, j = H, L$ .

There are two periods,  $t = 0, 1$  and 2. Let us denote the income of insureds when he is alive as  $W_t$  and the income is the same for all insureds. If the insured dies, then the income is zero.

The discount factor is denoted as  $\rho$ . At time 0, insureds purchase insurance paying premium  $Q_{ij}$  given death benefit  $D$ . At  $t = 1$ , insureds face income loss  $y$  with probability  $q_j$ .

Surrender value is denoted as  $S_{ij}$ . We assume that  $S_{ij} \geq 0$  since negative surrender value is unrealistic. We also assume that  $S_{ij} \leq \rho p_i D$ . A death event occurs at  $t = 2$  with probability  $p_i$ . Then insurance premium is denoted as follows.

$$Q_{ij} = \rho q_j S_{ij} + \rho^2 p_i (1 - q_j) D \quad (2.1)$$

We suppose that settlement market is also competitive and perfect market. For technical simplicity, we suppose that the settlement investors are risk neutral. That is, settlement price is equal to the actuarial fair value of insurance contract,  $\rho p_i D$ .<sup>1</sup>

The utility for insured is decomposed into two parts following DHL, FK and HS. The insured receives  $u(W)$  when he is alive and consumes  $W$ . If he dies, then his dependent's receive death benefit  $D$  and the utility is  $v(D)$ . For simplicity, we assume that  $u(0) = v(0) = 0$ . Note that the dependents have death benefit only when the insureds retain the insurance contract. Both utility functions are strictly concave.

When liquidity needs occur due to income loss, the insured can choose surrender (or settlement) or keeping insurance. The utility when surrendering is  $u(W_1 - y + S_{ij})$ , while the utility when keeping insurance is  $u(W_1 - y) + \rho p_i v(D)$ . We suppose that

$u(W_1 - y + S_{ij}) > u(W_1 - y) + \rho p_i v(D)$  to rule out the case that the insureds choose to keeping insurance even if he faces liquidity needs. We also suppose that

$u(W_1 + S_{ij}) < u(W_1) + \rho p_i v(D)$  to rule out the case that the insureds choose to surrender even if he does not face liquidity needs.

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<sup>1</sup> As was pointed by Hong and Seog (2018), even if the investors are risk averse and thus risk premium is positive, the assumption can be applied when CAPM holds.

The insurer offers a contract with  $(Q_{ij}, S_{ij})$ . The expected utility of insured when settlement is not allowed is:

$$Eu = u(W_0 - Q_{ij}) + \rho q_j u(W_1 - y + S_{ij}) + \rho(1 - q_j)u(W_1) + \rho^2(1 - q_j)p_i v(D) + \rho^2(1 - p_i)u(W_2) \quad (2.2)$$

The slope of the indifference curve on  $(Q, S)$  plane is as follows:

$$\frac{dQ_{ij}}{dS_{ij}} = \rho q_j \frac{u'(W_1 - y + S_{ij})}{u'(W_0 - Q_{ij})} \quad (2.3)$$

At a given point  $(Q, S)$ , the slope of the indifference curve for high liquidity risk is steeper than that for low liquidity risk since  $q_H > q_L$ . As a benchmark case, let us first investigate the no adverse selection case in which insurers can observe the risk of insureds. For notational simplicity,  $V_{ij}(C)$  is defined as the expected utility of insured  $ij$  given contract  $C$  composed of  $(Q, S)$ . The problem is as follows:

[No adverse selection]

$$\begin{aligned} \text{Max}_{Q_{ij}, S_{ij}} V_{ij}(C_{ij}) = & u(W_0 - Q_{ij}) + \rho q_j u(W_1 - y + S_{ij}) + \rho(1 - q_j)u(W_1) + \rho^2(1 - q_j)p_i v(D) \\ & + \rho^2(1 - p_i)u(W_2) \end{aligned} \quad (2.4)$$

$$\text{s.t. } Q_{ij} = \rho q_j S_{ij} + \rho^2 p_i(1 - q_j)D$$

The Lagrangian is:

$$\begin{aligned} L = & u(W_0 - Q_{ij}) + \rho q_j u(W_1 - y + S_{ij}) + \rho(1 - q_j)u(W_1) + \rho^2(1 - q_j)p_i v(D) + \rho^2(1 - p_i)u(W_2) \\ & + \lambda[Q_{ij} - \rho q_j S_{ij} - \rho^2 p_i(1 - q_j)D] \end{aligned} \quad (2.5)$$

The first order conditions are:

$$L_{S_{ij}} = \rho q_j u'(W_1 - y + S_{ij}) - \rho q_j \lambda = 0 \quad (2.6)$$

$$L_{Q_{ij}} = -u'(W_0 - Q_{ij}) + \lambda = 0 \quad (2.7)$$

Combining these FOCs, we have following lemma 1.

**Lemma 1.** (No adverse selection). Suppose that settlement is not allowed. The optimal insurance contract is satisfied following conditions.

- (1)  $u'(W_0 - Q_{ij}) = u'(W_1 - y + S_{ij}), i = H, L, j = H, L$
- (2) For  $i$ ,  $(S_{iH} > S_{iL}, Q_{iH} < Q_{iL})$
- (3) For  $j$ ,  $(S_{Hj} < S_{Lj}, Q_{Hj} > Q_{Lj})$

Proof. See the Appendix.//

The equilibrium contracts are depicted in following figure 2.1. The straight lines are the zero-profit lines for each risk type and the lines are defined by (2.1). In equilibrium, each indifference curve is tangential at a point that lies on each zero-profit line. Recall that the slope of the zero-profit line is  $\rho q_{ij}$  from (2.1). The location of intercept may depend on the relative size of  $p_i$  and  $q_j$ . Figure 2.1 shows the case in which  $S_{HH} < S_{LL}, Q_{HH} > Q_{LL}$ . The opposite case is also possible.

Figure 2.1 around here.

However, if the information of both risk types is private then above outcome cannot be an equilibrium since for  $i$ , the contract  $C_{iH}$  is preferred over  $C_{iL}$  and for  $j$ , contract  $C_{Lj}$  is preferred over  $C_{Hj}$ .

Now, let us consider settlement market. The investors can observe mortality risk through the insurance contract insureds choose. Thus, investors offer  $\rho p_i D$  as settlement price to insureds with risk  $i$ . Consequently, insurers also offer the contract  $C_H$  and  $C_L$  responding settlement. Note that the contract  $C_i$  is composed of  $(Q_{ij} = \rho^2 p_i D, S_{ij} = \rho p_i D)$ . This result is summarized in following lemma 2 and figure 2.2.

Figure 2.2 around here.

Lemma 2. (No adverse selection). Suppose that settlement is allowed. The optimal insurance contract is satisfied following conditions.

- (1) Optimal contract  $C_i$  for  $ij$  is  $(Q_{ij} = \rho^2 p_i D, S_{ij} = \rho p_i D)$ .
- (2) The utilities of insureds decrease..

Proof. See the Appendix.//

Lemma 2 indicates that consumer welfare decreases when settlement is allowed. This result is intuitively clear since insurers offer insurance contract under the additional restriction that  $S_{ij} \geq \rho p_i D$ .

Our next task is to identify the relevant range of equilibrium under Rothschild and Stiglitz condition and Wilson condition when the information of risk type is private. In this case, the effect of settlement on the equilibrium is also different.

### III. Rothschild and Stiglitz Model

#### 3.1. Settlement is not allowed.

Let us investigate the equilibrium under Rothschild and Stiglitz equilibrium conditions when settlement is not allowed. We call the equilibrium as RS equilibrium. Recall that the indifference curve of type H for liquidity risk  $j$  is steeper than that of L for  $j$  on the same contract. Let us denote Marginal rate of substitution (MRS) of insurance surrender value for insurance premium as  $MRS_{ij}$ . Then by (2.3),  $MRS_{Hj} = MRS_{Lj}$  at a given point  $(Q, S)$ . That is, insurers offer the insurance contract on each line pooling of the same liquidity risks but

different mortality risk . The proportion of each risk type is denoted as  $\lambda_{ij}$ . The pooling line is defined as follows:

$$Q = \rho q_j S + \rho^2 p^j (1 - q_j) D, j=H,L$$

$$\text{Where } p^H = \frac{\lambda_{LH}}{\lambda_{HH} + \lambda_{LH}} p_H + \frac{\lambda_{HH}}{\lambda_{HH} + \lambda_{LH}} p_L, \quad p^L = \frac{\lambda_{LL}}{\lambda_{HL} + \lambda_{LL}} p_H + \frac{\lambda_{HL}}{\lambda_{HL} + \lambda_{LL}} p_L \quad (2.8)$$

Let us also denote the contract of liquidity risk j as  $(Q_j, S_j)$ . Using (2.8), we have following program. The program is maximized the utility of the insureds with liquidity risk H.

[RS equilibrium without settlement]

$$\begin{aligned} \text{Max}_{Q_j, S_j} V_H(C_H) = & u(W_0 - Q_H) + \rho q_H u(W_1 - y + S_H) + \rho(1 - q_H)u(W_1) + \rho^2(1 - q_H)p_i v(D) \\ & + \rho^2(1 - p_i)u(W_2) \end{aligned} \quad (2.9)$$

$$\begin{aligned} \text{s.t. } Q_j = & \rho q_j S_j + \rho^2 p^j (1 - q_j) D \\ u(W_0 - Q_L) + \rho q_L u(W_1 - y + S_L) + \rho(1 - q_L)u(W_1) + \rho^2(1 - q_L)p_i v(D) + \rho^2(1 - p_i)u(W_2) \\ \geq & u(W_0 - Q_H) + \rho q_L u(W_1 - y + S_H) + \rho(1 - q_L)u(W_1) + \rho^2(1 - q_L)p_i v(D) + \rho^2(1 - p_i)u(W_2) \end{aligned}$$

RS equilibrium is obtained by solving above program. The characteristic of RS equilibrium is summarized in proposition 1.

Proposition 1. Suppose that settlement is not allowed. Then following results hold.

- (1) The condition for equilibrium condition depends on the relative proportion of insureds. When equilibrium exists, the equilibrium is RS semi-pooling equilibrium at which types with the same mortality risk s but different liquidity risks are pooled, while different mortality risk s are separated.
- (2) For liquidity risk L,  $u'(W_0 - Q_L) = u'(W_1 - y + S_L)$  while for liquidity risk H,  $u'(W_0 - Q_H) > u'(W_1 - y + S_H)$  at the equilibrium.

Proof. See the Appendix.//

We discuss diagrammatically the possible scenarios if there are hidden characteristics about mortality and liquidity risks in the relevant region of the contract space. This technical requirement is explained below in more detail.

Figure 3.1.1 around here.

The derivation of the equilibrium contract is an extension of the discussion in Rothschild and Stiglitz model. As was argued by Rothschild and Stiglitz, equilibrium may or may not exist depending on the relative portion of each risk type. In addition, the contract at which all insureds are pooled cannot be an equilibrium. In figure 3.1.2, the line which cuts  $\rho^2 kD$  is zero-profit pooling line for all insureds. Let us suppose that insurers offer the contract  $C_P$ . There exists the contract  $C_A$  as in figure 3.1.2 which is located between both difference curves

and this contract attracts liquidity risks H and insurers make positive profit.

### 3.2. Settlement is allowed.

Now, we suppose that settlement is allowed. Let us denote the proportion of mortality risk H and L as  $\lambda_H$  and  $\lambda_L$  respectively. Then  $\lambda_i = \lambda_{iH} + \lambda_{iL}$ ,  $i=H,L$ . The settlement investors cannot observe mortality risk exactly. The investors may offer pooling price  $\rho \bar{p}D$  where  $\bar{p} = \lambda_H p_H + \lambda_L p_L$ . In this case, insurers cannot retain origin contract since all insureds choose settlement rather than surrender and insurers suffer a loss. In addition, insurers cannot offer  $C_H$  and  $C_L$  since insureds with low mortality risk choose  $C_H$ . That is, insurers also offer pooling contract  $C_S^{RS}$  composed of  $(Q = \rho^2 \bar{p}D, S = \rho \bar{p}D)$ . Geometrically,  $C_S^{RS}$  is the point at which the lines presented by (2.8) meet. As a result, at RS equilibrium, all insureds utilities decrease. This result is depicted in proposition 2 and figure 3.2.

Proposition 2. Suppose that settlement is allowed. Then following results holds.

- (1) RS equilibrium is pooling contract and the utilities of insureds decrease.
- (2) Settlement price is equal to  $\rho \bar{p}D$  where  $\bar{p} = \lambda_H p_H + \lambda_L p_L$ ,  $\lambda_i = \lambda_{iH} + \lambda_{iL}$ ,  $i=H,L$ .

Proof. See the figure 3.2.//

Figure 3.2 around here.

## IV. Wilson Model

### 4.1. Settlement is not allowed.

Now, we investigate the equilibrium under Wilson equilibrium conditions. We call this equilibrium as Wilson equilibrium. Under Wilson conditions, insurers have less incentive to provide a new contract than under RS conditions since insureds withdraw the contract which is not profitable. Thus, RS semi-pooling equilibrium in previous section is also Wilson equilibrium if it exists. In addition, pooling Wilson equilibrium may exist. Contrary to Wilson model, the utility of high liquidity risks is maximized like figure 3.1.2. Note that there is no incentive that insurers offer the contract on the left side of  $C_P$  and in between both indifference curves, since the contract only attracts low liquidity risks and insurer suffer the loss. On the contrary, if some insurers offer  $C_A$ , then other insurers withdraw  $C_P$  and consequently insureds with low liquidity risks choose  $C_A$  as well. Then, insurers suffer the loss since  $C_A$  is below the zero-profit pooling line. Thus, insurers have no incentive to offer the contract like  $C_A$ .

Proposition 3. Suppose that settlement is allowed. Then Wilson equilibrium may exist. Pooling equilibrium constitutes Wilson equilibrium.

Proof. See the figure 3.1.2.//

### 4.2. Settlement is allowed.

When settlement is allowed, the investors cannot still observe the mortality risk in both

semi-pooling and pooling contract. Thus, the investors offer pooling price,  $\rho \bar{p}D$  and insurers also offer the contract  $C_S^{RS}$  composed of  $(Q = \rho^2 \bar{p}D, S = \rho \bar{p}D)$ . Then the utilities of all insureds decrease like figure 3.1.2.

Proposition 4. Suppose that settlement is allowed. Then Wilson equilibrium is pooling equilibrium and the utilities of insureds decrease.

Proof. See the figure 3.1.2.//

## V. Conclusion

We examine the effect of life insurance settlement on insurance market with two-dimensional asymmetric information on mortality risk and liquidity risk. We first show that under Rothschild and Stiglitz equilibrium condition (1976), semi-pooling equilibrium can exist without settlement. When settlement is allowed, the equilibrium is pooling equilibrium and the utilities of insureds decrease. Second, under the Wilson conjecture (1977), pooling equilibrium at which the utility of high mortality risk is maximized is possible without settlement. When settlement is allowed, pooling equilibrium is possible and the utilities of insureds decrease.

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## Appendix

### 1. Proof of Lemma 1.

(1) From (2.6) and (2.7), we have  $\lambda = u'(W_0 - Q_{ij}) = u'(W_1 - y + S_{ij})$ .

(2) Let us suppose that the insurer offer the contract that  $(S_{iH} \leq S_{iL}, Q_{iH} \geq Q_{iL})$ . Then following results hold.

$$Q_{iH} = \rho q_H S_{iH} + \rho^2 (1 - q_H) p_i D \geq Q_{iL} = \rho q_L S_{iL} + \rho^2 (1 - q_L) p_i D$$

$$\Leftrightarrow \rho q_H S_{iH} - \rho q_L S_{iL} - \rho^2 (q_H - q_L) p_i D > 0$$

It is contradictory if  $S_{iH} \leq S_{iL}$ , since

$$\rho q_H S_{iH} - \rho q_L S_{iL} - \rho^2 (q_H - q_L) p_i D \leq \rho (q_H - q_L) (S_{iH} - \rho p_i D) < 0.$$

(3) One can prove the statement by applying the similar logic as in the case for  $i$ . //

2. Proof of Lemma 2.

(1) When settlement is allowed, then the problem (2.4) is changed as follows:

$$\begin{aligned} \text{Max}_{Q_{ij}, S_{ij}} V_{ij}(C_{ij}) &= u(W_0 - Q_{ij}) + \rho q_j u(W_1 - y + S_{ij}) + \rho(1 - q_j)u(W_1) + \rho^2(1 - q_j)p_i v(D) \\ &\quad + \rho^2(1 - p_i)u(W_2) \end{aligned} \quad (\text{A.1})$$

$$\text{s.t. } Q_{ij} = \rho q_j S_{ij} + \rho^2 p_i (1 - q_j) D$$

$$S_{ij} \geq \rho p_i D$$

Then the Lagrangian is:

$$\begin{aligned} L &= u(W_0 - Q_{ij}) + \rho q_j u(W_1 - y + S_{ij}) + \rho(1 - q_j)u(W_1) + \rho^2(1 - q_j)p_i v(D) + \rho^2(1 - p_i)u(W_2) \\ &\quad + \lambda [Q_{ij} - \rho q_j S_{ij} - \rho^2 p_i (1 - q_j) D] + \mu [S_{ij} - \rho p_i D] \end{aligned} \quad (\text{A.2})$$

The first order conditions are:

$$L_{S_{ij}} = \rho q_j u'(W_1 - y + S_{ij}) - \rho q_j \lambda + \mu = 0 \quad (\text{A.3})$$

$$L_{Q_{ij}} = -u'(W_0 - Q_{ij}) + \lambda = 0 \quad (\text{A.4})$$

Combining these FOCs, we have:

$$\rho q_j [-u'(W_0 - Q_{ij}) + u'(W_1 - y + S_{ij})] = \mu \quad (\text{A.4})$$

By lemma 1, we have  $\mu > 0$ . Thus, we also have  $S_{ij} = \rho p_i D$  by complementary slackness condition.

(2) See the figure 2.2 //

3. Proof of proposition 1.

4.

$$\begin{aligned} \text{Max}_{Q_j, S_j} V_H(C_H) &= u(W_0 - Q_H) + \rho q_H u(W_1 - y + S_H) + \rho(1 - q_H)u(W_1) + \rho^2(1 - q_H)p_H v(D_H) \\ &\quad + \rho^2(1 - p_H)u(W_2) \end{aligned} \quad (\text{2.9})$$

$$\begin{aligned}
s.t. \quad Q_j &= \rho q_j S_j + \rho^2 p^j (1 - q_j) D \\
u(W_0 - Q_L) + \rho q_L u(W_1 - y + S_L) + \rho(1 - q_L)u(W_1) + \rho^2(1 - q_L)p_i v(D) + \rho^2(1 - p_i)u(W_2) \\
&\geq u(W_0 - Q_H) + \rho q_L u(W_1 - y + S_H) + \rho(1 - q_L)u(W_1) + \rho^2(1 - q_L)p_i v(D) + \rho^2(1 - p_i)u(W_2)
\end{aligned}$$

The Lagrangian is:

$$\begin{aligned}
L &= u(W_0 - Q_H) + \rho q_H u(W_1 - y + S_H) + \rho(1 - q_H)u(W_1) + \rho^2(1 - q_H)p_i v(D) + \rho^2(1 - p_i)u(W_2) \\
&\quad + \sum_j \lambda_j [Q_j - \rho q_j S_j - \rho^2 p^j (1 - q_j) D] \\
&\quad + \mu [u(W_0 - Q_L) + \rho q_L u(W_1 - y + S_L) - u(W_0 - Q_H) - \rho q_L u(W_1 - y + S_H)] \quad (A.5)
\end{aligned}$$

The first order conditions are:

$$L_{Q_H} = -u'(W_0 - Q_H) + \lambda_H + \mu u'(W_0 - Q_H) = 0 \quad (A.6)$$

$$L_{S_H} = \rho q_H u'(W_1 - y + S_H) - \lambda_H \rho q_H - \mu \rho q_L u'(W_1 - y + S_H) = 0 \quad (A.7)$$

$$L_{Q_L} = \lambda_L - \mu u'(W_0 - Q_L) = 0 \quad (A.8)$$

$$L_{S_L} = -\rho q_L \lambda_L + \mu \rho q_L u'(W_1 - y + S_L) = 0 \quad (A.9)$$

From (A.6) and (A.7),

$$\begin{aligned}
(\rho q_H - \mu \rho q_H) u'(W_0 - Q_H) &= (\rho q_H - \mu \rho q_L) u'(W_1 - y + S_H) \\
\Rightarrow (\rho q_H - \mu \rho q_H) < (\rho q_H - \mu \rho q_L) &\Leftrightarrow u'(W_0 - Q_H) > u'(W_1 - y + S_H) \quad (A.10)
\end{aligned}$$

Thus, we have:

$$u'(W_0 - Q_H) > u'(W_1 - y + S_H) \quad (A.11)$$

$$u'(W_0 - Q_L) = u'(W_1 - y + S_L) \quad (A.12)$$

By (2.3), the relevant region of the contracts is depicted in figure 3.1. The risk type of HH, LH and LL are pooled while HL is separated. //

Figure 2.1. No adverse selection case when settlement is not allowed.

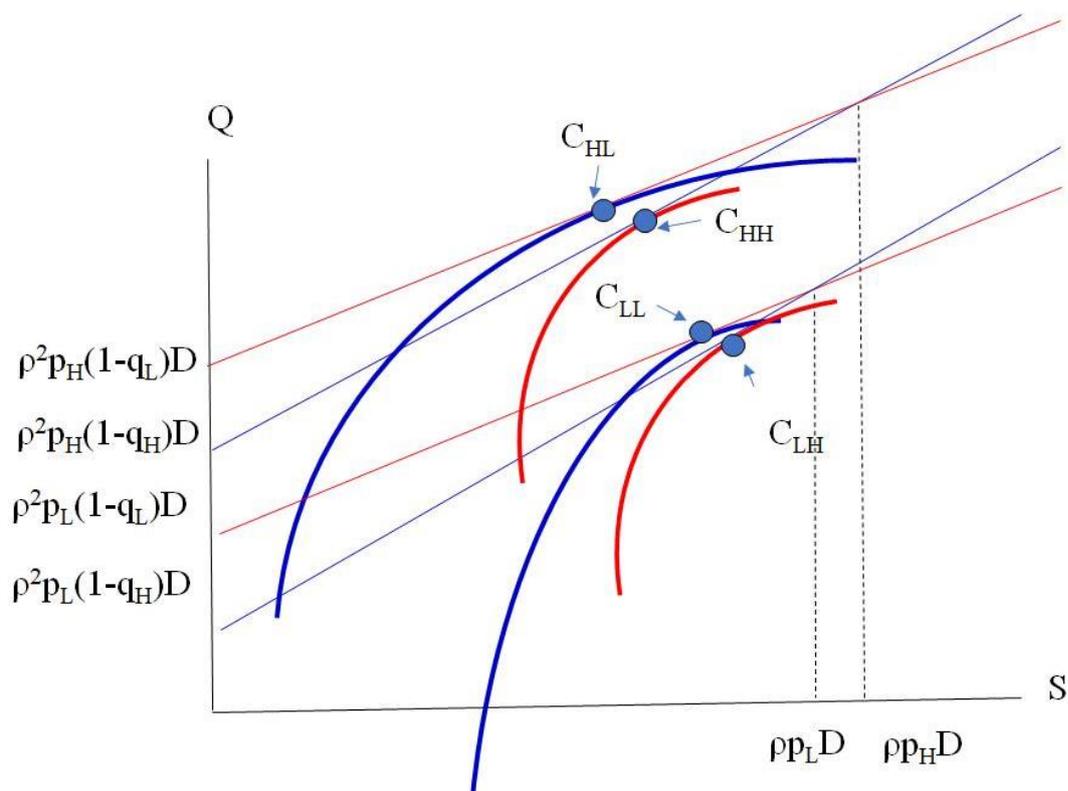


Figure 2.2. The case of no adverse selection when settlement is allowed

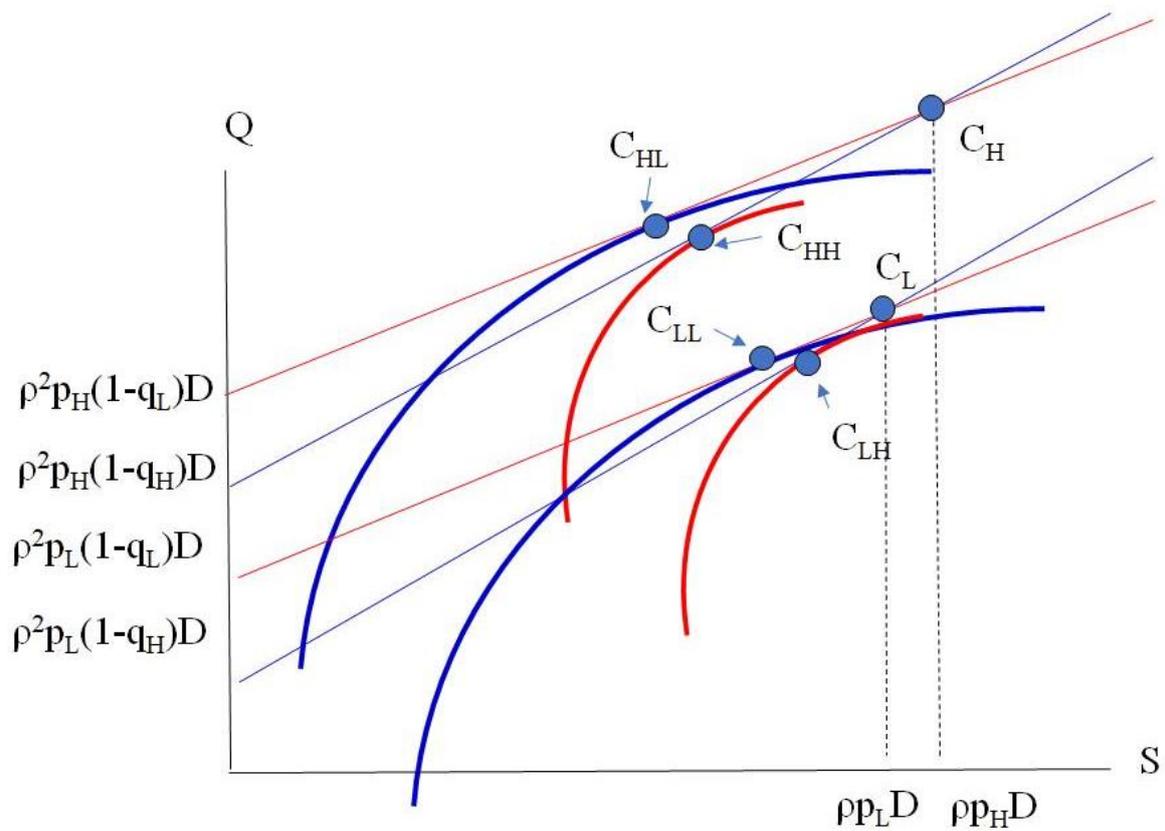


Figure 3.1.1. RS equilibrium when settlement is not allowed

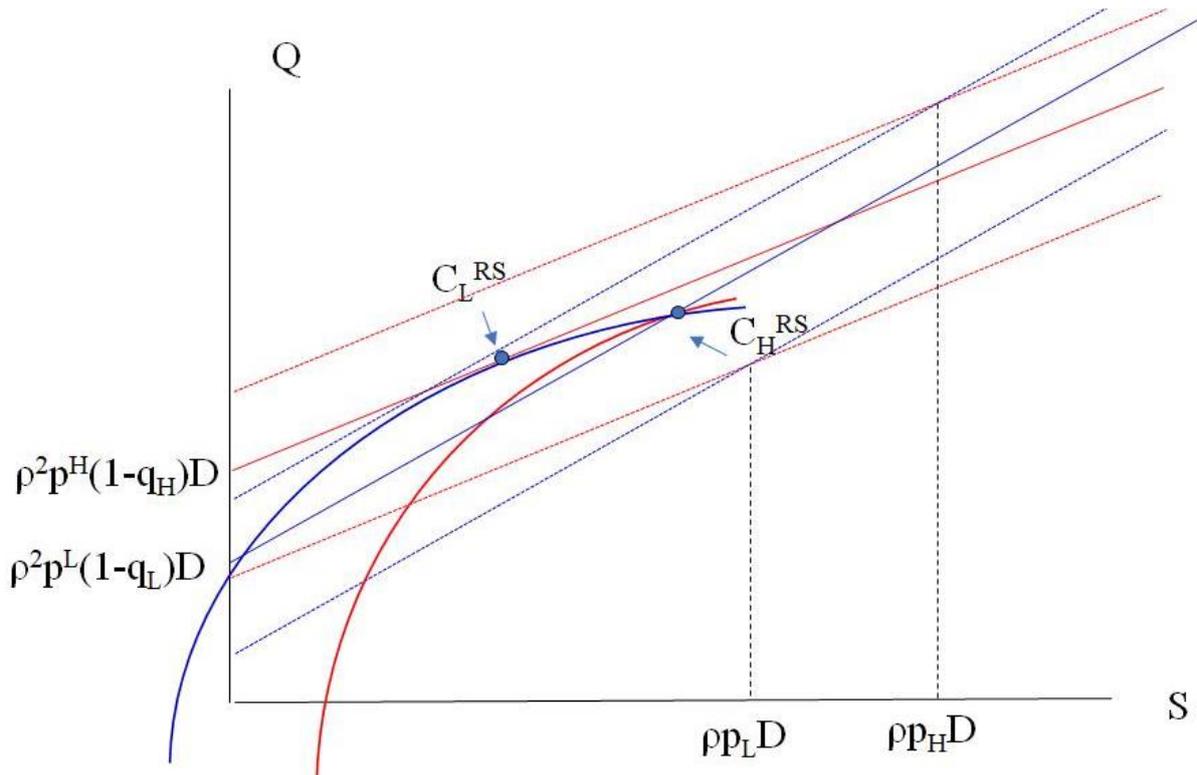


Figure 3.1.2. Pooling contract is not RS equilibrium when settlement is not allowed

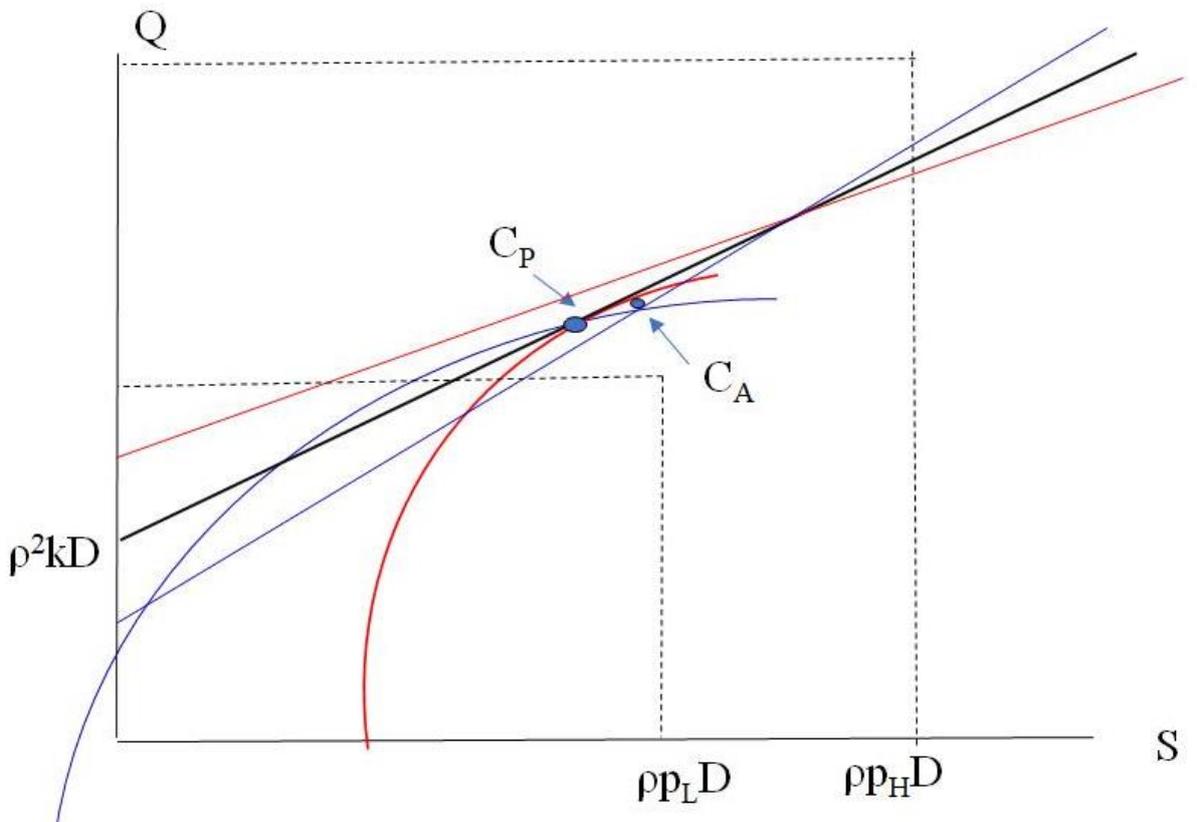


Figure 3.2. RS equilibrium when settlement not allowed

