Could Insurance-Linked Lotto Improve Social Welfare?

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Abstract

It is well known that rank dependent expected utility theory (RDEU) can explain why people purchase both insurance and lotteries. Inspired by the successful prediction of RDEU on human behavior, our paper proposes an insurance-linked lotto as a new tool of government intervention in insurance markets. The lotto is independent of the occurrence of the insurable risk, but the prize and/or the probability of winning the prize is dependent on the insurance demand. We show that when an individual's preferences is characterized by RDEU, the insurance-linked lotto could improve social welfare and enhance the equilibrium demand for insurance.

Keywords: Rank-dependent preferences; lotto; social welfare; demand for insurance; regulation

JEL classification: G22, H24, D50

1 Introduction

The literature has a long debate on regulation of competitive markets. One rationale for the support of the government as a paternalist is that individuals make certain decisions that cannot reach their own best interests due to market inefficiency or behavioral concerns. Market inefficiency could be caused by asymmetric information or bankruptcy, while behavioral concerns include self-control problems (O'Donoghue and Rabin, 2003), altruism (Jacobsson, Johannesson, and Borgquist, 2007), overconfidence (Sandroni and Squintani, 2007), or regret aversion (Huang, Muermann, and Tzeng, 2014), etc. Thus, the government can increase social welfare by correcting the decisions of individuals via providing information, taxation, or even limiting choices.[1]

Our paper aims at designing a new tool of government intervention into insurance markets under a behavioral model such that social welfare can be improved. We employ the rank dependent expected utility theory (RDEU) proposed by Quiggin (1982) to capture the behavior of policyholders. RDEU has been shown to have better prediction on human behavior and can explain many paradoxes which violate the expected utility (EU) theory. For example, RDEU can explain why people purchase both insurance and lotteries but EU cannot.

The new tool of the proposed intervention is named "insurance-linked lotto." This lotto provides policyholders a contingent prize, which is dependent on the insurance coverage purchased by the policyholders and is independent of the occurrence of the loss. The insurance-linked lotto can be self-financed through a lump-sum tax. Furthermore, we equate the policymaker's and agent's objective functions in the model, i.e., the objective of the government is to maximize the individual's rank dependent expected utility.

We examine the necessary and sufficient conditions for the government to provide this type of insurance-linked lotto. We further provide conditions when insurance-linked lotto could increase the equilibrium demand for insurance. The underlined rationale settles on that RDEU can explain why individuals purchase both insurance and lotteries. Since the prize of the lotto is proportional to the amount of the insurance coverage, increasing the

[1] For example, see Gruber and Köszegi (2004), Thaler and Benartzi (2004), O'Donoghue and Rabin (2006), Glaeser (2006), Huang and Tzeng (2007), Huang, Tsai, and Tzeng (2008) and Huang, Muermann and Tzeng (2014).

insurance demand has two effects: one is to hedge the insurable losses and the other is to have a higher gain on gambling. Both effects are preferred by RDEU individuals. Thus, the insurance-linked lotto could not only make the policyholders purchase more coverage, but also enhance social welfare.

In addition, we propose an alternative insurance-linked lotto: the probability of winning the prize is proportional to insurance coverage and the prize is a fixed amount. In practice, this type of lotto can be implemented by issuing more lottery tickets to the policyholders who purchase high insurance coverage. We find that no matter how the lotto is linked to insurance, either through the prize amount or the probability of winning the reward, both types of insurance-linked lotto could enhance insurance purchasing as well as improve social welfare.

It is worthy to note that if the policyholders's preferences are characterized by EU, the insurance-linked lotto could never improve the individual's welfare. The presence of the lotto creates a second-degree stochastic dominance deterioration. Thus, under EU theory, the risk-averse individuals' utility can not be improved. In other words, if some EU individuals exist in society, insurance-linked lotto could make social welfare worse off.

To take care of this concern, we suggest a self-selection mechanism to achieve Pareto improvement. The self-selection mechanism gives individuals a chance to choose whether or not to join the insurance-linked lotto after it has been announced by the government. Since the welfare of EU maximizer is worse off under insurance-linked lotto, they would not join it. On the other hand, the utility of RDEU individuals could be improved via this type of lotto. Thus, a Pareto improvement could be reached.

The remainder of this paper is organized as follows. Section 2 introduces the insurance-linked lotto. Section 3 proposes an alternative type of insurance-linked lotto and discusses in more detail self-selection mechanism. Section 4 concludes the paper. All the proofs are in the appendix.

2 Insurance-Linked Lotto under Rank Dependent Utility

We consider a two-stage model with full information. In the first stage, the government designs an insurance-linked lotto. The prize of the lotto depends on the amount of insurance coverage. In the second stage, based on the lotto system, the individuals with rank dependent expected utilities choose the optimal insurance demand in a competitive private insurance market. Two questions are examined: (1) Are individuals better off under the insurance-linked lotto? and (2) Will this lotto enhance the demand for insurance in the private insurance market?

Now, let us introduce the insurance-linked lotto. Suppose that the government can launch a lotto by announcing a multiplier $\alpha \geq 0$. The prize of the lotto is αQ with probability θ. Note that Q is the insurance coverage determined by the individual in the second stage. Assume that the insurance-linked lotto is self-financed by a lump-sum tax T, which is equal to $(1+\lambda_g)\,\theta\alpha Q$, where λ_g denotes the expense loading of the government. The government chooses the value of α to maximize the individual's utility. This lottery is named as insurance-linked lotto because the prize is designed to be positively relative to the insurance demand Q. Also note that the prize of this lotto is independent of the insurable risk.

In the second stage, the individual with endowment W can purchase insurance with coverage Q from a competitive insurance market to hedge a fixed amount loss L, where $Q \leq L$. Let π be the loss probability. The insurance premium is PQ where $P = (1+\lambda_c)\,\pi$ denotes the premium rate and λ_c is the expense loading of the insurance company.

Assume that the individual's preferences can be characterized by RDEU proposed by Quiggin (1982). Let u denote the utility function with $u' > 0$ and $u'' < 0$, which captures the aversion of facing a mean-preserving spread of payoff. Let $q : [0,1] \to [0,1]$ be a probability distortion (or weighting) function with $q(0) = 0$, $q(1) = 1$ and $q' > 0$. This probability distortion function depends on the relative ranking of the outcomes to which they are associated.

The final wealth of the individual is:

$$
\begin{aligned}
W_1 &= W - PQ - T - L + Q \text{ with probability } \pi(1-\theta);\\
W_2 &= W - PQ - T - L + Q + \alpha Q \text{ with probability } \pi\theta;\\
W_3 &= W - PQ - T \text{ with probability } (1-\pi)(1-\theta);\\
W_4 &= W - PQ - T + \alpha Q \text{ with probability } (1-\pi)\,\theta.
\end{aligned}
$$

Since whether the government should provide an insurance-linked lotto to the market is examined, the starting point of the analysis settles on the case where the insurance-linked lotto does not exist, i.e., $\alpha = 0$. Since $Q \leq L$, we have $\alpha Q \leq L - Q$ as α approaches 0. Thus, the final wealth in each state can be ranked as

$$W_1 \leq W_2 \leq W_3 \leq W_4.$$

Given the insurance-linked lotto, the individual's rank dependent expected utility can be written as

$$RDEU = q_1 u\left(W_1\right) + \left(q_2 - q_1\right) u\left(W_2\right) + \left(q_3 - q_2\right) u\left(W_3\right) + \left(1 - q_3\right) u\left(W_4\right), \tag{1}$$

where

$$\begin{aligned} q_1 &= q\left(\pi\left(1-\theta\right)\right); \\ q_2 &= q\left(\pi\right); \\ q_3 &= q\left(\pi + \left(1-\pi\right)\left(1-\theta\right)\right), \end{aligned}$$

with $q_1 \leq q_2 \leq q_3$.

This two-stage problem can be solved by backward induction. In the second stage, the individual's objective is to maximize Equation (1) by choosing the insurance coverage Q. The first-order condition (FOC) for the optimal solution Q^* is:

$$\begin{aligned} \frac{dRDEU}{dQ} &= q_1\left(1-P\right)u'\left(W_1\right) + \left(q_2 - q_1\right)\left(1 - P + \alpha\right)u'\left(W_2\right) \\ &\quad + \left(q_3 - q_2\right)\left(-P\right)u'\left(W_3\right) + \left(1 - q_3\right)\left(-P + \alpha\right)u'\left(W_4\right) \\ &= 0. \end{aligned} \tag{2}$$

The second-order condition (SOC) holds under the assumption that u is concave. The optimal solution Q^* is a function of α. If the lotto is absent, we have $\alpha = 0$ and Equation (2) becomes

$$\begin{aligned} \left.\frac{dRDEU}{dQ}\right|_{\alpha=0} &= q_2\left(1-P\right)u'\left(W - PQ - L + Q\right) + \left(1 - q_2\right)\left(-P\right)u'\left(W - PQ\right) \\ &= 0. \end{aligned} \tag{3}$$

Let Q_0^* denote the optimal coverage under $\alpha = 0$. To examine the case where $0 < Q_0^* \leq L$, we assume the insurance premium rate P satisfies

$$q_2 < P = (1 + \lambda_c)\,\pi \leq \frac{q_2 u'(W - L)}{(1 - q_2)\,u'(W) + q_2 u'(W - L)}. \tag{4}$$

In the first stage, the government's objective function is as follows:

$$\max_{\alpha} RDEU^* = q_1 u(W_1^*) + (q_2 - q_1)\,u(W_2^*) + (q_3 - q_2)\,u(W_3^*) + (1 - q_3)\,u(W_4^*)\,, \tag{5}$$

where $RDEU^*$ and W_i^* respectively denote $RDEU$ and W_i under Q^*, $i = 1, 2, 3, 4$. The government's FOC for the optimal solution α^* is

$$\begin{aligned}
\frac{dRDEU^*}{d\alpha} &= q_1 \left(-\frac{dT}{d\alpha}\right) u'(W_1^*) + (q_2 - q_1)\left(Q^* - \frac{dT}{d\alpha}\right) u'(W_2^*) \\
&\quad + (q_3 - q_2)\left(-\frac{dT}{d\alpha}\right) u'(W_3^*) + (1 - q_3)\left(Q^* - \frac{dT}{d\alpha}\right) u(W_4^*) \\
&= 0,
\end{aligned} \tag{6}$$

where

$$\frac{dT}{d\alpha} = (1 + \lambda_g)\,\theta Q^*. \tag{7}$$

The SOC also holds since u is concave. Evaluating the above FOC at $\alpha = 0$ yields

$$\begin{aligned}
\left.\frac{dRDEU^*}{d\alpha}\right|_{\alpha=0} &= \left[(q_2 - q_1)\,Q_0^* + q_2\left(-\left.\frac{dT}{d\alpha}\right|_{\alpha=0}\right)\right] u'(W - PQ_0^* - L + Q_0^*) \\
&\quad + \left[(1 - q_3)\,Q_0^* + (1 - q_2)\left(-\left.\frac{dT}{d\alpha}\right|_{\alpha=0}\right)\right] u'(W - PQ_0^*)\,.
\end{aligned} \tag{8}$$

The following proposition provides the sufficient and necessary condition for $\left.\frac{dRDEU^*}{d\alpha}\right|_{\alpha=0} > 0$, given Equation (3).

Proposition 1 *Suppose that the individual's preference is characterized by rank dependent expected utility. The government can provide an insurance-linked lotto to increase the individual's utility if and only if*

$$(1 + \lambda_g)\,\theta < \left(1 - \frac{q_1}{q_2}\right) P + \frac{1 - q_3}{1 - q_2}\,(1 - P)\,. \tag{9}$$

In the second stage, as shown in Equation (3), under $\alpha = 0$, the optimal insurance demand Q_0^* satisfies

$$q_2 (1 - P) u' (W - PQ_0^* - L + Q_0^*) = (1 - q_2) Pu' (W - PQ_0^*) . \tag{10}$$

In the first stage, by Equations (7) and (8), the government should provide the lotto if and only if

$$\begin{aligned} &[(q_2 - q_1) Q_0^* - q_2 (1 + \lambda_g) \theta Q_0^*] u' (W - PQ_0^* - L + Q_0^*) \\ &+ [(1 - q_3) Q_0^* - (1 - q_2) (1 + \lambda_g) \theta Q_0^*] u' (W - PQ_0^*) > 0. \end{aligned} \tag{11}$$

Substituting Equation (10) into the above equation and rearranging the condition yield

$$\frac{1 - q_2}{1 - P} (1 + \lambda_g) \theta Q_0^* < \left[(q_2 - q_1) \frac{(1 - q_2) P}{q_2 (1 - P)} + (1 - q_3) \right] Q_0^* \tag{12}$$

due to the assumption $u' > 0$. The left-hand side of the above Equation can be viewed as the marginal effect of α on the lump-sum tax T and the right-hand side is the marginal effect of α on the lotto prize αQ_0^*. When $Q_0^* > 0$, re-arranging Equation (12) yields Equation (9). Thus, Proposition 1 shows that when the marginal effect of α on the lump-sum tax is smaller than that of the prize, the government can use this type of lotto to further improve the RDEU of an individual's utility.

Note that when $q(z) = z$, RDEU is reduced to the expected utility. The following corollary shows that this type of lotto should not be implemented if an individual's preferences is characterized by the traditional expected utility.

Corollary 1 *If $q(z) = z$, then $\frac{dRDEU^*}{d\alpha}\Big|_{\alpha=0} \leq 0$.*

This is because the right-hand side of Equation (9) becomes

$$\frac{(q_2 - q_1)}{q_2} P + \frac{(1 - q_3)}{(1 - q_2)} (1 - P) = \theta,$$

and Equation (9) can never hold if $\lambda_g \geq 0$.

This insurance-linked lotto creates a mean-reduction spread on wealth. Thus, for individuals whose preferences satisfy the traditional expected utility theory, none of those with concave utility functions would prefer this type of lotto. Although RDEU individuals still have concave utility functions, they distort the probability as well, with both extremely good and extremely bad events being overweighted. This insurance-linked lotto creates a extremely good events, and thus could be attractive to RDEU individuals.

Now, let us check whether an insurance-linked lotto could enhance insurance purchasing. The insurance decision is determined in the second stage. From Equation (2), we know that Q^* for individuals with rank dependent expected utility preferences satisfies

$$\mathcal{L} = \left.\frac{dRDEU}{dQ}\right|_{Q=Q^*} = 0.$$

Since the SOC holds, by the implicit function theorem, we have

$$sgn\left\{\frac{\partial Q^*}{\partial \alpha}\right\} = sgn\left\{\left.\frac{\partial \mathcal{L}}{\partial \alpha}\right|_{Q=Q^*}\right\}.$$

Note that,

$$\begin{aligned} \left.\frac{\partial \mathcal{L}}{\partial \alpha}\right|_{Q=Q^*} = {} & (q_2 - q_1)\, u'(W_2^*) + (1 - q_3)\, u'(W_4^*) \\ & + \left[(q_2 - q_1)(1 - P + \alpha)\, u''(W_2^*) + (1 - q_3)(-P + \alpha)\, u''(W_4^*)\right] Q^* \\ & - \left\{q_1 (1 - P)\, u''(W_1^*) + (q_2 - q_1)(1 - P + \alpha)\, u''(W_2^*)\right. \\ & \left. + (q_3 - q_2)(-P)\, u''(W_3^*) + (1 - q_3)(-P + \alpha)\, u''(W_4^*)\right\} \times \left.\frac{\partial T}{\partial \alpha}\right|_{Q=Q^*} \end{aligned} \tag{13}$$

This effect of an increase in α on $\mathcal{L}$ can be decomposed into three parts: (1) the marginal effect from the change in prize but keeping the wealth level unchanged, i.e.

$$(q_2 - q_1)\, u'(W_2^*) + (1 - q_3)\, u'(W_4^*);$$

(2) the marginal wealth effect from the change in prize, i.e.,

$$\left[(q_2 - q_1)(1 - P + \alpha)\, u''(W_2^*) + (1 - q_3)(-P + \alpha)\, u''(W_4^*)\right] Q^*;$$

(3) the marginal wealth effect from the change in lump-sum tax, i.e.,

$$-\left\{q_1(1-P)u''(W_1^*)+(q_2-q_1)(1-P+\alpha)u''(W_2^*)\right.$$
$$\left.+(q_3-q_2)(-P)u''(W_3^*)+(1-q_3)(-P+\alpha)u''(W_4^*)\right\}\times\left.\frac{\partial T}{\partial\alpha}\right|_{Q=Q^*}.$$

Since $u'>0$, the first effect is always positive, the second effect undetermined, and third effect depends on individual risk preferences. The following proposition provides a sufficient condition for $\frac{\partial Q^*}{\partial\alpha}>0$.

Proposition 2 *Suppose that the individual is characterized by rank dependent expected utility and the risk preferences of the individual exhibits DARA or CARA. Given $\alpha\leq P$, the insurance-linked lotto increases the equilibrium demand for insurance if*

$$\alpha\leq\frac{(1-q_3)Pu''(W_4^*)-(q_2-q_1)(1-P)u''(W_2^*)}{(q_2-q_1)u''(W_2^*)+(1-q_3)u''(W_4^*)}.\tag{14}$$

When the individual's preference exhibits DARA or CARA, the effect of α on $\mathcal{L}$ would be positive if the marginal wealth effect from the change in prize is positive, i.e.,

$$\left[(q_2-q_1)(1-P+\alpha)u''(W_2^*)+(1-q_3)(-P+\alpha)u''(W_4^*)\right]Q^*\geq0.$$

If $Q^*>0$, rearranging the above condition yields Equation (14).

3 Discussions

3.1 Multiple Preference Types of Individuals

Suppose that within society exists certain types of individuals. One is characterized by rank dependent expected utility with probability weighting function $q(z)$ while the other is characterized by expected utility. All information is assumed to be common knowledge of both the government and individuals. On basis of Proposition 1 and Corollary 1, the above insurance-linked lotto will never be a Pareto optimal solution since it will decrease the utility of the individuals characterized by expected utility.

We propose that to achieve Pareto optimality, the lotto can be designed as optional. In the first stage, the government designs the prize multiplier α and announces to launch an insurance-linked lotto. In the second stage, the individuals decide wether or not to join the lotto and decide on their insurance demand. Then, the government charges the lump-sum tax T for individuals who are willing to participate in the game. By the self-selection mechanism, we know that the rank dependent expected utility maximizer would join the insurance-linked lotto policy, but the expected utility maximizer would not. All participants will not be worse off and Pareto efficiency can be achieved.

3.2 Other Types of Insurance-Linked Lotto

In the previous type of lotto, the insurance is linked through the lotto prize. Here, we provide another type of lotto where the insurance is linked through the probability of winning a fixed prize.

Suppose that the government provides this other type of insurance-linked lotto, where the amount of the reward is M, and the probability of winning the reward is $\beta\frac{Q}{L}$. Assume that β is given and $M \geq 0$ is the decision variable for the government. The insurance-linked lotto is also financed by a lump-sum tax T, which is equal to $(1+\lambda_g)\,\beta\frac{Q}{L}M$, where λ_g denotes the expense loading of the government.

We first provide the condition for the government such that the presence of the lotto can improve the individual's utility. In this type of lotto, the final wealth of the individual is

$$\begin{aligned}
W_1 &= W - PQ - T - L + Q \text{ with probability } \pi\left(1-\beta Q/L\right); \\
W_2 &= W - PQ - T - L + Q + M \text{ with probability } \pi\beta Q/L; \\
W_3 &= W - PQ - T \text{ with probability } \left(1-\pi\right)\left(1-\beta Q/L\right); \\
W_4 &= W - PQ - T + M \text{ with probability } \left(1-\pi\right)\beta Q/L.
\end{aligned}$$

The starting point of our analysis settles on the case where the insurance-linked lotto does not exist, i.e., $M = 0$. Further, assume that $Q \leq L$. Therefore, we can conclude that $M \leq L - Q$. So the final wealth in each state can be ranked as

$$W_1 \leq W_2 \leq W_3 \leq W_4.$$

Given the insurance-linked lotto, the individual's rank dependent expected utility can be written as

$$RDEU = q_1 u(W_1) + (q_2 - q_1) u(W_2) + (q_3 - q_2) u(W_3) + (1 - q_3) u(W_4), \tag{15}$$

where

$$\begin{aligned} q_1 &= q(\pi(1 - \beta Q/L)) \\ q_2 &= q(\pi) \\ q_3 &= q(\pi + (1-\pi)(1 - \beta Q/L)). \end{aligned}$$

with $q_1 \leq q_2 \leq q_3$.

The following proposition provides the sufficient and necessary condition for $\frac{dRDEU}{dM}\big|_{M=0} \geq 0$.

Proposition 3 *If the individual is characterized by rank dependent expected utility, the government maximizes the individual's rank dependent expected utility and provides an insurance-linked lotto to the market if and only if*

$$(1+\lambda_g)\beta\frac{Q_0^*}{L} \leq P\left(1 - \frac{q_1}{q_2}\right) + (1-P)\frac{1-q_3}{1-q_2},$$

where Q_0^* *is the optimal insurance coverage in the absence of the insurance-linked lotto.*

The proof is similar to that of Proposition 1 and thus is omitted. The following corollary provides a necessary condition for Proposition 3 and further elaborates it.

Corollary 2 *If* $q(z) = z$*, then* $\frac{dRDEU}{dM}\big|_{M=0} \leq 0$.

Knowing that when $q(z) = z$, rank dependent expected utility is reduced to the expected utility. Thus Corollary 2 shows that if the individual is characterized by the traditional expected utility, the government should never provide an insurance-linked lotto.

The insight remains the same as shown by Corollaries 1 and 2. That is, a self-financed insurance-linked lotto creates a mean-preserving spread and makes expected utility maximizers worse off, but could improve the welfare of individuals characterized by rank dependent utility.

4 Conclusions

As pointed out by Chetty (2015), "behavioral economics offers new policy tools." In this paper, we had proposed a new policy tool, "insurance-linked lotto", under a RDEU model. By assuming full information, we demonstrated that implementing the insurance-linked lotto could improve social welfare and increase the equilibrium demand for insurance when an individual's preferences are characterized by RDEU.

It is well known that state-dependent utility and range-dependent utility (Kontek and Lewandowski, 2018) can also explain the coexistence of gambling and insurance. Thus, examining the effects of the insurance-linked lotto under state-dependent utility or range-dependent utility would be fruitful for future studies.

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Appendix

A.1 Proof of Proposition 1

From Equations (3), we have

$$\begin{aligned} \left.\frac{dRDEU}{dQ}\right|_{\alpha=0} &= q_2(1-P)u'(W-PQ_0^*-L+Q_0^*)+(1-q_2)(-P)u'(W-PQ_0^*) \\ &= 0, \end{aligned}$$

or

$$u'(W-PQ_0^*-L+Q_0^*)=\frac{(1-q_2)}{q_2}\frac{P}{(1-P)}u'(W-PQ_0^*).$$

Note that to have an internal solution, we assume $1-P=1-(1+\lambda_c)\pi>0$. Furthermore, we have

$$\left.\frac{dT}{d\alpha}\right|_{\alpha=0}=(1+\lambda_g)\theta Q_0^*.$$

Thus, Equation (8) can be rewritten as

$$\begin{aligned} \left.\frac{dRDEU^*}{d\alpha}\right|_{\alpha=0} &= \left[(q_2-q_1)Q_0^*+q_2\left(-\left.\frac{dT}{d\alpha}\right|_{\alpha=0}\right)\right]u'(W-PQ_0^*-L+Q_0^*) \\ &\quad+\left[(1-q_3)Q_0^*+(1-q_2)\left(-\left.\frac{dT}{d\alpha}\right|_{\alpha=0}\right)\right]u'(W-PQ_0^*) \\ &= \left[(q_2-q_1)-q_2(1+\lambda_g)\theta\right]\frac{(1-q_2)}{q_2}\frac{P}{(1-P)}Q_0^*u'(W-PQ_0^*) \\ &\quad+\left[(1-q_3)-(1-q_2)(1+\lambda_g)\theta\right]Q_0^*u'(W-PQ_0^*). \end{aligned} \tag{A.1}$$

Since $Q_0^*>0$, we have $\left.\frac{dRDEU^*}{d\alpha}\right|_{\alpha=0}>0$ if and only if

$$(1+\lambda_g)\theta<\frac{(q_2-q_1)}{q_2}P+\frac{(1-q_3)}{(1-q_2)}(1-P).$$

A.2 Proof of Proposition 2

From Equations (2) and (13), we have

$$\begin{aligned}\frac{dRDEU}{dQ} &= q_1(1-P)u'(W_1)+(q_2-q_1)(1-P+\alpha)u'(W_2)\\ &\quad+(q_3-q_2)(-P)u'(W_3)+(1-q_3)(-P+\alpha)u'(W_4)\\ &= 0. \end{aligned} \tag{A.2}$$

and

$$\begin{aligned}\left.\frac{\partial\mathcal{L}}{\partial\alpha}\right|_{Q=Q^*} &= (q_2-q_1)u'(W_2^*)+(1-q_3)u'(W_4^*)\\ &\quad+\left[(q_2-q_1)(1-P+\alpha)u''(W_2^*)+(1-q_3)(-P+\alpha)u''(W_4^*)\right]Q^*\\ &\quad-\left\{q_1(1-P)u''(W_1^*)+(q_2-q_1)(1-P+\alpha)u''(W_2^*)\right.\\ &\quad\left.+(q_3-q_2)(-P)u''(W_3^*)+(1-q_3)(-P+\alpha)u''(W_4^*)\right\}\times\left.\frac{\partial T}{\partial\alpha}\right|_{Q=Q^*} \end{aligned} \tag{A.3}$$

Note that the effect of initial wealth on Q^* depends on

$$\begin{aligned}\left.\frac{\partial\mathcal{L}}{\partial W}\right|_{Q=Q^*} &= q_1(1-P)u''(W_1^*)+(q_2-q_1)(1-P+\alpha)u''(W_2^*)\\ &\quad+(q_3-q_2)(-P)u''(W_3^*)+(1-q_3)(-P+\alpha)u''(W_4^*). \end{aligned} \tag{A.4}$$

Let

$$R(W_i^*) = -\frac{u''(W_i^*)}{u'(W_i^*)},\ i=1,2,3,4.$$

Equation (A.4) can be rewritten as

$$\begin{aligned}\left.\frac{\partial\mathcal{L}}{\partial W}\right|_{Q=Q^*} &= -\left[q_1(1-P)u'(W_1^*)R(W_1^*)+(q_2-q_1)(1-P+\alpha)u'(W_2^*)R(W_2^*)\right.\\ &\quad\left.+(q_3-q_2)(-P)u'(W_3^*)R(W_3^*)+(1-q_3)(-P+\alpha)u'(W_4^*)R(W_4^*)\right]. \end{aligned}$$

If the risk preference of the individual is DARA or CARA, we have $R(W_1^*) \geq R(W_2^*) \geq$

$R(W_3^*) \geq R(W_4^*)$. Note that given the assumption $\alpha < P$ and $u' \geq 0$, we have

$$\begin{aligned}
q_1 (1-P) u'(W_1^*) &\geq 0, \\
(q_2-q_1)(1-P+\alpha) u'(W_2^*) &\geq 0, \\
(q_3-q_2)(-P) u'(W_3^*) &\leq 0, \text{ and} \\
(1-q_3)(-P+\alpha) u'(W_4^*) &\leq 0.
\end{aligned}$$

Thus, if $R(W_1^*) \geq R(W_2^*) \geq R(W_3^*) \geq R(W_4^*)$, then we have

$$\begin{aligned}
& q_1 (1-P) u'(W_1^*) R(W_1^*) + (q_2-q_1)(1-P+\alpha) u'(W_2^*) R(W_2^*) \\
& + (q_3-q_2)(-P) u'(W_3^*) R(W_3^*) + (1-q_3)(-P+\alpha) u'(W_4^*) R(W_4^*) \\
\geq\ & q_1 (1-P) u'(W_1^*) R(W_3^*) + (q_2-q_1)(1-P+\alpha) u'(W_2^*) R(W_3^*) \\
& + (q_3-q_2)(-P) u'(W_3^*) R(W_3^*) + (1-q_3)(-P+\alpha) u'(W_4^*) R(W_3^*) \\
\geq\ & 0,
\end{aligned}$$

The last inequality holds from Equation (A.2). In other words, the last therm of Equation (A.3) is positive. Thus, by assuming DARA and CARA, if

$$\left[(q_2-q_1)(1-P+\alpha) u''(W_2^*) + (1-q_3)(-P+\alpha) u''(W_4^*)\right] Q^* \geq 0,$$

or, equivalently,

$$\alpha \leq \frac{(1-q_3) P u''(W_4^*) - (q_2-q_1)(1-P) u''(W_2^*)}{(q_2-q_1) u''(W_2^*) + (1-q_3) u''(W_4^*)},$$

then $\frac{\partial \mathcal{L}}{\partial \alpha}\big|_{Q=Q^*} \geq 0$. By the implicit function theorem, we have

$$\frac{\partial Q^*}{\partial \alpha} \geq 0.$$