The Efficiency of Voluntary Risk Classification in Insurance Markets

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Abstract

It has been established that categorical discrimination based on observable characteristics such as gender, age, or ethnicity enhances efficiency. We consider a different form of risk classification where there exists some costless yet imperfectly informative test on risk types, with the test outcome unknown to the agents *ex-ante*. We show that a voluntary risk classification where agents are given the option to take the test always increases efficiency compared with no risk classification. Moreover, voluntary risk classification also Pareto dominates the regime of compulsory risk classification in which all agents are required to take the test.

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1 Introduction

When insurers face customers who have private information about their propensities for suffering insurable losses, the resulting problem of adverse selection is widely understood to have pernicious effects on insurance markets. A common response by insurers is to engage in risk classification, a process in which the contracts offered to each customer are conditioned on observable characteristics associated with that customer which are known to be correlated with the individual's underlying risk. Employed in this fashion, risk classification may be viewed as a tool insurers use against privately informed consumers to mitigate their informational disadvantage. In contrast to the traditional approach in which insurers make the decision to engage in risk classification, there has recently been a move in some settings to permit the insured individual to choose whether or not to be classified. This paper develops a model of voluntary classification and demonstrates that, by increasing the dimensionality of the screening space, such classification always is welfare enhancing. Moreover, we find that voluntary classification is always preferred to a regime of compulsory classification in which all customers are subject to classification.

The use of risk classification by insurers on the basis of immutable customer characteristics that are both costlessly observable and imperfectly correlated with underlying risk, such as sex, age or race, is first examined by Hoy (1982). Using an equilibrium analysis, he concludes that the effect of risk classification depends on the particular equilibrium concept employed and that there may be winners as well as losers, so that the net effect of permitting such classification is ambiguous. In contrast, Crocker and Snow (1986) examine costless but imperfect categorization in the context of an efficiency analysis and demonstrate that to permit such classification would shift the utility possibilities frontier outward, resulting in Pareto improvements. Bond and Crocker (1991) examine the use of risk classification by insurers on the basis of observable consumer choices that either directly cause higher risk, such as smoking and heart disease, or that simply have a correlation with risk, such as students with good grades who tend to be more careful drivers. They find that the insurers can harness the consumption choices made by consumers to design more efficient contracts in the insurance market. Finally, Rothschild (2011) examines the case of risk categorization on the basis of immutable characteristics that are costly to observe and concludes that to prohibit such classification would be inefficient.[1] The common threads in this received literature on risk classification are that the decision to categorize is made by the insurer, so the insured has no say in the matter, and that the characteristics upon which the classification is based, such as gender, age or ethnicity, are observable to everyone *ex-ante*. Put differently, the insured

[1] Specifically, Rothschild (2011) demonstrates that the government can design and implement a partial social insurance policy that provides insurers with the incentive to engage in costly categorization only when it is socially efficient to do so.

faces no uncertainty regarding the outcome of the classification test.[2]

In recent years, voluntary classification has become increasingly common in the context of automobile insurance. Perhaps the best known example is the "Snapshot" program offered by Progressive in which drivers may opt to install a telematic device in their car that, for six months, will track several parameters (such as hard braking) of driving behavior.[3] Drivers that are determined by the insurer to be "good" are given a discount that remains even after the device is removed, and those who are determined to be "bad" may see rate increases in some states. Similar usage-based programs are offered by The Hartford ("Trulane"), Nationwide ("SmartRide"), Safeco ("Rewind") and Travelers ("Intellidrive").[4] What all of these programs have in common is that the customers decide whether to opt in, the device is removed after several months, and good drivers are given discounts that continue even after the device is removed.[5] The underlying assumption of the insurers offering these programs is that most people have repetitive driving behaviors that a plug-in device is sufficient to identify, so that continual monitoring is not necessary.

This paper examines the welfare effect of voluntary risk classification under which the customer is given the option of taking a costless and imperfectly informative test, the result from which—uncertain to both parties before the test—will be used to classify the individual for insurance purposes. We find that the use of such voluntary categorization is always efficient. This is because the use of the test increases the dimensionality of the screening space, so that the informationally constrained insurer can now sort customers based not only upon their willingness to accept a deductible, but also upon their willingness to take the test. Perhaps more surprisingly, we also show that voluntary classification is more efficient than a regime of compulsory classification in which all customers are required to take the test. The intuition for this result is that, by making the test mandatory, the insurer loses the information contained in a customer's voluntary *choice* to be classified.

The paper proceeds as follows. The next section provides a model of voluntary risk classification and demonstrates that it is efficient. The following section compares the regime of voluntary classification to that of compulsory classification, and concludes that the former dominates, in an efficiency sense, the latter. A final section contains concluding remarks.

[2]See Dionne and Rothschild (2014) for a recent survey of research on efficiency effects of insurance risk classification.

[3]For details of the program, see https://www.progressive.com/auto/discounts/snapshot/.

[4]See e.g. https://www.nerdwallet.com/blog/insurance/comparing-drivewise-snapshot-usage-based-insurance/ for a comparison of different usage-based auto insurance plans that are currently available in the US.

[5]There are also usage-based programs that require non-stop use of the telematic device, for example see Kremslehner and Muermann (2016) who use European telematic data in which the device remains in the vehicle and find correlation between driving behavior and contract choice as well as risk type. Similarly, in the US, Allstate Insurance has the "Drivewise" program using a telematic device that remains in the vehicle.

2 Voluntary Risk Classification

We consider the canonical adverse selection model of insurance markets first described by Rothschild and Stiglitz (1976). Agents in the model are each endowed with an initial wealth W, and face a random insurable loss of D. Agents differ from each other with respect to the probability of incurring the loss. In particular, there are only two types of agents: "high-risk" (H-) types with the probability p^H of suffering the loss, and "low-risk" (L-) types with the probability $p^L < p^H$ of suffering the loss. We assume that the H-types are a proportion λ of the entire population, and that the value of λ is known to everyone. Each agent's risk type, however, cannot be directly observed by the insurer as it is private information known only to the agent.

We introduce the possibility of risk classification by the application of a test that is a costless but imperfect indicator of underlying risk type. We assume that the outcome of this test can be either "good" (G) or "bad" (B). The accuracy of the test can be characterized by the following conditional probabilities: $P(B|H) \equiv \alpha^H$, and $P(G|L) \equiv \alpha^L$. Naturally, a higher α^H or α^L indicates more informative classification. We restrict the range of α^H and α^L in the following assumption.

Assumption 1. $1 < \alpha^H + \alpha^L < 2$.

Requiring $\alpha^H + \alpha^L < 2$ eliminates from consideration the trivial case where $\alpha^H = \alpha^L = 1$, since then the costless classification is perfect and thus completely removes the information asymmetry from the model. To see the importance of the other side of the inequality, we note that $\alpha^H + \alpha^L > 1 \Leftrightarrow \alpha^H \alpha^L > (1 - \alpha^H)(1 - \alpha^L) \Leftrightarrow {}^{\alpha^H}/_{1-\alpha^H} > {}^{1-\alpha^L}/_{\alpha^L} \Leftrightarrow {}^{P(B|H)}/_{P(G|H)} > {}^{P(B|L)}/_{P(G|L)}$. The condition is therefore equivalent to the odds ratio $\frac{P(B|H)/P(G|H)}{P(B|L)/P(G|L)} > 1$, which indicates that the test is (at least partially) informative since H-types are then more likely to be classified in category B than L-types. In contrast, if $\alpha^H + \alpha^L = 1$, then the test would be uninformative because both H-types and L-types would be equally likely to be classified in either category.[6]

An insurance contract is denoted by $C \equiv (R, M)$, where R is the insurance premium paid by the agent and M is the indemnity paid by the insurer in the event of loss. Assume that each agent has the same risk-averse preference based on the von Neumann-Morgenstern utility function $U(\cdot)$, where U is twice differentiable, strictly increasing ($U' > 0$) and strictly concave ($U'' < 0$). The utility is based on the agent's final wealth in each state, that is, after choosing an insurance contract C, the agent has utility $U(W-R)$ if she does not experience the loss, and utility $U(W-D-R+M)$

[6] While it is somewhat counterintuitive to assume $\alpha^H + \alpha^L < 1$, we can show that when this is the case, all findings in the paper remain the same by interchanging G and B. Indeed, when the test is systematically flawed in the sense that it places more H-types into group G, we can simply remedy this by recognizing/relabeling group G as the "bad" group, and group B as the "good" group.

if she does experience the loss. Her expected utility is then defined as

$$V(p, R, M) = (1 - p) \times U(W - R) + p \times U(W - D - R + M),$$

with p as the probability of suffering the loss.

We first consider as a baseline for comparison the no-classification case by characterizing efficient contracts in the absence of risk classification.[7] In this case, the insurer offers a set of two contracts: $\mathbb{C}_{NC} \equiv \{C_H, C_L\}$, with $C_H = (R_H, M_H)$ and $C_L = (R_L, M_L)$ for H-types and L-types, respectively while satisfying standard resource, incentive compatibility (IC) and individual rationality (IR) constraints. The class of efficient contracts in the regime of no-classification can be formally characterized as a solution to the following problem:

$$\max_{(C_H, C_L)} V(p^L, R_L, M_L) \tag{NC}$$

subject to

$$\lambda \times (R_H - p^H M_H) + (1 - \lambda) \times (R_L - p^L M_L) \geq 0, \tag{1}$$

$$V(p^H, R_H, M_H) \geq V(p^H, R_L, M_L), \tag{2}$$

$$V(p^L, R_L, M_L) \geq V(p^L, R_H, M_H), \tag{3}$$

$$V(p^H, R_H, M_H) \geq \bar{V}^H. \tag{4}$$

A solution to optimization problem (NC) satisfies the necessary conditions of Theorem 1 in Crocker and Snow (1986).[8] In particular, H-types always obtain full insurance ($M_H = D$) and L-types receive less than full insurance ($M_L < D$); the resource constraint (1) binds; for H-types the incentive constraint (2) binds; and the incentive constraint for L-types (3) is slack. In this traditional treatment, the L-types have a lower cost of bearing the deductible ($M_L < D$) than do the H-types, and this is used by the insurer as a tool to separate the different agent types.

We now consider the efficiency problem with voluntary classification, where we assume that the agents can choose to take the test (that is, to participate in the risk classification) or not. For those who voluntarily choose to be classified, the insurer can offer different contracts based on the *ex-post* publicly observed outcome of the test (G or B). In practice, this can be implemented by

[7] While this case has been thoroughly studied by Crocker and Snow (1985, 1986) and we offer no new findings here, we explicitly write down the optimization problem and discuss some necessary conditions for its solution in order to have a clearer comparison with other cases that consider risk classification.

[8] As do Crocker and Snow (1986), we will restrict our attention in the analysis that follows to only consider cases in which $\bar{V}^H < V(p^H, (\lambda p^H + (1 - \lambda)p^L) \times D, D)$. That is, we cap the welfare, $\bar{V}^H$, of H-types at the equal per capita full insurance level which is denoted as the F point in the standard Rothschild and Stiglitz analysis. Setting $\bar{V}^H = V(p^H, (\lambda p^H + (1 - \lambda)p^L) \times D, D)$ in problem (NC) yields as a solution the first-best equal per capita full insurance policy that cannot be improved upon by any type of risk classification.

the insurer first offering a low premium to those signing up to take the test (e.g. to use a telematic plug-in device), with the understanding that the insurer may impose additional surcharges in the event of adverse classification outcome, as is seen in the Snapshot case. In a setting where some agents elect to be tested and others do not, the insurer can in principle offer three types of contracts: $\mathbb{C}_{VC} \equiv \{C_N, C_G, C_B\}$, with $C_N = (R_N, M_N)$ for the non-participating agents; $C_G = (R_G, M_G)$ for the participating agents who are subsequently classified in group G; and $C_B = (R_B, M_B)$ for the participating agents who are subsequently classified in group B. In the analysis that follows, we characterize optimal contracts in which only the L-types choose to take the test and H-types elect not to be classified.

The class of efficient contracts with voluntary classification only chosen by L-types can be formally characterized by solving the following problem:

$$\max_{(C_N, C_G, C_B)} \alpha^L \times V(p^L, R_G, M_G) + (1-\alpha^L) \times V(p^L, R_B, M_B) \tag{VC}$$

subject to

$$\lambda \times (R_N - p^H M_N) + (1-\lambda) \times \{\alpha^L (R_G - p^L M_G) + (1-\alpha^L)(R_B - p^L M_B)\} \geq 0, \tag{5}$$

$$V(p^H, R_N, M_N) \geq (1-\alpha^H) V(p^H, R_G, M_G) + \alpha^H V(p^H, R_B, M_B), \tag{6}$$

$$\alpha^L V(p^L, R_G, M_G) + (1-\alpha^L) V(p^L, R_B, M_B) \geq V(p^L, R_N, M_N), \tag{7}$$

$$V(p^H, R_N, M_N) \geq \bar{V}^H.\text{[9]} \tag{8}$$

Since the test is imperfectly informative, the L-types who take the test do not know the outcome beforehand, and so anticipate receiving the contracts C_G with probability α^L and C_B with probability $1 - \alpha^L$ as reflected in the objective function. In order to choose to be voluntarily classified, the L-types must prefer the expected outcome of electing to take the test over the alternative of not taking the test where they receive the contract C_N, which is assured by the incentive constraint (7). The H-types, were they to choose to take the test, would receive the contracts C_B with probability α^H and C_G with probability $1 - \alpha^H$, whereas by choosing not to be classified they would receive the contract C_N with certainty. The incentive constraint (6) guarantees that the H-types prefer not to be classified, so that their individual rationality condition (8) is formulated in the same way as for the no-classification problem.

In the following theorem, we examine the necessary conditions for a solution to the optimization problem (VC). The proof of the theorem is contained in the Appendix.

[9]As in the no-classification case, we restrict our analysis by only considering cases with $\bar{V}^H < V(p^H, (\lambda p^H + (1-\lambda)p^L) \times D, D)$.

Theorem 1. *A solution to the voluntary classification problem* (VC) *satisfies the following necessary conditions:*

$$\lambda \times (R_N - p^H M_N) + (1-\lambda) \times \{\alpha^L (R_G - p^L M_G) + (1-\alpha^L)(R_B - p^L M_B)\} = 0, \quad \text{(a)}$$

$$M_N = D, \quad \text{(b)}$$

$$V(p^H, R_N, M_N) = (1-\alpha^H) V(p^H, R_G, M_G) + \alpha^H V(p^H, R_B, M_B), \quad \text{(c)}$$

$$M_G < D, \quad \text{(d)}$$

$$M_B < D, \quad \text{(e)}$$

$$R_B > R_G, \quad \text{(f)}$$

$$M_G - R_G > M_B - R_B, \quad \text{(g)}$$

$$\alpha^L V(p^L, R_G, M_G) + (1-\alpha^L) V(p^L, R_B, M_B) > V(p^L, R_N, M_N). \quad \text{(h)}$$

Proof. See Appendix. □

Condition (a) indicates that the resource constraint (5) holds with equality at a solution. Condition (h) implies that constraint (7) is slack, so that L-types strictly prefer to take the test, and condition (c) implies that constraint (6) holds with equality, so that H-types opt not to be classified. Subsequently, (b) implies that the untested H-types receive full insurance, while conditions (d) and (e) indicate that the L-types, who elect to take the test, will receive partial coverage irrespective of the outcome of the test. Finally, condition (f) implies that the premium paid by those receiving the "bad" outcome of the test is higher than those who receive the "good" outcome, and (g) further indicates that the coverage net of premium ($M-R$) is higher for those who are classified as "good" rather than "bad".

An immediate implication of conditions (f) and (g) is that

$$\begin{aligned} (1-p^L)R_G - p^L(M_G - R_G) &< (1-p^L)R_B - p^L(M_B - R_B) \\ \Rightarrow \quad R_G - p^L M_G &< R_B - p^L M_B, \end{aligned}$$

so that, within the L-risk types, there is an internal cross-subsidization from those being classified in group B to those being classified in group G.

The next result is that efficient contracts with voluntary classification treat those who elect to take the test, and receive different test outcomes, differently whenever the test is partially informative.

Corollary 1. $C_B = C_G$ *if and only if* $\alpha^H + \alpha^L = 1$.

Proof. See Appendix. □

Now, let $C_H^* = (R_H^*, D)$ and $C_L^* = (R_L^*, M_L^*)$ denote efficient contracts from the no-classification problem. Consider the set of contracts $\bar{\mathbb{C}}_{VC} = \{\bar{C}_N, \bar{C}_G, \bar{C}_B\}$ under voluntary classification in which the insurer offers $\bar{C}_N \equiv C_H^* = (R_H^*, D)$ to the non-participating agents, and $\bar{C}_G = \bar{C}_B \equiv C_L^* = (R_L^*, M_L^*)$ to the participating agents regardless of the test outcome. It is easy to show that the set of contracts $\bar{\mathbb{C}}_{VC}$ satisfies all constraints ((5) through (8)) of the voluntary classification problem (VC), and so is a feasible solution to the problem.[10] But we know from Corollary 1 that, whenever the test is informative, a solution to problem (VC) necessarily treats those taking the test differently based upon the outcome of the test. Therefore, the set of contracts $\bar{\mathbb{C}}_{VC}$ that implements the efficient contracts in the no-classification regime is feasible for, but is not a solution of, the voluntary classification optimization problem. The following result is immediate.

Theorem 2. *Voluntary classification dominates no classification whenever $\alpha^H + \alpha^L > 1$, that is, when the test is (at least partially) informative.*

A point worth noting at this juncture is that the efficient contracts penalize those who have the "bad" outcome from the imperfectly informative test even though the insurer knows that agents have been perfectly sorted by the combination of the voluntary test and the deductibles so that only the L-risks elect to take the test. While this may seem to be counterintuitive, the reason for this result is that, by committing to punish those who fail the test, the option to take the test becomes less attractive to H-types since they have a higher probability of being classified into the "bad" group. As a result, committing to penalize any agent who does not pass the test enables the insurer to screen the H- and L-types more efficiently, and also makes L-types better off *ex-ante*.

3 Compulsory Risk Classification

In this section, we consider an alternative regime in which all agents are required to take the test and thereby to participate in a compulsory risk classification. We first show that, after the agents take the test, the optimization problem with compulsory risk classification is identical to the one described in Crocker and Snow (1986). We then demonstrate that the voluntary risk classification Pareto dominates such compulsory risk classification.

Define $\lambda_G \equiv P(H|G)$ and $\lambda_B \equiv P(H|B)$. With the compulsory risk classification imposed upon the entire population, the proportions λ_G and λ_B are the proportions of H-types in groups G and B, respectively. The following lemma characterizes the effect of the application of the imperfectly informative test.

[10] Put differently, when the contracts $\bar{\mathbb{C}}_{VC}$ are substituted into constraints (5)-(8) of problem (VC), they collapse to constraints (1)-(4) of the (NC) problem.

Lemma 1. $0 < \lambda^G < \lambda < \lambda^B < 1$ *if and only if* $\alpha^H + \alpha^L > 1$.

Proof. See Appendix. □

Under compulsory classification, all agents are subject to the test and, as a result, all are assigned to either group G or group B. Moreover, Lemma 1 indicates that when $\alpha^H + \alpha^L > 1$, so that when the classification is imperfectly informative, it is more likely to find H-types in group B than in group G. Therefore, after the application of the compulsory test, the insurer faces two distinct groups of agents that have different proportions of H-types. As a result, the insurer can tailor its insurance policies so that each group is offered a different menu of contracts.

This is precisely the environment faced by an insurer in the model of categorical discrimination by Crocker and Snow (1986). As in that setting, the insurer offers a set of four different contracts: $\mathbb{C}_{CC} \equiv \{C_{LG}, C_{HG}, C_{LB}, C_{HB}\}$. For those who are classified in group G, they can choose between either $C_{LG} = (R_{LG}, M_{LG})$ or $C_{HG} = (R_{HG}, M_{HG})$. Similarly, for those who are classified in group B, they can choose between either $C_{LB} = (R_{LB}, M_{LB})$ or $C_{HB} = (R_{HB}, M_{HB})$. Following Crocker and Snow (1986), one can characterize the class of efficient contracts as a solution of the following optimization problem:

$$\max_{(C_{LG}, C_{HG}, C_{LB}, C_{HB})} V(p^L, R_{LB}, M_{LB}) \tag{CC}$$

subject to

$$\lambda \times \{\alpha^H(R_{HB} - p^H M_{HB}) + (1-\alpha^H)(R_{HG} - p^H M_{HG})\} + (1-\lambda) \times \{\alpha^L(R_{LG} - p^L M_{LG}) + (1-\alpha^L)(R_{LB} - p^L M_{LB})\} = 0, \tag{9}$$

$$V(p^H, R_{HG}, M_{HG}) \geq V(p^H, R_{LG}, M_{LG}), \tag{10}$$

$$V(p^L, R_{LG}, M_{LG}) \geq V(p^L, R_{HG}, M_{HG}), \tag{11}$$

$$V(p^H, R_{HB}, M_{HB}) \geq V(p^H, R_{LB}, M_{LB}), \tag{12}$$

$$V(p^H, R_{HG}, M_{HG}) \geq V(p^H, R_H^*, D), \tag{13}$$

$$V(p^H, R_{HB}, M_{HB}) \geq V(p^H, R_H^*, D), \tag{14}$$

$$V(p^L, R_{LG}, M_{LG}) \geq V(p^L, R_L^*, M_L^*), \tag{15}$$

where (R_H^*, D) and (R_L^*, M_L^*) are, as before, efficient contracts from the no-classification regime. Theorem 2 in Crocker and Snow (1986) characterizes the necessary conditions a solution to problem (CC) satisfies. In particular, the H-types again obtain full insurance irrespective of the classification outcome. They also show that when such classification is costless, the utility possibilities frontier of the classification regime lies outside, and nowhere inside, of the the utility possibilities frontier of the no-classification regime, so that categorical discrimination is indeed efficient. We denote a solution to the optimization problem (CC) as $\mathbb{C}^*_{CC} = \{C^*_{LG}, C^*_{HG}, C^*_{LB}, C^*_{HB}\}$.

Before proceeding, it is useful to highlight an important difference between the compulsory classification that is the topic of this section, and the model of categorical discrimination considered by Crocker and Snow (1986). In the latter setting, the risk classification was conditioned on a costlessly *observable* characteristic, such as gender, age or ethnicity, that was imperfectly correlated with underlying risk types (H or L). As a result, the insurance customers were endowed initially with the knowledge of their group membership. In contrast, when evaluating compulsory classification, the customers only know that they are to be tested, and that the result of the imperfectly informative test will be used to place them in one group or the other with a known probability. Put differently, to evaluate the welfare effects of compulsory classification, one must consider the *ex-ante* expected utility of consumers who anticipate being tested.

In the case of compulsory classification, the expected utilities based on a set of efficient contracts $\mathbb{C}^*_{CC}$ are denoted by

$$EUT^L_{CC} = \alpha^L V(p^L, R^*_{LG}, M^*_{LG}) + (1-\alpha^L)V(p^L, R^*_{LB}, M^*_{LB})$$

for the L-types, and

$$\begin{aligned} EUT^H_{CC} &= \alpha^H V(p^H, R^*_{HB}, M^*_{HB}) + (1-\alpha^H)V(p^H, R^*_{HG}, M^*_{HG}) \\ &= \alpha^H U(W - R^*_{HB}) + (1-\alpha^H)U(W - R^*_{HG}) \end{aligned}$$

for the H-types.

Now, we construct a set of contracts under the regime of voluntary classification as $\hat{\mathbb{C}}_{VC} = \{\hat{C}_N, \hat{C}_G, \hat{C}_B\}$, where $\hat{C}_N = (\hat{R}_N, \hat{M}_N) \equiv (\alpha^H \times R^*_{HB} + (1-\alpha^H) \times R^*_{HG}, D)$, $\hat{C}_G = C^*_{LG}$, and $\hat{C}_B = C^*_{LB}$. This contract assigns to H-types opting out of the test a single contract that is full insurance with a premium being the average of the premiums assigned to the H-types under the compulsory categorization contracts $\mathbb{C}^*_{CC}$. The L-types are assigned contracts that are equivalent to what they would receive under the compulsory classification contracts $\mathbb{C}^*_{CC}$. The following lemma shows that such set of contracts $\hat{\mathbb{C}}_{VC}$ meets all constraints but the I.C. constraint for L-types (7) for the optimization problem (VC).

Lemma 2. *The set of contracts $\hat{\mathbb{C}}_{VC}$ satisfies* (5), (6) *and* (8) *for the voluntary classification efficiency problem as characterized in Equation* (VC)*, with the IR threshold $\bar{V}^H = V(p^H, \hat{R}_N, \hat{M}_N)$.*

Proof. See Appendix. □

The following theorem summarizes the main finding of the paper with regard to the welfare comparison between the two regimes of classification.

Theorem 3. *$\hat{\mathbb{C}}_{VC}$ dominates compulsory classification, but is itself dominated by voluntary classification. So voluntary classification always dominates compulsory classification.*

Proof. See Appendix. □

While it is difficult to directly compare efficient contracts under voluntary and compulsory classification to draw a welfare inference, Theorem 3 shows that we can use the set of contracts $\hat{\mathbb{C}}_{VC}$ as an intermediary and with its help make indirect comparisons between the two regimes. First of all, by keeping the contracts for the L-types intact, they will be indifferent between $\hat{\mathbb{C}}_{VC}$ and any set of efficient contracts from compulsory classification. Lemma 2 further demonstrates that $\hat{\mathbb{C}}_{VC}$ is a feasible solution to a relaxed optimization problem that is equivalent to (VC) only without the constraint (7) that we know, by Theorem 1, is slack at a solution to (VC). We then prove that $\hat{\mathbb{C}}_{VC}$ violates a necessary condition identified by Theorem 1, so that it cannot be a solution to that problem. Therefore, $\hat{\mathbb{C}}_{VC}$ is strictly less preferred by the L-types than *the* optimal set of efficient contracts to such relaxed problem, which can further be shown to be identical to the optimal solution from the standard problem (VC).[11] On the other hand, by setting an *average* contract defined in $\hat{C}_N$ as the IR threshold (8) for the H-types of the voluntary classification problem, we know that a set of efficient contracts from voluntary classification will always be (weakly) preferred by the H-types to any set of efficient contracts from compulsory classification, due to the direct application of Jensen's inequality. To sum up, for any set of efficient contracts under compulsory classification, we show that there always exists a set of efficient contracts under voluntary classification that will be strictly preferred by at least one risk type. Voluntary classification therefore Pareto dominates compulsory classification.

When requiring all agents to take the test, the insurer gains more information on the direct outcome of the test, which is however only *imperfectly* informative of the underlying risk types. The cost of doing so, is that the insurer can no longer use the willingness to participate in such test as a tool to *perfectly* sort out high risk types from low risk types. While both cases are themselves welfare-improving from a no-classification regime, we show in this section that the informational gain from an increase in the screening space dimensionality outweighs the informational gain from an imperfect system-wise test imposed within the current screening space. This makes voluntary classification a better choice over compulsory classification.

[11] As is detailed in the Appendix and is evident in condition (h) in Theorem 1, constraint (7) is always slack in a set of efficient contracts to problem (VC). Therefore, the set of contracts is also efficient in a similar optimization problem (VC') but without constraint (7).

4 Conclusion

In this paper, we examine the welfare effect when there exists some costless yet imperfectly informative test on the hidden risk types. We first show that a voluntary risk classification, that is, giving agents the option to take the test is always more effective when compared with not utilizing such test. We note that the use (or no use) of the voluntary test creates an additional dimension in the screening space that helps the insurer to separate different risk types more effectively. We further show that a voluntary risk classification is also always more effective when compared with a compulsory risk classification, one that requires all agents to take the test. The intuition for this result is that the insurer can gain more information from observing agents' choice on whether or not to participate, than from simply collecting imperfect test outcomes by imposing it to all agents.

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Appendix

This appendix provides proofs of Theorems 1, 3, Corollary 1, and Lemmas 1, 2.

Proof of Theorem 1

We denote the Lagrange multipliers for (5), (6), (7), and (8) as γ, μ_H, μ_L, and δ, respectively. Furthermore, for the simplicity of expression, define state 1 as the state with no loss, and state 2 as the state with loss, we then have $W_N^1 = W - R_N$; $W_N^2 = W - R_N - D + M_N$; $W_G^1 = W - R_G$; $W_G^2 = W - R_G - D + M_G$; $W_B^1 = W - R_B$; $W_B^2 = W - R_B - D + M_B$. In the proof, we start by solving the maximization problem (VC) without considering the I.C. constraint of the L-types (Inequality (7)), and then show that under the solved efficient contracts, constraint (7) is indeed slack and therefore irrelevant.

Without (7), the Lagrangian function for problem (VC) can be expresses as:

$$\begin{aligned}
\mathcal{L} \;=\;& \alpha^L(1-p^L)U(W_G^1) + \alpha^L p^L U(W_G^2) + (1-\alpha^L)(1-p^L)U(W_B^1) + (1-\alpha^L)p^L U(W_B^2) \\
&+\gamma\left\{\lambda \times (R_N - p^H M_N) + (1-\lambda) \times \{\alpha^L(R_G - p^L M_G) + (1-\alpha^L)(R_B - p^L M_B)\}\right\} \\
&+\mu_H\left\{(1-p^H)U(W_N^1) + p^H U(W_N^2) - (1-\alpha^H)(1-p^H)U(W_G^1) - (1-\alpha^H)p^H U(W_G^2)\right. \\
&\qquad \left. -\alpha^H(1-p^H)U(W_B^1) - \alpha^H p^H U(W_B^2)\right\} \\
&+\delta\left\{(1-p^H)U(W_N^1) + p^H U(W_N^2) - \bar{V}^H\right\},
\end{aligned}$$

and the first order conditions for an interior solution to the optimization problem are

$$\frac{\partial \mathcal{L}}{\partial R_N} = \gamma\lambda - (\mu_H + \delta)\{(1-p^H)U'(W_N^1) + p^H U'(W_N^2)\} = 0, \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial M_N} = \gamma\lambda p^H - (\mu_H + \delta)\{p^H U'(W_N^2)\} = 0, \tag{17}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_G} \;=\;& \gamma(1-\lambda)\alpha^L + \{-\alpha^L(1-p^L) + \mu_H(1-\alpha^H)(1-p^H)\}U'(W_G^1) \\
&+\{-\alpha^L p^L + \mu_H(1-\alpha^H)p^H\}U'(W_G^2),
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_B} \;=\;& \gamma(1-\lambda)(1-\alpha^L) + \{-(1-\alpha^L)(1-p^L) + \mu_H\alpha^H(1-p^H)\}U'(W_B^1) \\
&+\{-(1-\alpha^L)p^L + \mu_H\alpha^H p^H\}U'(W_B^2),
\end{aligned} \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial M_G} = -\gamma(1-\lambda)\alpha^L p^L + \{\alpha^L p^L - \mu_H(1-\alpha^H)p^H\}U'(W_G^2), \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial M_B} = -\gamma(1-\lambda)(1-\alpha^L)p^L + \{(1-\alpha^L)p^L - \mu_H\alpha^H p^H\}U'(W_B^2). \tag{21}$$

Non-satiation implies that $\gamma > 0$ and condition (a) holds. Also, from (16) and (17), it is easy to verify that $U'(W_N^1) = U'(W_N^2)$, that is, H-types always obtain full insurance and condition (b)

$(M_N = D)$ holds.

i. $\mu_H > 0$

Equations (18) through (21) can be rewritten as:

$$\gamma(1-\lambda) = \{1 - p^L - \frac{\mu_H(1-\alpha^H)(1-p^H)}{\alpha^L}\}U'(W_G^1) + \{p^L - \frac{\mu_H(1-\alpha^H)p^H}{\alpha^L}\}U'(W_G^2), \quad (18')$$

$$\gamma(1-\lambda) = \{1 - p^L - \frac{\mu_H\alpha^H(1-p^H)}{1-\alpha^L}\}U'(W_B^1) + \{p^L - \frac{\mu_H\alpha^H p^H}{1-\alpha^L}\}U'(W_B^2), \quad (19')$$

$$\gamma(1-\lambda) = \{1 - \frac{\mu_H(1-\alpha^H)p^H}{\alpha^L p^L}\}U'(W_G^2), \quad (20')$$

$$\gamma(1-\lambda) = \{1 - \frac{\mu_H\alpha^H p^H}{(1-\alpha^L)p^L}\}U'(W_B^2). \quad (21')$$

From (18') and (20'), we can show that

$$\gamma(1-\lambda) = \{1 - \frac{\mu_H(1-\alpha^H)(1-p^H)}{\alpha^L(1-p^L)}\}U'(W_G^1). \quad (22)$$

Similarly, from (19') and (21'), we can show that

$$\gamma(1-\lambda) = \{1 - \frac{\mu_H\alpha^H(1-p^H)}{(1-\alpha^L)(1-p^L)}\}U'(W_B^1). \quad (23)$$

From (20'), (21'), (22), (23), and with $\gamma > 0$, we finally have

$$\begin{aligned} 0 \;<\; \gamma(1-\lambda) &= \{1 - \frac{\mu_H(1-\alpha^H)p^H}{\alpha^L p^L}\}U'(W_G^2) = \{1 - \frac{\mu_H\alpha^H p^H}{(1-\alpha^L)p^L}\}U'(W_B^2) \\ &= \{1 - \frac{\mu_H(1-\alpha^H)(1-p^H)}{\alpha^L(1-p^L)}\}U'(W_G^1) = \{1 - \frac{\mu_H\alpha^H(1-p^H)}{(1-\alpha^L)(1-p^L)}\}U'(W_B^1). \quad (24) \end{aligned}$$

If $\mu_H = 0$, we obtain from Equation (24) that $W_G^2 = W_B^2 = W_G^1 = W_B^1$, which further gives $R_G = R_B$, and $M_G = M_B = D$. In this case, the regime of voluntary classification reduces to the regime of no classification. With conditions (a) and (b), it can be shown that the only contracts that do not violate both I.C. constraints are when $R_N = R_G = R_B = (\lambda p^H + (1-\lambda)p^L) \times D$, under which both I.C. constraints are binding simultaneously. However, as already shown in the proof of Theorem 1 in Crocker and Snow (1986), in the regime of no classification, it is only possible that both I.C. constraints hold with equality when $\bar{V}^H = V(p^H, (\lambda p^H + (1-\lambda)p^L) \times D, D)$. In other words, when $\bar{V}^H < V(p^H, (\lambda p^H + (1-\lambda)p^L) \times D, D)$, we must have $\mu_H > 0$, which implies condition (c).

ii. *Partial and different contracts when choosing to be classified*

With $\alpha^L + \alpha^H > 1$, $p^H > p^L$, we have

$$\frac{1-\alpha^H}{\alpha^L} < 1 < \frac{\alpha^H}{1-\alpha^L},$$

and

$$\frac{p^H}{p^L} > 1 > \frac{1-p^H}{1-p^L}.$$

Now with $\mu_H > 0$, we obtain the following inequalities:

$$\frac{\mu_H(1-\alpha^H)(1-p^H)}{\alpha^L(1-p^L)} < \frac{\mu_H(1-\alpha^H)p^H}{\alpha^L p^L} < \frac{\mu_H \alpha^H p^H}{(1-\alpha^L)p^L},$$

and

$$\frac{\mu_H(1-\alpha^H)(1-p^H)}{\alpha^L(1-p^L)} < \frac{\mu_H \alpha^H(1-p^H)}{(1-\alpha^L)(1-p^L)} < \frac{\mu_H \alpha^H p^H}{(1-\alpha^L)p^L}.$$

Applying above inequalities into (24), for any concave utility function $U(\cdot)$ we obtain

$$W_G^1 > W_G^2 > W_B^2,$$

and

$$W_G^1 > W_B^1 > W_B^2,$$

which can be converted into conditions (d) through (g). In particular, when the L-types (voluntarily) choose to be classified, they would obtain partial insurance in both groups *G* and *B*. Furthermore, the coverage (both premium and net-indemnity) will be distinctly different in the two groups.

iii. $\mu_L = 0$

With $M_N = D$, it is trivial to show that $V(p^H, R_N, M_N) = V(p^L, R_N, M_N)\ (= U(W_N^1))$. Replacing $V(p^L, R_N, M_N)$ with $V(p^H, R_N, M_N)$ on the right hand side of (7) and using condition (c), we find that the constraint (7) is equivalent to

$$\alpha^L V(p^L, R_G, M_G) + (1-\alpha^L)V(p^L, R_B, M_B) \geq (1-\alpha^H)V(p^H, R_G, M_G) + \alpha^H V(p^H, R_B, M_B). \quad (25)$$

The left hand side of Inequality (25) subtracts its right hand side can be further rewritten as

$$\begin{aligned}&\{\alpha^L p^L - (1-\alpha^H)p^H\}U(W_G^2) + \{\alpha^L(1-p^L) - (1-\alpha^H)(1-p^H)\}U(W_G^1)\\ &\quad + \{(1-\alpha^L)p^L - \alpha^H p^H\}U(W_B^2) + \{(1-\alpha^L)(1-p^L) - \alpha^H(1-p^H)\}U(W_B^1).\end{aligned}$$

For convenience, we define $a \equiv \alpha^L p^L - (1-\alpha^H)p^H$, $b \equiv \alpha^L(1-p^L) - (1-\alpha^H)(1-p^H)$, $c \equiv (1-\alpha^L)p^L - \alpha^H p^H$, and $d \equiv (1-\alpha^L)(1-p^L) - \alpha^H(1-p^H)$. It immediately follows that $a+b+c+d=0$. With $\alpha^L + \alpha^H > 1$ and $p^H > p^L$, it is also easy to verify that $b > 0$, $c < 0$, $a+b > 0$, and $b+d > 0$. What we want to show is that $aU(W_G^2) + bU(W_G^1) + cU(W_B^2) + dU(W_B^1)$ is always positive, that is, (7) is always slack under a set of efficient contracts for the optimization problem (VC) solved without imposing it. We discuss based on following cases:

1. $a \geq 0$ **and** $d \geq 0$**:** In this case, we have

$$\begin{aligned} aU(W_G^2) + bU(W_G^1) + cU(W_B^2) + dU(W_B^1) &> (a+b+c+d)U(W_B^2)\\ &= 0.\end{aligned}$$

2. $a < 0$ **and** $d < 0$**:** In this case, we have

$$\begin{aligned} aU(W_G^2) + bU(W_G^1) + cU(W_B^2) + dU(W_B^1) &> (a+b+c+d)U(W_G^1) \\ &= 0. \end{aligned}$$

3. $a \geq 0$ **and** $d < 0$**:** In this case, we have

$$\begin{aligned} aU(W_G^2) + bU(W_G^1) + cU(W_B^2) + dU(W_B^1) &> (a+c)U(W_B^2) + (b+d)U(W_G^1) \\ &= (b+d)(U(W_G^1) - U(W_B^2)) \\ &> 0. \end{aligned}$$

4. $a < 0$ **and** $d \geq 0$**:** In this case, we have

$$\begin{aligned} aU(W_G^2) + bU(W_G^1) + cU(W_B^2) + dU(W_B^1) &> (a+b)U(W_G^1) + (c+d)U(W_B^2) \\ &= (a+b)(U(W_G^1) - U(W_B^2)) \\ &> 0. \end{aligned}$$

Therefore, we show that under the efficient contracts that satisfy conditions (a) through (g), the constraint (7) is always slack, which implies $\mu_L = 0$ and condition (h).

Proof of Corollary 1

i) $\alpha^H + \alpha^L = 1 \Rightarrow C_B = C_G$

When $\alpha^H + \alpha^L = 1$, from Equation (24) we obtain $W_G^2 = W_B^2$ and $W_G^1 = W_B^1$, which further gives $R_G = R_B$ and $M_G = M_B$. Therefore, when the test is not informative at all, a set of efficient contracts under the regime of voluntary classification reduces to the case of no-classification: The L-types will receive the same contract ($C_B = C_G$) regardless of the outcome of such non-informative test.

ii) $C_B = C_G \Rightarrow \alpha^H + \alpha^L = 1$

When $C_B = C_G$, we immediately have $W_G^2 = W_B^2$ and further $U'(W_G^2) = U'(W_B^2)$. This, together with Equations (20') and (21'), and with $\gamma > 0$ and $\mu_H > 0$ as shown in the proof of Theorem 1, imply that ${}^{(1-\alpha^H)p^H}/_{\alpha^L p^L} = {}^{\alpha^H p^H}/_{(1-\alpha^L)p^L} \Rightarrow \alpha^H + \alpha^L = 1$. Therefore, if the L-types receive the same contract from participating in a voluntary classification regardless of the test outcome, it must again be the case that the test itself is non-informative at all, that is, $\alpha^H + \alpha^L = 1$.

Proof of Lemma 1

Using conditional probabilities, we have

$$\begin{aligned} \lambda_G &= P(H|G) = \frac{P(H,G)}{P(G)} = \frac{P(H,G)}{P(H,G) + P(L,G)} \\ &= \frac{P(G|H) \times P(H)}{P(G|H) \times P(H) + P(G|L) \times P(L)} \end{aligned}$$

$$= \frac{(1-\alpha^H)\lambda}{(1-\alpha^H)\lambda + \alpha^L(1-\lambda)} \in (0,1),$$

and

$$\begin{aligned}
\lambda_B &= P(H|B) = \frac{P(H,B)}{P(B)} = \frac{P(H,B)}{P(H,B)+P(L,B)} \\
&= \frac{P(B|H)\times P(H)}{P(B|H)\times P(H) + P(B|L)\times P(L)} \\
&= \frac{\alpha^H\lambda}{\alpha^H\lambda + (1-\alpha^L)(1-\lambda)} \in (0,1).
\end{aligned}$$

Note that $\lambda_G P(G) + \lambda_B(1-P(G)) = P(H|G)P(G) + P(H|B)P(B) = P(H) = \lambda$. Therefore, condition $\lambda_G < \lambda < \lambda_B$ is equivalent to $\lambda_G < \lambda_B$. We further have

$$\begin{aligned}
\frac{\lambda_B}{\lambda_G} &= \frac{\alpha^H\lambda\left[(1-\alpha^H)\lambda + \alpha^L(1-\lambda)\right]}{(1-\alpha^H)\lambda\left[\alpha^H\lambda + (1-\alpha^L)(1-\lambda)\right]} \\
&= \frac{\alpha^H(1-\alpha^H)\lambda + \alpha^H\alpha^L(1-\lambda)}{\alpha^H(1-\alpha^H)\lambda + (1-\alpha^H)(1-\alpha^L)(1-\lambda)}.
\end{aligned}$$

With every term in the above expression being non-negative, ${}^{\lambda_B}/_{\lambda_G} > 1$ is equivalent to $\alpha^H\alpha^L(1-\lambda) > (1-\alpha^H)(1-\alpha^L)(1-\lambda) \Leftrightarrow \alpha^H\alpha^L > (1-\alpha^H)(1-\alpha^L)$, which can be further simplified to $\alpha^H + \alpha^L > 1$. Therefore, we show that the condition $\alpha^H + \alpha^L > 1$ is equivalent to $0 < \lambda_G < \lambda < \lambda_B < 1$.

Proof of Lemma 2

It is trivial to verify that the resource constraint (5) holds. Constraint (8) is binding since the threshold is chosen right at $\bar{V}^H = V(p^H, \hat{R}_N, \hat{M}_N)$. Finally, for (6), we have

$$\begin{aligned}
V(p^H, \hat{R}_N, \hat{M}_N) &= V(p^H, \alpha^H \times R^*_{HB} + (1-\alpha^H)\times R^*_{HG}, D) \\
&= U(W - \alpha^H \times R^*_{HB} - (1-\alpha^H)\times R^*_{HG}) \\
&\underset{\text{Jensen's ineq.}}{\geq} \alpha^H U(W - R^*_{HB}) + (1-\alpha^H)U(W - R^*_{HG}) \\
&= \alpha^H V(p^H, R^*_{HB}, M^*_{HB}) + (1-\alpha^H)V(p^H, R^*_{HG}, M^*_{HG}) \\
&= \alpha^H V(p^H, R^*_{LB}, M^*_{LB}) + (1-\alpha^H)V(p^H, R^*_{LG}, M^*_{LG}). \qquad (26)
\end{aligned}$$

The inequality is the direct application of Jensen's inequality on the concave utility function $U(\cdot)$. The second last equation is obtained since $M^*_{HB} = M^*_{HG} = D$. The last equation is obtained from conditions (bA) and (bB) in Theorem 2 of Crocker and Snow (1986), which shows that the I.C. conditions of H-types in both groups are binding.

Therefore, for any set of efficient contracts $\mathbb{C}^*_{CC} = \{C^*_{LG}, C^*_{HG}, C^*_{LB}, C^*_{HB}\}$ from the regime of compulsory classification, we verify that all constraints except for (7) in the regime of voluntary categorization are satisfied when the insurer offers $\hat{\mathbb{C}}_{VC} = \{\hat{C}_N, \hat{C}_G, \hat{C}_B\}$, where $\hat{C}_N = (\hat{R}_N, \hat{M}_N) = (\alpha^H \times R^*_{HB} + (1-\alpha^H)\times R^*_{HG}, D)$, $\hat{C}_G = C^*_{LG}$, and $\hat{C}_B = C^*_{LB}$.

Proof of Theorem 3

We first denote a set of efficient contracts under voluntary classification as $\mathbb{C}^*_{VC} = \{C^*_N, C^*_G, C^*_B\}$. From condition (h) of Theorem 1, we know that with $\mathbb{C}^*_{VC}$, the I.C. constraint of L-types (7) is never binding. Put differently, the set $\mathbb{C}^*_{VC}$ is also the solution to an optimization problem similar to (VC) but without constraint (7). We denote such optimization problem as (VC'). Lemma 2 shows that the set of contracts $\hat{\mathbb{C}}_{VC}$ is feasible for (VC').

i) When $R^*_{HG} \neq R^*_{HB}$

In this case, we have

$$
\begin{aligned}
V(p^H, R^*_N, M^*_N) &\underset{(8)}{\geq} \bar{V}_H \\
&= V(p^H, \hat{R}_N, \hat{M}_N) \\
&= V(p^H, \alpha^H \times R^*_{HB} + (1-\alpha^H) \times R^*_{HG}, D) \\
&= U(W - \alpha^H \times R^*_{HB} - (1-\alpha^H) \times R^*_{HG}) \\
&\underset{\text{Jensen's ineq.}}{>} \alpha^H U(W - R^*_{HB}) + (1-\alpha^H) U(W - R^*_{HG}) \\
&= EUT^H_{CC}.
\end{aligned}
$$

Therefore, for any efficient contracts C^*_{HG} and C^*_{HB} offered in the regime of compulsory classification, the H-types will strictly prefer efficient contracts $\mathbb{C}^*_{VC}$ offered in the regime of voluntary classification.

For the L-types, first of all they will be indifferent between choosing $\mathbb{C}^*_{CC}$ or $\hat{\mathbb{C}}_{VC}$ as they face identical contracts ($\hat{C}_G = C^*_{LG}$, $\hat{C}_B = C^*_{LB}$). We now move to compare $\mathbb{C}^*_{VC}$ and $\hat{\mathbb{C}}_{VC}$ as both are feasible under the (slightly relaxed) optimization problem (VC'). We note that with $R^*_{HG} \neq R^*_{HB}$, we obtain

$$
\begin{aligned}
V(p^H, \hat{R}_N, \hat{M}_N) &> \alpha^H U(W - R^*_{HB}) + (1-\alpha^H) U(W - R^*_{HG}) \\
&= \alpha^H V(p^H, R^*_{HB}, M^*_{HB}) + (1-\alpha^H) V(p^H, R^*_{HG}, M^*_{HG}) \\
&\underset{(10)\,\&(12)}{\geq} \alpha^H V(p^H, R^*_{LB}, M^*_{LB}) + (1-\alpha^H) V(p^H, R^*_{LG}, M^*_{LG}) \\
&= \alpha^H V(p^H, \hat{R}_B, \hat{M}_B) + (1-\alpha^H) V(p^H, \hat{R}_G, \hat{M}_G),
\end{aligned}
$$

suggesting that the set of contracts $\hat{\mathbb{C}}_{VC} = \{\hat{C}_N, \hat{C}_G, \hat{C}_B\}$ violates condition (c) in Theorem 1, and therefore cannot be a set of efficient contracts for problem (VC'). In other words, the welfare of L-types under the set of efficient contracts $\mathbb{C}^*_{VC}$ in the regime of voluntary classification will be further increased, compared with the one under the also admissible set of contracts $\hat{\mathbb{C}}_{VC}$. Therefore, for L-types, $\hat{\mathbb{C}}_{VC}$ is indifferent to $\mathbb{C}^*_{CC}$, and both will be strictly dominated by $\mathbb{C}^*_{VC}$.

To sum up, when $R^*_{HG} \neq R^*_{HB}$, we show that voluntary classification strictly dominates compulsory classification for both H- and L-types.

ii) When $R^*_{HG} = R^*_{HB}$

In this case, the H-types will still (weakly) prefer $\mathbb{C}^*_{VC}$ over $\mathbb{C}^*_{CC}$ from the IR constraint (8).

Furthermore, we have

$$V(p^H, R^*_{LG}, M^*_{LG}) \stackrel{(bA)}{=} V(p^H, R^*_{HG}, M^*_{HG} = D) = V(p^H, R^*_{HB}, M^*_{HB} = D) \stackrel{(bB)}{=} V(p^H, R^*_{LB}, M^*_{LB}),$$

where the first and last equation is the direct application of conditions (bA) and (bB) in Theorem 2 of Crocker and Snow (1986). Therefore, for the set $\hat{\mathbb{C}}_{VC} = \{\hat{C}_N, \hat{C}_G, \hat{C}_B\}$, we have

$$V(p^H, \hat{R}_G, \hat{M}_G) = V(p^H, \hat{R}_B, \hat{M}_B).$$

However, from conditions (f) and (g) in our Theorem 1, we know that for a set of efficient contracts $\mathbb{C}^*_{VC}$ under voluntary classification,

$$\begin{aligned} V(p^H, R^*_G, M^*_G) &= p^H U(W - D - R^*_G + M^*_G) + (1 - p^H) U(W - R^*_G) \\ &\stackrel{\text{(f)\&(g)}}{>} p^H U(W - D - R^*_B + M^*_B) + (1 - p^H) U(W - R^*_B) \\ &= V(p^H, R^*_B, M^*_B). \end{aligned}$$

Once again, this suggests that the set of contracts $\hat{\mathbb{C}}_{VC}$ violates necessary conditions in Theorem 1, and therefore is not efficient under voluntary classification. The expected utility for L-types under a set of efficient contracts $\mathbb{C}^*_{VC}$ in the regime of voluntary classification will be strictly higher than the one under $\hat{\mathbb{C}}_{VC}$, which is again the same to $\mathbb{C}^*_{CC}$.

To sum up, when $R^*_{HG} = R^*_{HB}$, we show that voluntary classification (weakly) dominates compulsory classification for H-types, and once again strictly dominates compulsory classification for L-types.